Spin in Particle Physics

Motivated by recent dramatic developments in the field, this book provides a thorough introduction to spin and its role in elementary particle physics. Starting with a simple pedagogical introduction to spin and its relativistic generalization, the author successfully avoids the obscurity and impenetrability of traditional treatments of the subject. The book surveys the main theoretical and experimental developments of recent years, as well as discussing exciting plans for the future. Emphasis is placed on the importance of spin-dependent measurements in testing QCD and the Standard Model.

This book will be of value to graduate students and researchers working in all areas of quantum physics and particularly in elementary particle and high energy physics. It is suitable as a supplementary text for graduate courses in theoretical and experimental particle physics. This title, first published in 2001, has been reissued as an Open Access publication on Cambridge Core.

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Preface

Spin is an essential and fascinating complication in the physics of elementary particles. The spin of a particle is a quantum mechanical attribute. Questions about the spin dependence of reactions therefore tend to probe the underlying theoretical structures very deeply.

Spin plays a dramatic Jekyll and Hyde rôle in the theatre of elementary particle physics, acting sometimes as the harbinger of the demise of a current theory, sometimes as a powerful tool in the confirmation and verification of such a theory.

Witness, for example, the parameters of the Standard Model. The world's most precise measurement of the Weinberg angle,

$$\sin^2\theta_W^{\rm eff} = 0.23061 \pm 0.00047,$$

comes from the SLD experiment at Stanford, where the use of a polarized electron beam turns out to be equivalent to gaining a factor of 25 in the statistics compared with the unpolarized situation. Or take the LEP collider at CERN. Even though there has never been a serious spin programme there, nonetheless the most precise determination of the beam energy comes from a measurement of the resonant depolarization of the beams. And spin measurements have played a key rôle in elucidating the structure of the weak interactions and in demonstrating the $V-A$ form of the weak Lagrangian, and several exquisite and delicate experiments (e.g. the parity-violating optical rotation in bismuth and the longitudinal polarization asymmetry in electron–proton scattering) have had a profound effect upon our fundamental view of the electroweak interaction.

On the 'destructive' side witness the theory of J/Ψ production in hadronic collisions. Measured cross-sections were long ago found to be more than an order of magnitude larger than the predictions of the colour-singlet QCD calculations. So colour-octet enhancement was introduced, thereby apparently providing a successful theory of J/Ψ production. Now it turns out from more refined measurements, wherein the state of polarization of the J/Ψ particles is determined, that there is a serious disagreement between theory and experiment.

On a longer time scale take the case of Regge pole theory. There, an entire and beautiful theoretical structure, highly successful on many fronts, was severely shaken in the face of an accumulating mass of spin-dependent data in contradiction with its predictions.

Spin, because it has no classical correspondence limit to aid our intuition, has tended to be regarded with trepidation and to be seen as surrounded by dangerous pitfalls epitomized by the Thomas precession, which is always mentioned, but rarely explained, in textbooks on quantum mechanics. Indeed there is an unconscious element of witchcraft in the oft found statement that a purely relativistic effect produces a 50% correction to the calculation of the $\mathbf{L} \cdot \mathbf{S}$ coupling in a hydrogenic atom!

Our opening sentence was inspired by a much loved slogan of the 1960s that 'spin is an inessential complication', a view that lent some practical relief in wrestling with the analytic properties of scattering amplitudes and the Mandelstam representation; this was an approach that seemed to offer, for the first time, the possibility of significant results in strong interaction theory. But here too later developments demonstrated clearly that spin could not be ignored and that the high energy behaviour of Feynman diagrams is much influenced by the spin of the virtual particles.

During the 1970s and early 1980s spin physics drifted into a relatively tranquil state of activity, from which it was rudely awakened in 1987 by the extraordinary results of the European Muon Collaboration's experiment, at CERN, on deep inelastic lepton–hadron scattering, using a longitudinally polarized lepton beam on a longitudinally polarized target. Interpreted in simple parton model terms the experiment implied, loosely speaking, that the sum of the spins carried by the quarks in a proton added up to only about one eighth of the proton's spin — a most counter-intuitive result.

The EMC publication became the most-cited experimental paper in the field for the following three years and catalysed an enormous theoretical effort to re-examine, at a more fundamental level, the whole theory of spin effects in deep inelastic scattering. Once again it was found that the explanation of spin-dependent phenomena poses a more profound challenge to a theory than the mere prediction of event rates. The theory of the spin-dependent structure function $g_1(x)$ is much more subtle than expected in the simple parton model and is linked to a deep aspect of field theory, the axial anomaly. And the structure function $g_2(x)$ turns out to have no explanation at all in the simple parton model and requires essential field-theoretic generalizations of the parton model.

The EMC experiment also stimulated massive experimental programmes at SLAC, CERN and DESY, which, in turn, have stimulated the major contemporary experiments, COMPASS at CERN, HERMES at HERA and RHIC, which has just come into operation at Brookhaven.

The information gleaned from decades of unpolarized deep inelastic scattering experiments has played a seminal rôle in our understanding of the internal structure of hadrons and in the testing of certain aspects of quantum chromodynamics. The depth and breadth of this information owes much to the fact that unpolarized deep inelastic scattering can be studied using both charged lepton beams $(e^{\pm}, \mu^{\pm})$ and neutral ones $(\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu)$, the latter requiring gigantic kilotonne targets. The polarized case, by comparison, suffers from the lack of neutrino data – one does not know how to polarize a battleship! But, most extraordinary, it now appears that it may be possible to construct a neutrino factory, based upon a muon storage ring, that produces neutrino fluxes 10^3 or 10^4 times greater than ever before, thus making polarized targets feasible. With this, one can contemplate a new era of polarized deep inelastic scattering, with profound implications for our understanding of the internal spin structure of hadrons.

In purely hadronic physics, too, there are tantalizing questions regarding spin dependence. There exists a whole array of semi-inclusive experiments like $p^{\uparrow}p \to \pi X$, with a transversely polarized proton beam or target, or $pp \to \text{hyperon} + X$, with an unpolarized initial state in which huge hyperon spin asymmetries or polarizations – at the 30%–40% level! – are observed. These experiments are very hard to explain within the framework of QCD. The asymmetries all vanish at the partonic level and one has to invoke soft, non-perturbative mechanisms. All such mechanisms predict that the asymmetries must die out as the momentum transfer increases, yet there is no sign in the present data of such a decrease.

In exclusive reactions like $pp \to pp$ the disagreement between the data on the analysing power at large momentum transfer and the naive QCD asymptotic predictions is even more severe, but here at least there is an escape clause: the theory of exclusive reactions in QCD is horrendously difficult.

On the practical side, the technology of spin measurements has improved dramatically over the past few years. Improvements in polarized sources suggest that proton beams of almost 100% polarization, and with nearly the same intensity as present-day unpolarized beams, will eventually be available. Polarized-target construction is also improving. A highly successful polarized gas cell is in operation in the circulating electron beam at HERA. Experiments using a polarized gas-jet target in a circulating proton beam have been carried out. Polarized electrons and positrons in e^+e^- colliders are commonplace.

Our aim in this book is threefold.

(1) We hope to offer a simple pedagogical treatment of spin in relativistic physics that strips it of its unnecessary mystery. Our approach, based upon the helicity formalism, leads to a unified general treatment for arbitrary exclusive and inclusive reactions at a level that, we hope, should make it of interest to both theorists and experimentalists.
(2) While admitting a lack of expertise in the matter, we have tried, with the help and advice of experimental colleagues, to present and explain some of the absolutely dramatic achievements on the experimental side of spin physics, a continuing endeavour which seems to be part science, part art.
(3) We wish to highlight the importance of spin-dependent measurements in testing QCD and in providing a highly refined probe of the structure of the Standard Model of electroweak interactions. We survey the rich and challenging physics results that have emerged from the major spin-physics experiments of the past few years, EMC and SMC at CERN, E142, E143, E154 and E155 at SLAC, and HERMES at HERA. And we discuss some of the exciting physics that will be explored in the new generation of experiments, COMPASS at CERN and RHIC-SPIN at the RHIC collider at Brookhaven. RHIC will be unique, exploring a formerly undreamed-of regime of spin physics, with its colliding beams of polarized 250 GeV protons.

Looking further ahead, the HERA-$\vec{\mathrm{N}}$ project to polarize the proton beam at HERA would provide a marvellous facility to explore an entirely new regime in polarized deep inelastic lepton–hadron scattering and would, with a fixed polarized nucleon target, offer an experimental set-up beautifully complementary to RHIC in terms of the reactions it could study with high efficiency. We can only hope that a positive decision will be taken to proceed with the project.

In the appendices we have gathered together a large number of useful results, e.g. on the representations of the rotation and Lorentz groups, on Dirac spinors and matrix elements and various representations of the γ-matrices, on the Feynman rules for QCD and on the linearly independent helicity amplitudes and spin-dependent observables for several reactions.

Acknowledgements

I am greatly indebted to a group of colleagues who share my belief in the excitement and importance of spin-dependent measurements in elementary particle physics and from whose advice and expertise I have often benefited: Xavier Artru, Mauro Anselmino, Daniel Boer, Elena Boglione, Claude Bourrely, Gerry Bunce, Nigel Buttimore, Don Crabb, Tolya Efremov, Thomas Gehrmann, Sergey Goloskokov, Rainer Jakob, Alan Krisch, Peter Kroll, Yousef Makdisi, Gerhard Mallot, Brian Montague, Piet Mulders, Francesco Murgia, Aldo Penzo, Phil Ratcliffe, Oleg Selyugin, Jacques Soffer, Dimiter Stamenov, Oleg Teryaev, Larry Trueman and Werner Vogelsang. In particular the earlier chapters of this book owe much to work done in collaboration with Claude Bourrely and Jacques Soffer.

I am grateful to Elena Boglione for help with diagrams and numerical computations, and to Philip Burrows, Jim Clendenin, Michel Düren, Jim Johnson, Jean-Pierre Koutchouk, Wolfang Lorenzon, Livio Piemontese and Morris Swartz for information about and diagrams of experimental apparatus.

Finally I wish to thank Pasquale Iannelli for his efficient typing of my not always legible manuscript.

Notational conventions

Units

Natural units $\hbar = c = 1$ are used throughout. For the basic unit of charge we use the *magnitude* of the charge of the electron: $e > 0$.

Relativistic conventions

Our notation generally follows that of Bjorken and Drell (1964), in *Relativistic Quantum Mechanics.*

The metric tensor is

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Space–time points are denoted by the contravariant 4-vector x^μ ($\mu = 0, 1, 2, 3$), where

$$x^\mu = (t, \mathbf{x}) = (t, x, y, z),$$

and the 4-momentum vector for a particle of mass m is

$$p^\mu = (E, \mathbf{p}) = (E, p_x, p_y, p_z),$$

where

$$E = \sqrt{\mathbf{p}^2 + m^2}.$$

Using the equation for the metric tensor, the scalar product of two 4-vectors A, B is defined as

$$A \cdot B = A_\mu B^\mu = g_{\mu\nu} A^\mu B^\mu = A^0 B^0 - \mathbf{A} \cdot \mathbf{B}.$$

γ-matrices
The γ matrices for spin-1/2 particles satisfy

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

and we use a representation in which

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \gamma^j = \begin{pmatrix} 0 & \sigma_j \\ -\sigma_j & 0 \end{pmatrix}, \quad j = 1, 2, 3,$$

where σ_j are the usual Pauli matrices. We define

$$\gamma^5 = \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

In this representation one has, for the transpose of the γ-matrices,

$$\gamma^{jT} = \gamma^j \qquad \text{for } j = 0, 2, 5,$$

but

$$\gamma^{jT} = -\gamma^j \qquad \text{for } j = 1, 3.$$

For the hermitian conjugates one has

$$\gamma^{0\dagger} = \gamma^0, \qquad \gamma^{5\dagger} = \gamma^5,$$

but

$$\gamma^{j\dagger} = -\gamma^j \qquad \text{for } j = 1, 2, 3.$$

The combination

$$\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

is often used.

The scalar product of the γ matrices and any 4-vector A is defined as

$$\not{A} \equiv \gamma^\mu A_\mu = \gamma^0 A^0 - \gamma^1 A^1 - \gamma^2 A^2 - \gamma^3 A^3.$$

For further details and properties of the γ-matrices see Appendix A of Bjorken and Drell (1964).

Spinors and normalization
The particle spinors u and the antiparticle spinors v, which satisfy the Dirac equations

$$(\not{p} - m)u(p) = 0$$
$$(\not{p} + m)v(p) = 0$$

respectively, are related by

$$v = i\gamma^2 u^*$$
$$\bar{v} = -iu^T\gamma^0\gamma^2$$

where $\bar{v} \equiv v^\dagger\gamma^0$; similarly $\bar{u} \equiv u^\dagger\gamma^0$.

Note that our spinor normalization differs from Bjorken and Drell. We utilize

$$u^\dagger u = 2E, \qquad v^\dagger v = 2E,$$

the point being that this normalization can be used equally well for massive fermions and for neutrinos. For a massive fermion or antifermion the above implies

$$\bar{u}u = 2m, \qquad \bar{v}v = -2m.$$

Cross-sections
With our normalization the cross-section formula (B.1) of Appendix B in Bjorken and Drell (1964) holds for both mesons and fermions, massive or massless.

Fields
Often a field such as $\psi_\mu(x)$ for the muon is simply written $\mu(x)$ or just μ if there is no danger of confusion.

In fermion lines in Feynman diagrams the arrow indicates the direction of flow of *fermion number.*

Group symbols and matrices
In dealing with the electroweak interactions and QCD the following symbols often occur.

- n_f is the number of flavours.
- N specifies the gauge group $SU(N)$. Note that $N = 3$ for the colour gauge group QCD.
- The Pauli matrices are written either as σ_j or τ_j $(j = 1, 2, 3)$.
- The Gell-Mann $SU(3)$ matrices are denoted by λ^a $(a = 1, \ldots, 8)$.
- For a group G with structure constants f_{abc} one defines $C_2(G)$ via

  $$\delta_{ab} C_2(G) \equiv f_{acd} f_{bcd}$$

  and one writes

  $$C_A \equiv C_2[SU(3)] = 3.$$

If there are n_f multiplets of particles, each multiplet transforming according to some representation R under the gauge group, wherein the group generators are represented by matrix $\mathbf{t}^a$, then $T(R)$ is defined by

$$\delta_{ab} T(R) \equiv n_f \; Tr \; (\mathbf{t}^a \mathbf{t}^b).$$

For $SU(3)$ and the triplet (quark) representation one has $\mathbf{t}^a = \lambda^a/2$ and

$$T \equiv T(SU(3);\ \text{triplet}) = \tfrac{1}{2} n_f.$$

For the above representation R one defines $C_2(R)$ analogously to $C_2(G)$ via

$$\delta_{ij} C_2(R) \equiv t^a_{ik} t^a_{kj}.$$

For $SU(3)$ and the triplet representation one has

$$C_F \equiv C_2(SU(3); \text{triplet}) = \tfrac{4}{3}.$$

Colour sums in weak and electromagnetic currents
Since the weak and electromagnetic interactions are 'colour-blind' the colour label on a quark field is almost never shown explicitly when dealing with electroweak interactions. In currents involving quark field operators a *colour sum is always implied*. For example, the electromagnetic current of a quark of flavour f and charge Q_f (in units of e) is written

$$J^{\mu}_{\text{em}}(x) = Q_f \bar{q}_f(x) \gamma^{\mu} q_f(x)$$

but if the colour of the quark is labelled j ($j = 1, 2, 3$) then what is implied is

$$J^{\mu}_{\text{em}}(x) = Q_f \sum_{\substack{\text{colours}\\ j}} \bar{q}_{f_j}(x) \gamma^{\mu} q_{f_j}(x).$$

Subscripts referring to the laboratory frame (Lab)
Normally a subscript upper-case 'L' is used, e.g. p_{L}. However, sometimes the subscript 'Lab' is used, for further clarification.

Errata

Page 14: the line above Eqn. (1.2.27) should read:- " .... frame S_A obtained from S via ..."

Page 22: the equations (2.2.7) and (2.2.8) for the effect of a general Lorentz transformation are incorrect. The correct expressions are:

$$|\boldsymbol{p}\,;\lambda\rangle_{S^{l(\beta)}} = e^{i\lambda'\eta'}\, d_{\lambda'\lambda}(\theta_{Wick})\, e^{-i\lambda\eta}\,|\boldsymbol{p'};\,\lambda'\rangle \qquad (2.2.7)$$

where $\boldsymbol{p'} = l^{-1}\boldsymbol{p} = (p', \theta', \varphi')$, θ_{Wick} is given by (2.2.6), and η and η' are given by

$$\begin{aligned} \cos\eta &= \frac{\cos\theta_\beta \sin\theta - \sin\theta_\beta \cos\theta \cos(\varphi_\beta - \varphi)}{\sin\delta} \\ \sin\eta &= \frac{\sin\theta_\beta \sin(\varphi_\beta - \varphi)}{\sin\delta} \qquad (2.2.8a) \end{aligned}$$

and

$$\begin{aligned} \cos\eta' &= \frac{\cos\theta_\beta \sin\theta' + \sin\theta_\beta \cos\theta' \cos(\varphi_\beta - \varphi')}{\sin\delta'} \\ \sin\eta' &= \frac{\sin\theta_\beta \sin(\varphi_\beta - \varphi')}{\sin\delta'} \qquad (2.2.8b) \end{aligned}$$

where, as in (2.2.6), δ is the angle between $\boldsymbol{\beta}$ and $\boldsymbol{p}$, and δ' is the angle between $\boldsymbol{\beta}$ and $\boldsymbol{p'}$.

Page 119: Eqn. (5.6.12) should read:

$$\begin{aligned} \frac{d^2\sigma}{dt\,d\phi} = \frac{1}{2\pi}\frac{d\sigma}{dt}\{ & 1 + A^{(A)}(\mathscr{P}^A_y \cos\phi - \mathscr{P}^A_x \sin\phi) \\ & -A^{(B)}(\mathscr{P}^B_y \cos\phi - \mathscr{P}^B_x \sin\phi) \\ & +A_{xx}[\cos^2\phi\, \mathscr{P}^A_x \mathscr{P}^B_x + \sin^2\phi\, \mathscr{P}^A_y \mathscr{P}^B_y \\ & +\cos\phi \sin\phi\, (\mathscr{P}^A_x \mathscr{P}^B_y + \mathscr{P}^A_y \mathscr{P}^B_x)] \\ & -A_{yy}[\sin^2\phi\, \mathscr{P}^A_x \mathscr{P}^B_x + \cos^2\phi\, \mathscr{P}^A_y \mathscr{P}^B_y \\ & -\cos\phi \sin\phi\, (\mathscr{P}^A_x \mathscr{P}^B_y + \mathscr{P}^A_y \mathscr{P}^B_x)] \\ & -A_{zz}\, \mathscr{P}^A_z \mathscr{P}^B_z + A_{zx}\, \mathscr{P}^A_z(\cos\phi\, \mathscr{P}^B_x + \sin\phi\, \mathscr{P}^B_y) \\ & -A_{xz}\, \mathscr{P}^B_z(\cos\phi\, \mathscr{P}^A_x + \sin\phi\, \mathscr{P}^A_y)\} \end{aligned}$$

Page 121: Eqn. (5.6.20) should read:

$$
\begin{aligned}
t^{lL}_{mM}(C,D)\,\frac{d^2\sigma}{dt d\phi} \quad = \quad & \frac{1}{2\pi}\left[t^{lL}_{mM}(C,D)\frac{d\sigma}{dt}\right]_{unpol.}\\
&+\frac{1}{2\pi}\frac{d\sigma}{dt}\big\{\mathscr{P}^A_x[\cos\phi\,(X0|f)-\sin\phi\,(Y0|f)]\\
&+\tilde{\mathscr{P}}^B_x\,[\cos\phi\,(0X|f)+\sin\phi\,(0Y|f)]\\
&+\mathscr{P}^A_y\,[\cos\phi\,(Y0|f)+\sin\phi\,(X0|f)]\\
&+\tilde{\mathscr{P}}^B_y\,[\cos\phi\,(0Y|f)-\sin\phi\,(0X|f)]\\
&+\mathscr{P}^A_z\,(Z0|f)+\tilde{\mathscr{P}}^B_z\,(0Z|f)+\mathscr{P}^A_z\tilde{\mathscr{P}}^B_z\,(ZZ|f)\\
&+\mathscr{P}^A_x\tilde{\mathscr{P}}^B_x\,\big[\cos^2\phi\,(XX|f)-\sin^2\phi\,(YY|f)\\
&+\cos\phi\,\sin\phi\,\langle(XY|f)-(YX|f)\rangle\big]\\
&+\mathscr{P}^A_y\tilde{\mathscr{P}}^B_y\,\big[\cos^2\phi\,(YY|f)-\sin^2\phi\,(XX|f)\\
&+\cos\phi\,\sin\phi\,\langle(XY|f)-(YX|f)\rangle\big]\\
&\mathscr{P}^A_x\tilde{\mathscr{P}}^B_y\,\big[\cos^2\phi\,(XY|f)+\sin^2\phi\,(YX|f)\\
&-\cos\phi\,\sin\phi\,\langle(XX|f)+(YY|f)\rangle\big]\\
&\mathscr{P}^A_y\tilde{\mathscr{P}}^B_x\,\big[\cos^2\phi\,(YX|f)+\sin^2\phi\,(XY|f)\\
&+\cos\phi\,\sin\phi\,\langle(XX|f)+(YY|f)\rangle\big]\\
&+\mathscr{P}^A_x\tilde{\mathscr{P}}^B_z\,[\cos\phi\,(XZ|f)-\sin\phi\,(YZ|f)]\\
&+\mathscr{P}^A_z\tilde{\mathscr{P}}^B_x\,[\cos\phi\,(ZX|f)+\sin\phi\,(ZY|f)]\\
&+\mathscr{P}^A_y\tilde{\mathscr{P}}^B_z\,[\cos\phi\,(YZ|f)+\sin\phi\,(XZ|f)]\\
&+\mathscr{P}^A_z\tilde{\mathscr{P}}^B_y\,[\cos\phi\,(ZY|f)-\sin\phi\,(ZX|f)]\big\}
\end{aligned}
$$

Pages 173/174: for a particle on the closed orbit, the vector $\boldsymbol{n}(\theta)$ should more correctly be called $\boldsymbol{n}_0(\theta)$ to conform with current usage in the field. For a modern treatment of this topic, see G.H.Hoffstätter, *A modern view of high energy polarized proton beams* (Springer; to be published) and D.P.Barber, *Electron and proton spin polarization in storage rings—an introduction* , 15th Advanced ICFA Beam Dynamics Workshop: Quantum Aspects of Beam Dynamics, Monterey, California, January 1998, Ed. Pisin Chen. (World Scientific, 1999, p67)

Page 188, Eqn. (8.1.5): one factor of α should be removed from the last