

## Spin in Particle Physics

Motivated by recent dramatic developments in the field, this book provides a thorough introduction to spin and its role in elementary particle physics. Starting with a simple pedagogical introduction to spin and its relativistic generalization, the author successfully avoids the obscurity and impenetrability of traditional treatments of the subject. The book surveys the main theoretical and experimental developments of recent years, as well as discussing exciting plans for the future. Emphasis is placed on the importance of spin-dependent measurements in testing QCD and the Standard Model.

This book will be of value to graduate students and researchers working in all areas of quantum physics and particularly in elementary particle and high energy physics. It is suitable as a supplementary text for graduate courses in theoretical and experimental particle physics. This title, first published in 2001, has been reissued as an Open Access publication on Cambridge Core.

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# SPIN IN PARTICLE PHYSICS



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comes from the SLD experiment at Stanford, where the use of a polarized electron beam turns out to be equivalent to gaining a factor of 25 in the statistics compared with the unpolarized situation. Or take the LEP collider at CERN. Even though there has never been a serious spin programme there, nonetheless the most precise determination of the beam energy comes from a measurement of the resonant depolarization of the beams. And spin measurements have played a key rôle in elucidating the structure of the weak interactions and in demonstrating the  $V - A$  form of the weak Lagrangian, and several exquisite and delicate experiments (e.g. the parity-violating optical rotation in bismuth and the longitudinal polarization asymmetry in electron–proton scattering) have had a profound effect upon our fundamental view of the electroweak interaction.

On the ‘destructive’ side witness the theory of  $J/\psi$  production in hadronic collisions. Measured cross-sections were long ago found to be more than an order of magnitude larger than the predictions of the colour-singlet QCD calculations. So colour-octet enhancement was introduced, thereby apparently providing a successful theory of  $J/\psi$  production. Now

$$\sin^2 \theta_{\text{eff}}^W = 0.23061 \pm 0.00047,$$

world’s most precise measurement of the Weinberg angle, Witness, for example, the parameters of the Standard Model. The verification of such a theory.

current theory, sometimes as a powerful tool in the confirmation and particle physics, acting sometimes as the harbringer of the demise of a Spin plays a dramatic rôle in the theatre of elementary the underlying theoretical structures very deeply.

Questions about the spin dependence of reactions therefore tend to probe tary particles. The spin of a particle is a quantum mechanical attribute. Spin is an essential and fascinating complication in the physics of elemen-

## Preface

it turns out from more refined measurements, wherein the state of polarization of the  $J/\Psi$  particles is determined, that there is a serious disagreement between theory and experiment.

On a longer time scale take the case of Regge pole theory. There, an entire and beautiful theoretical structure, highly successful on many fronts, was severely shaken in the face of an accumulating mass of spin-dependent data in contradiction with its predictions.

Spin, because it has no classical correspondence limit to aid our intuition, has tended to be regarded with trepidation and to be seen as surrounded by dangerous pitfalls epitomized by the Thomas precession, which is always mentioned, but rarely explained, in textbooks on quantum mechanics. Indeed there is an unconscious element of witchcraft in the oft found statement that a purely relativistic effect produces a 50% correction to the calculation of the  $L \cdot S$  coupling in a hydrogenic atom!

Our opening sentence was inspired by a much loved slogan of the 1960s that 'spin is an inessential complication, a view that lent some practical relief in wrestling with the analytic properties of scattering amplitudes and the Mandelstam representation; this was an approach that seemed to offer, for the first time, the possibility of significant results in strong interaction theory. But here too later developments demonstrated clearly that spin could not be ignored and that the high energy behaviour of Feynman diagrams is much influenced by the spin of the virtual particles. During the 1970s and early 1980s spin physics drifted into a relatively tranquil state of activity, from which it was rudely awakened in 1987 by the extraordinary results of the European Muon Collaboration's experiment, at CERN, on deep inelastic lepton-hadron scattering, using a longitudinally polarized lepton beam on a longitudinally polarized target. Interpreted in simple parton model terms the experiment implied, loosely speaking, that the sum of the spins carried by the quarks in a proton added up to only about one eighth of the proton's spin — a most counter-intuitive result.

The EMC publication became the most-cited experimental paper in the field for the following three years and catalysed an enormous theoretical effort to re-examine, at a more fundamental level, the whole theory of spin effects in deep inelastic scattering. Once again it was found that the explanation of spin-dependent phenomena poses a more profound challenge to a theory than the mere prediction of event rates. The theory of the spin-dependent structure function  $g_1(x)$  is much more subtle than expected in the simple parton model and is linked to a deep aspect of field theory, the axial anomaly. And the structure function  $g_2(x)$  turns out to have no explanation at all in the simple parton model and requires essential field-theoretic generalizations of the parton model.

The EMC experiment also stimulated massive experimental programmes at SLAC, CERN and DESY, which, in turn, have stimulated the major contemporary experiments, COMPASS at CERN, HERMES at HERA and RHIC, which has just come into operation at Brookhaven. The information gleaned from decades of unpolarized deep inelastic scattering experiments has played a seminal rôle in our understanding of the internal structure of hadrons and in the testing of certain aspects of quantum chromodynamics. The depth and breadth of this information owes much to the fact that unpolarized deep inelastic scattering can be studied using both charged lepton beams ( $e^\pm, \mu^\pm$ ) and neutral ones ( $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$ ), the latter requiring gigantic kilotonne targets. The polarized case, by comparison, suffers from the lack of neutrino data — one does not know how to polarize a battleship! But, most extraordinarily, it now appears that it may be possible to construct a neutrino factory, based upon a muon storage ring, that produces neutrino fluxes  $10^3$  or  $10^4$  times greater than ever before, thus making polarized targets feasible. With this, one can contemplate a new era of polarized deep inelastic scattering, with profound implications for our understanding of the internal spin structure of hadrons.

In purely hadronic physics, too, there are tantalizing questions regarding spin dependence. There exists a whole array of semi-inclusive experiments like  $p \uparrow p \rightarrow \pi X$ , with a transversely polarized proton beam or target, or  $pp \rightarrow \text{hyperon} + X$ , with an unpolarized initial state in which huge hyperon spin asymmetries or polarizations — at the 30%–40% level! — are observed. These experiments are very hard to explain within the framework of QCD. The asymmetries all vanish at the partonic level and one has to invoke soft, non-perturbative mechanisms. All such mechanisms predict that the asymmetries must die out as the momentum transfer increases, yet there is no sign in the present data of such a decrease.

In exclusive reactions like  $pp \rightarrow dp$  the disagreement between the data on the analysing power at large momentum transfer and the naive QCD asymptotic predictions is even more severe, but here at least there is an escape clause: the theory of exclusive reactions in QCD is horrendously difficult.

On the practical side, the technology of spin measurements has improved dramatically over the past few years. Improvements in polarized sources suggest that proton beams of almost 100% polarization, and with nearly the same intensity as present-day unpolarized beams, will eventually be available. Polarized-target construction is also improving. A highly successful polarized gas cell is in operation in the circulating electron beam at HERA. Experiments using a polarized gas-jet target in a circulating proton beam have been carried out. Polarized electrons and positrons in  $e^+e^-$  colliders are commonplace.



I am greatly indebted to a group of colleagues who share my belief in the excitement and importance of spin-dependent measurements in elementary particle physics and from whose advice and expertise I have often benefited: Xavier Artur, Mauro Anselmino, Daniel Boer, Elena Boglione, Claude Bourrelly, Gerry Bunce, Nigel Buttimore, Don Crabb,

### Acknowledgements

In the appendices we have gathered together a large number of useful results, e.g. on the representations of the rotation and Lorentz groups, on Dirac spinors and matrix elements and various representations of the  $\gamma$ -matrices, on the Feynman rules for QCD and on the linearly independent helicity amplitudes and spin-dependent observables for several reactions.

be taken to proceed with the project.  
 study with high efficiency. We can only hope that a positive decision will be beautifully complementary to RHIC in terms of the reactions it could would, with a fixed polarized nucleon target, offer an experimental set-up tirelessly new regime in polarized deep inelastic lepton-hadron scattering and beam at HERA would provide a marvellous facility to explore an en- Looking further ahead, the HERA-N project to polarize the proton

of polarized 250 GeV protons.  
 formerly undreamed-of regime of spin physics, with its colliding beams the RHIC collider at Brookhaven. RHIC will be unique, exploring a generation of experiments, COMPASS at CERN and RHIC-SPIN at we discuss some of the exciting physics that will be explored in the new E142, E143, E154 and E155 at SLAC, and HERMES at HERA. And physics experiments of the past few years, EMC and SMC at CERN, and challenging physics results that have emerged from the major spin- of the Standard Model of electroweak interactions. We survey the rich in testing QCD and in providing a highly refined probe of the structure (3) We wish to highlight the importance of spin-dependent measurements science, part art.

side of spin physics, a continuing endeavour which seems to be part some of the absolutely dramatic achievements on the experimental the help and advice of experimental colleagues, to present and explain (2) While admitting a lack of expertise in the matter, we have tried, with should make it of interest to both theorists and experimentalists.

arbitrary exclusive and inclusive reactions at a level that, we hope, upon the helicity formalism, leads to a unified general treatment for physics that strips it of its unnecessary mystery. Our approach, based (1) We hope to offer a simple pedagogical treatment of spin in relativistic

Our aim in this book is threefold.

$$A \cdot B = A_{\mu} B^{\mu} = g^{\mu\nu} A_{\mu} B_{\nu} = A_0 B_0 - \mathbf{A} \cdot \mathbf{B}.$$

Using the equation for the metric tensor, the scalar product of two 4-vectors  $A, B$  is defined as

$$E = \sqrt{\mathbf{p}^2 + m^2}.$$

where

$$p^{\mu} = (E, \mathbf{p}) = (E, p_x, p_y, p_z),$$

and the 4-momentum vector for a particle of mass  $m$  is

$$x^{\mu} = (t, \mathbf{x}) = (t, x, y, z),$$

Space-time points are denoted by the contravariant 4-vector  $x^{\mu}$  ( $\mu = 0, 1, 2, 3$ ), where

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The metric tensor is

*Relativistic Quantum Mechanics*.

Our notation generally follows that of Bjorken and Drell (1964), in *Relativistic Conventions*

we use the magnitude of the charge of the electron:  $e > 0$ .

Natural units  $\hbar = c = 1$  are used throughout. For the basic unit of charge

Units

### Notational conventions

not always legible manuscript.

Finally I wish to thank Pasquale Iannelli for his efficient typing of my apparatus.

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*Notational conventions*

*$\gamma$ -matrices*

The  $\gamma$  matrices for spin-1/2 particles satisfy

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

and we use a representation in which

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_j = \begin{pmatrix} 0 & -\sigma_j \\ \sigma_j & 0 \end{pmatrix}, \quad j = 1, 2, 3,$$

where  $\sigma_j$  are the usual Pauli matrices. We define

$$\gamma_5 = \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

In this representation one has, for the transpose of the  $\gamma$ -matrices,

$$\gamma_{jT} = \gamma_j \quad \text{for } j = 0, 2, 5,$$

but

$$\gamma_{jT} = -\gamma_j \quad \text{for } j = 1, 3,$$

For the hermitian conjugates one has

$$\gamma_0^\dagger = \gamma_0, \quad \gamma_5^\dagger = \gamma_5,$$

but

$$\gamma_{jT}^\dagger = -\gamma_j \quad \text{for } j = 1, 2, 3.$$

The combination

$$\sigma_{\mu\nu} \equiv \frac{i}{2} [\gamma_\mu, \gamma_\nu]$$

is often used.

The scalar product of the  $\gamma$  matrices and any 4-vector  $A$  is defined as

$$\not{A} \equiv \gamma_\mu A^\mu = \gamma_0 A_0 - \gamma_1 A_1 - \gamma_2 A_2 - \gamma_3 A_3.$$

For further details and properties of the  $\gamma$ -matrices see Appendix A of Bjorken and Drell (1964).

*Spinors and normalization*

The particle spinors  $u$  and the antiparticle spinors  $v$ , which satisfy the Dirac equations

$$\not{p} u = 0 \quad \not{p} v = 0$$

respectively, are related by

$$v = i\gamma_2 n^* \bar{u} \\ \bar{v} = -i\gamma_2 n^T u$$

where  $\bar{u} \equiv u^\dagger \gamma_0$ ; similarly  $\bar{v} \equiv v^\dagger \gamma_0$ .

$$T \equiv T(SU(3); \text{triplet}) = \frac{2}{3}n_f.$$

For  $SU(3)$  and the triplet (quark) representation one has  $\mathbf{t}^a = \lambda^a/2$  and

$$\delta^{ab}T(R) \equiv n_f T^r(\mathbf{t}^a \mathbf{t}^b).$$

If there are  $n_f$  multiplets of particles, each multiplet transforming according to some representation  $R$  under the gauge group, wherein the group generators are represented by matrix  $\mathbf{t}^a$ , then  $T(R)$  is defined by

$$C^r \equiv C^r[SU(3)] = 3.$$

and one writes

$$\delta^{ab}C^2(G) \equiv f^{acdf}bcd$$

- For a group  $G$  with structure constants  $f^{abc}$  one defines  $C^2(G)$  via
- The Gell-Mann  $SU(3)$  matrices are denoted by  $\lambda^a$  ( $a = 1, \dots, 8$ ).
- The Pauli matrices are written either as  $\sigma_j$  or  $\tau_j$  ( $j = 1, 2, 3$ ).
- colour gauge group QCD.
- $N$  specifies the gauge group  $SU(N)$ . Note that  $N = 3$  for the
- $n_f$  is the number of flavours.

In dealing with the electroweak interactions and QCD the following symbols often occur.

*Group symbols and matrices*

of flow of fermion number.

In fermion lines in Feynman diagrams the arrow indicates the direction if there is no danger of confusion. Often a field such as  $\psi^\mu(x)$  for the muon is simply written  $\mu(x)$  or just  $\mu$

*Fields*

massless. With our normalization the cross-section formula (B.1) of Appendix B in Bjorken and Drell (1964) holds for both mesons and fermions, massive or

*Cross-sections*

$$u\bar{u} = 2m, \quad \bar{v}v = -2m.$$

the above implies

the point being that this normalization can be used equally well for massive fermions and for neutrinos. For a massive fermion or antifermion

$$u^\dagger u = 2E, \quad v^\dagger v = 2E,$$

utilize

Note that our spinor normalization differs from Bjorken and Drell. We

*Notational conventions*

Subscripts referring to the laboratory frame ( $Lab$ ) Normally a subscript upper-case 'L' is used, e.g.  $p_L$ . However, sometimes the subscript 'Lab' is used, for further clarification.

$$J_n^{cm}(x) = \sum_{\text{colours } f} \bar{q}_f(x) \gamma_n q_f(x).$$

is but if the colour of the quark is labelled  $f$  ( $f = 1, 2, 3$ ) then what is implied

$$J_n^{cm}(x) = \bar{Q}_f \gamma_n q_f(x)$$

Colour sums in weak and electromagnetic currents Since the weak and electromagnetic interactions are 'colour-blind' the colour label on a quark field is almost never shown explicitly when dealing with electroweak interactions. In currents involving quark field operators a colour sum is always implied. For example, the electromagnetic current of a quark of flavour  $f$  and charge  $\bar{Q}_f$  (in units of  $e$ ) is written

$$C_f^T \equiv C_2(SU(3); \text{triplet}) = \frac{3}{4}.$$

For  $SU(3)$  and the triplet representation one has

$$\delta_{ij} C_2(R) \equiv t_a^i t_a^j.$$

For the above representation  $R$  one defines  $C_2(R)$  analogously to  $C_2(G)$  via

*Notational conventions*

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$$\frac{d^2\sigma}{d\Omega d\phi} = \frac{1}{4\pi} \left\{ 1 + A^{(A)} \cos\phi - A^{(B)} \cos\phi \right. \\
 + A^{xx} [\cos^2\phi \sin^2\theta + \sin^2\phi \cos^2\theta] + A^{xy} [\cos\phi \sin\theta \sin\phi \cos\theta + \sin\phi \cos\theta \cos\phi \sin\theta] \\
 - A^{yy} [\sin^2\phi \cos^2\theta + \cos^2\phi \sin^2\theta] + A^{zz} [\cos\phi \sin\theta \sin\phi \cos\theta + \sin\phi \cos\theta \cos\phi \sin\theta] \\
 \left. - A^{xz} \cos\phi \sin\theta \sin\phi \cos\theta + A^{zx} \cos\phi \sin\theta \sin\phi \cos\theta \right\}$$

Page 119: Eqn. (5.6.12) should read:

where, as in (2.2.6),  $\delta$  is the angle between  $\mathbf{p}$  and  $\mathbf{p}'$ , and  $\delta'$  is the angle between  $\mathbf{p}$  and  $\mathbf{p}'$ .

$$\cos\eta' = \frac{\sin\delta'}{\sin\theta \sin(\phi_\beta - \phi)}$$

$$\cos\eta = \frac{\sin\delta'}{\cos\theta \sin\theta' + \sin\theta \cos\theta' \cos(\phi_\beta - \phi)}$$

(2.2.8b)

and

$$\sin\eta = \frac{\sin\delta}{\sin\theta \sin(\phi_\beta - \phi)}$$

$$\cos\eta = \frac{\sin\delta}{\cos\theta \sin\theta - \sin\theta \cos\theta \cos(\phi_\beta - \phi)}$$

(2.2.8a)

where  $\mathbf{p}' = l^{-1}\mathbf{p} = (p', \theta', \phi')$ ,  $\theta^{wick}$  is given by (2.2.6), and  $\eta$  and  $\eta'$  are given by

$$|\mathbf{p}; \lambda\rangle_{S(\beta)} = e^{i\chi' \lambda' n'} p_{\lambda'} \chi' (\theta^{wick}) e^{-i\chi \lambda n} |\mathbf{p}; \lambda\rangle \quad (2.2.7)$$

Page 14: the line above Eqn. (1.2.27) should read: "... frame  $S_A$  obtained from  $S$  via ..."  
 Page 22: the equations (2.2.7) and (2.2.8) for the effect of a general Lorentz transformation are incorrect. The correct expressions are:

### Errata

Pages 173/174: for a particle on the closed orbit, the vector  $\mathbf{n}(\theta)$  should more correctly be called  $\mathbf{n}_0(\theta)$  to conform with current usage in the field. For a modern treatment of this topic, see G.H.Hoffstäter, *A modern view of high energy polarized proton beams* (Springer, to be published) and D.P.Barber, *Electron and proton spin polarization in storage rings—an introduction*, 15th Advanced ICFA Beam Dynamics Workshop: Quantum Aspects of Beam Dynamics, Monterey, California, January 1998, Ed. Pisin Chen. (World Scientific, 1999, p67)

Page 188, Eqn. (8.1.5): one factor of  $\alpha$  should be removed from the last

$$\frac{1}{2\pi} \frac{d\sigma}{d\phi} \left[ \cos \phi \sin \phi \left( \mathcal{P}_A^x \mathcal{P}_B^z \cos \phi \cos \phi \cos \phi \cos \phi \right) + \right. \\ \left. + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos \phi \sin \phi \cos \phi \sin \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos \phi \sin \phi \right] + \right. \\ \left. + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos \phi \sin \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos \phi \sin \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos \phi \sin \phi \right] \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] \\ + \mathcal{P}_A^z \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^z \mathcal{P}_B^x \left[ \cos^2 \phi \sin^2 \phi \right] + \mathcal{P}_A^x \mathcal{P}_B^z \left[ \cos^2 \phi \sin^2 \phi \right]$$

Page 121: Eqn. (5.6.20) should read:



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