

## FOUNDATIONS OF PERTURBATIVE QCD

The most non-trivial of the established microscopic theories of physics is QCD: the theory of the strong interaction. A critical link between theory and experiment is provided by the methods of perturbative QCD, notably the well-known factorization theorems. Giving an accurate account of the concepts, theorems, and their justification, this book is a systematic treatment of perturbative QCD.

As well as giving a mathematical treatment, the book relates the concepts to experimental data, giving strong motivations for the methods. It also examines in detail transverse-momentum-dependent parton densities, an increasingly important subject not normally treated in other books. Ideal for graduate students starting their work in high-energy physics, it will also interest experienced researchers wanting a clear account of the subject. This title, first published in 2011, has been reissued as an Open Access publication on Cambridge Core.

JOHN COLLINS is Distinguished Professor of Physics at Penn State University. He has long experience in perturbative QCD. He has proved a number of the fundamental theorems that form the main content of this book, and has a record of formulating and deriving novel results in QCD. During his career he has received several awards, including a Guggenheim fellowship, a Humboldt Research Award, a Mercator professorship, and the JJ Sakurai prize.

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To Mary, Dave, and George

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