1
Introduction

This book stems from lectures in different places and at different times. I would like to thank all those colleagues, graduate students and collaborators, who have patiently listened, commented upon and by insistent questioning given me insight into the physics described in this text.

You will find that the physics is described in a semi-classical language. I believe that my generation, the grandchildren of the wonderful generation that developed the tools of quantum mechanics, have largely learned to use semi-classical dynamical pictures while avoiding the quantum mechanical pitfalls. After having understood that the state density is different and that probabilities are not additive in quantum mechanics most of one's classical intuition can be used. I provide an example in Chapter 2 which shows that you can never fool Heisenberg's indeterminacy relations (i.e. position and conjugate momentum cannot be determined simultaneously with arbitrary precision). But you may choose your variables in such a way (rapidity and position for high-energy particles) that all the quantum mechanical rules are fulfilled and you may still transfer easily between the descriptions in terms of the different variable sets.

The material in the book has been chosen to stress the connections between different approaches to high-energy physics. The basic picture is nevertheless the one stemming from field theory as it is used in the Lund model. The Lund model has been successful in describing many of the dynamical features of multiparticle production because it contains so many relations to earlier and contemporary work, although often with very different dynamical starting points. I am very sorry that due to space limitations I have had to exclude many interesting and still-viable theoretical approaches to the physics of high-energy multiparticle production from this book.

It may at this point be useful to try to clarify what I mean by the Lund model in this book. There is some confusion because during the years many of the original contributors (and also people never working with the Lund Group) have provided a lot of material described as 'in accordance with the Lund model'. After chapters on *relativistic kinematics, field theory, renormalisation* and the *parton model*, all introduced to provide the notation as well as some useful formulas, I will consider *the Lund fragmentation model of quarks and gluons.*

This part of the Lund model (which was the first part produced and which, owing to lucky coincidences has not been changed very much over the years) makes use of the massless relativistic string as a model for the QCD color force fields. It provides a description of the transition from the partonic entities to the final-state observables in terms of the hadronic states. The model is described in detail in Chapters 6-15 and is implemented in the well-known Monte Carlo simulation program JETSET. The major achievements are

1 A consistent space-time and energy-momentum-space description leading to a unique (Markov) stochastic process for the breakup of the (string) field into hadrons. The process is described on the $(1+1)$-dimensional surface spanned by the string field during its periodic motion (and it is determined uniquely from the partons).

2 A highly nontrivial description of the partons, with the quarks (q-particles) and antiquarks ($\bar{q}$-particles) as endpoint excitations and the gluons (g) as internal excitations on the string field.

3 The breakup of the fields into 'new' $q\bar{q}$-pairs stems from a quantum mechanical tunnelling process. Although all the formulas of the model are derived in a semi-classical framework the final results can be interpreted within a consistent quantum mechanical scenario (and actually also within statistical mechanics, thereby providing the so-called Feynman-Wilson gas analogy).

4 It is possible within the model to account for the strong (transverse) polarisation effects observed and to describe more subtle quantum mechanical interference effects such as Bose-Einstein correlations.

There is secondly the *Lund dipole cascade model* (the DCM), which contains a description of the multiparton bremsstrahlung emissions in perturbative QCD, thereby providing the states for which the Lund fragmentation model may be applied. This is described in Chapters 16-18 and it is implemented in the ARIADNE Monte Carlo simulation program. A different approach, the *method of independent parton cascades*, has been implemented in the JETSET and, according to the Webber-Marchesini model, cf. Chapter 17, in the HERWIG Monte Carlo simulation programs.

There is finally (and this is a very recent advance) the *linked dipole chain model*, providing a description of the states occurring in deep inelastic scattering (DIS) events. I start with Chapter 19 on the 'ordinary' approach to DIS using the (double) leading-logarithm approximation as well as the results of approximating the matrix elements by the (major) lightcone singularities. The main problem is to describe the hadron structure functions, i.e. the partonic flux factors, stemming from the hadronic wave function, in accordance with perturbative QCD. The well-known Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations are derived and also the considerations behind the Balitsky-Fadin-Kuraev-Lipatov (BFKL) mechanism. Finally I have included a section on the recently developed Ciafaloni-Catani-Marchesini-Fiorani (CCMF) model, which contains a very ambitious effort to re-sum the large-order contributions to the perturbative QCD diagrams.

The linked dipole chain (LDC) model, described in Chapter 20, is a generalisation and simplification of the results of the CCFM model and just as for CCFM it interpolates between the DGLAP and BFKL results for the structure functions. It provides a general framework to describe all kinds of deep inelastic scattering events (besides the 'ordinary' parton-probe events that occur in accordance with perturbative QCD and the Feynman parton model there are boson-gluon fusion events, which contribute a large part of the present HERA cross section, and Rutherford scattering between the resolved probe structure and the hadron structure).

In this way the Lund model contains one common general feature at every level of the description of QCD, i.e. the occurrence of dipoles:

- An excitation in the vacuum, e.g. from an e^+e^- annihilation event, produces a color $q\bar{q}$-dipole, which decays via gluon bremsstrahlung according to the dipole cascade model into a set of color dipoles, spanned between the partons. This is known as a 'timelike' cascade because the original large excitation mass decays into smaller and smaller dipole masses. The dipoles move apart thereby producing a force field similar to the modes of the massless relativistic string.

- Afterwards the string field breaks up into hadrons, 'the ultimate dipoles', produced in the Lund fragmentation model from a quark and antiquark from adjacent breakup vertices together with the field in between.

- When such a hadron is probed the states can again be described as a set of dipoles, according to the linked dipole chain model, spanned between the color-adjacent gluons emitted in the ensuing bremsstrahlung. This is known as a 'spacelike' cascade because it corresponds to probing the hadron wave function up towards larger and larger 'virtualities', i.e. more and more spacelike momentum transfers, $-q^2 = Q^2$ (smaller wavelengths $\lambda \sim 1/Q$). The interaction with the probe brings the whole chain on-shell and then the dipoles again decay via the dipole cascade model to smaller dipoles and finally into hadrons via the Lund fragmentation model.

At this point I would like to make two remarks. Firstly there is a duality between descriptions of perturbative QCD in terms of dipoles and in terms of gluonic excitations. The gluons correspond to pointlike excitations in the color field while the dipoles are the (field) 'links' between these points. In other words the color from one dipole meets the anticolor from the adjacent one at a gluon 'corner' (note that the color-8 gluons can be considered as a combination of $3\bar{3}$ color charges).

My second remark is that the only solvable confined field theory we know of, $(1+1)$-dimensional QED (the Schwinger model described in Chapter 6) is just a theory of dipoles. The Lagrangian of the original fermion–antifermion field interacting with the connecting electric field can be transformed into the Lagrangian of a free field, corresponding to a dipole density of massive quanta composed of such a pair and the adjoining field. It should be stressed, however, that it is not known whether confinement implies a dipole picture of the charges and the fields.

Hadronic interactions *per se* have been investigated during a longer timespan than any other parts of multiparticle dynamics, but we are still very far from a consistent and useful description. I have at different places introduced some features, e.g. the S-matrix and unitarity, which are so general that they must be part of any future theory. But I have owing to space limitations decided to exclude all specific models, although some of them, like Gribov's Reggeon theory, have beauty and generality sufficient to redeem even a partial study.

I have also generally avoided to include experimental material. It should be stressed that no phenomenological work is alive without the necessary experimental checks on the approach. There have been, however, a large number of investigations, reviews and comparisons with experimental data in all the conference proceedings of the last decade. They are all in agreement with the general approach of the book. I will as a further excuse make use of the following sentence, which occurs in many places and must have been invented for just this situation: 'New experimental material is also coming in at such a rapid rate that the book would date unnecessarily quickly by including only the presently available data'. I admire my experimental colleagues for the fact that it is a true statement!

But we should always keep in mind what Bacon has pointed out (this is a free translation of the credo of phenomenology): 'You have not learned anything by being in agreement with data, because there are always other possible explanations. But if you put forward an idea, calculate inside the framework in an honest way and find disagreements with experiments *then you have learned something*, i.e. that this approach is not taken by Nature'. Or as one of my friends enthusiastically said during a heated conference discussion: *We must dare to be wrong!*

I have used the units conventional in today's high-energy physics putting the velocity of light c and Planck's constant $\hbar$ equal to unity thereby making energy dimensions inverse to length dimensions. In that connection it is useful to remember that a transfer between energy and length units is with this convention provided by the rather precise approximation $1\,\text{fm} \times 1\,\text{GeV} \simeq 5$.

In order to keep the reference list reasonably short I have taken the liberty of omitting references to phenomena like the parton model, Wick's theorem, the Ward identity etc., which nowadays are all part of our common physics heritage. I may have overdone it and if so I apologise to the authors. I would like to mention that material included in the books

J.D. Jackson, *Classical Electrodynamics*, John Wiley & Sons
H. Goldstein, *Classical Mechanics*, Addison-Wesley
E. Merzbacher, *Quantum Mechanics*, John Wiley & Sons

is referred to by these authors' name only. There is evidently a set of equally useful basic text-books where you can find the same material, but it is impossible to be exhaustive. When it comes to quantum field theory the subject has still not matured to the extent of these text-books. A rather formal description (containing, however, many useful references) is given by C. Itzykson and J.B. Zuber, *Quantum Field Theory*, McGraw-Hill, 1980. For perturbative QCD there is a recent book, Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, Basics of Perturbative QCD, Editions Frontières, 1991, which is very good. An early reference to the Lund model (as of 1982) is *Phys. Rep.* **97** 31, 1983.

2

Relativistic kinematics, electromagnetic fields and the method of virtual quanta

The dynamics of the massless relativistic string (which we will meet at very many different places in this book) is a delightful theoretical laboratory to study the properties of the theory of special relativity. To make the book self-contained and also to define our notation we will briefly review in this chapter some properties of special relativity, in particular with respect to its implications for high-energy particle kinematics.

We will also review some properties of electromagnetic fields with particular emphasis on the features we are going to make use of later in the book. We will end with a description of the *interaction ability of an electrically charged particle.*

This is the first but not the last example in this book of the *law of the conservation of useful dynamics.* This says that every new generation of theoretical physicists tends to reinvent, reuse (and usually also rename) the most useful results of earlier generations. One reason is evidently that there are few situations where it is possible to find a closed mathematical expression for the solution to a dynamical problem.

Here our basic aim is to describe the interactions between charged particles which are moving with very large velocities (as they do in high-energy physics). As a charged particle interacts via its field the question can be reformulated into finding a way to describe the field of a charged particle which is moving very fast. To account for quantum mechanics we need a way to describe the quantum properties of the charged particle's field and this problem can be solved even at a semi-classical level. It is possible to obtain a closed formula for the *flux of the field quanta* in this case.

Fermi addressed the problem in the 1920s, Weizsäcker and Williams found the method independently of Fermi and each other in the 1930s. After that it became a standard tool in connection with QED in terms of the *method of virtual quanta*, the MVQ. Later again Feynman made use of it in order to introduce the *parton model*. We will discuss that model repeatedly in this book, but it is useful to see how 'partons' emerge even at the semi-classical level in electromagnetism.

2.1 The Lorentz boost

Michelson and Morley demonstrated that the velocity of light, c, is independent of the direction of a light beam. Einstein interpreted this finding to imply that the velocity c is independent of the relative motion of the light source and the detector.

We are not going to dwell upon the many basic questions that are raised by this interpretation but simply accept that it has profound implications with respect to measurements of events in space and time. The resulting predictions have been tested repeatedly and always been found to be true. In this section we will briefly consider some of these predictions.

I *The Lorentz boost*. Consider two observers A and B, moving with respect to each other. We will suppose that they have calibrated their watches and decided upon a common origin in space and time as well as the directions of the coordinate axes in space. The arrangement will be that they move along their common x-axis so that B has the velocity v with respect to A. We will for simplicity use units such that the velocity of light $c = 1$. Then an event (1) which for A occurs at the space-time coordinates

$$(1) \equiv (t_{1A}, x_{1A}, y_{1A}, z_{1A}) \tag{2.1}$$

will for B, in his system, seem to occur at the time and space coordinates (with the corresponding index B):

$$\begin{aligned} t_{1B} &= \gamma(v)(t_{1A} - vx_{1A}) \\ x_{1B} &= \gamma(v)(x_{1A} - vt_{1A}) \\ y_{1B} &= y_{1A} \\ z_{1B} &= z_{1A} \end{aligned} \tag{2.2}$$

This transformation is termed a *boost along the x-axis* and $\gamma(v) = 1/\sqrt{1-v^2}$. The time- and the (*longitudinal*) x-coordinates get mixed by the transformation but the *transverse* coordinates, i.e. the y- and z-coordinates, are unaffected. Several boosts may be performed one after the other. It is easy to see that the final result does not depend upon the order and therefore the boosts along a single direction constitute a commutative (abelian) group.

More complex transformations also include rotations of the coordinate systems. Note that such rotations in general do not commute with each other or with the boost transformations. This means that the outcome of the total transformation depends upon the order in which each one of the rotations and boosts is done.

II *The proper time*. The coordinate and time values are all differences between the commonly agreed origin and the space-time point at which event (1) occurs. They are all *relative coordinates*. A and B will have different values for their measured t, x values for the event but there is one combination which they will agree upon,

$$t_{1A}^2 - x_{1A}^2 = t_{1B}^2 - x_{1B}^2 \equiv \tau_1^2 \tag{2.3}$$

The proper time of the event, τ_1, is evidently an *invariant* with respect to all boosts along the x-axis. This means that it does not contain any reference to the relative velocity of the observers along the x-axis.

The proper time is the value a watch would show if it started out from the origin (i.e. at $t = 0, x = 0$) in A's system and moved away with velocity $v_A = x_{1A}/t_{1A}$. Then it will arrive at x_{1A} at time t_{1A}, just when the event (1) occurs. To see this imagine that observer B had chosen the velocity $v = v_A$. It is therefore the time obtained in the *rest frame* of the watch. This is the frame in which both events occur at the same place, the space origin (make use of the second line in Eq. (2.2)!).

IIIA *Time dilation*. The observer A will conclude that the time difference in his system that corresponds to the proper time τ_1 would be (make use of the first line of Eq. (2.2)!)

$$t_{1A} = \frac{\tau_1}{\sqrt{1 - v_A^2}} \tag{2.4}$$

This means that to A it will seem that the time difference is larger, i.e. it will seem as if time is passing more slowly in the watch rest system. This effect is called time dilation.

This is a noticeable effect for the fast-moving fragments of a collision between cosmic ray elements and the atoms of the upper atmosphere. There are e.g. the μ-particles, very short-lived when we produce them basically at rest, in the laboratory on earth. The lifetime of a μ-particle is around 2×10^{-6} seconds. Therefore even if it was moving with the velocity of light it would only be able to cover about 600 metres!

Nevertheless the produced μ-particles survive a sufficiently long time to be able to go all the way from the top of the atmosphere down to earth, where we can find them in abundance.

To understand this effect we note that the decay time is related to the properties of the particle in its rest frame while the ‘survival time’ we observe is the time it will take a fast-moving particle (with velocity close to c) to move the distance δ from the top of the atmosphere (at a height of around 2×10^4 meters) to the observation point on earth. According to Eq. (2.4) this survival time is much longer and therefore many of the μ-particles survive to reach the ground.

IIIB *Lorentz contraction.* There is a corresponding effect for distances, which is called Lorentz contraction. For the surviving μ-particles, the distance δ, which to us is about 2×10^4 meters, will seem to be at most the 600 metres mentioned above. Considered from the rest system of the μ-particle the distance δ_{rest} is the length that the earth and its atmosphere moves towards it during its lifetime! From the Eq. (2.4) we conclude for the Lorentz contraction effect

$$\delta_{rest} = \delta\sqrt{1 - v^2} \tag{2.5}$$

IV *Covariance.* The scalar product of two ordinary vectors $\mathbf{a} \cdot \mathbf{b}$, written in terms of the coordinates as $a_x b_x + a_y b_y + a_z b_z$, is an invariant with respect to rotations. It is possible to write the invariant τ_1^2 as a (generalised) scalar product. The quantity

$$(1)(2) = t_1 t_2 - x_1 x_2 - y_1 y_2 - z_1 z_2 \tag{2.6}$$

will be invariant with respect to the general Lorentz transformations (i.e. boosts and rotations in any order) if the coordinates and times of the events (1) and (2) transform with respect to Lorentz boosts as in Eq. (2.2) (and $(\mathbf{1}) \equiv (x_1, y_1, z_1)$ and similarly $(\mathbf{2})$ transform as ordinary vectors under rotation).

Such quantities as (1) in Eq. (2.1) are called *four-vectors*. They transform as vectors with respect to the Lorentz transformations, in particular as in Eq. (2.2) for boosts along an axis. Besides the invariants, in the same way called *scalars* under the Lorentz transformations, and the four-vectors it is possible to define *four-tensors* (the electromagnetic field tensor is an example of such a quantity).

All these quantities are said to be *covariant*: they transform in a linear way with respect to the Lorentz transformations, i.e. the corresponding quantities in different Lorentz frames are related by means of linear equations.

V *The transformation of the velocity.* As an example of a quantity with more complex properties with respect to the Lorentz transformations we consider the velocity. We have already mentioned the velocity v_A measured in A's system. From B's point of view the corresponding velocity will be (use both the first and the second line of Eq. (2.2)!)

$$v_B = \frac{v_A - v}{1 - v_A v} \tag{2.7}$$

It is not difficult to show that if the velocities v_A, v do not exceed $c = 1$ then the velocity v_B will have the same property.

VI The *energy-momentum four-vector*. The classical (Newtonian) definition of momentum is the mass (m) times the velocity (v_p) of the particle. But from Eq. (2.7) it is obvious that the transformation properties of the velocity are complex under a Lorentz boost. In order to generalise the definition of momentum Einstein made use of the proper time of the particle motion in the following way.

The velocity of the particle is defined in terms of its trajectory $\mathbf{r}(t)$ (i.e. its space position $\mathbf{r}$ labelled by means of the time t) as

$$\mathbf{v}_p = \frac{d\mathbf{r}}{dt} \tag{2.8}$$

For every (massive) particle it is possible to imagine a rest frame in which the particle is always at the (space) origin. In this way it is possible to define the proper time τ for the particle's motion; it is the time in this, the particle's rest system.

Considered from any other Lorentz frame the proper time τ will be related to the 'ordinary' time t by means of the differential equation

$$d\tau = dt\sqrt{1 - \mathbf{v}_p^2} \tag{2.9}$$

according to Eqs. (2.3), (2.4).

The proper time $\tau(t)$ defined in this way is unique as soon as proper boundary conditions are given for the differential equation. (Its functional dependence upon the time t will in general be different in different Lorentz frames, however.)

We conclude that the corresponding *four-velocity* u defined by

$$u \equiv \left(\frac{dt}{d\tau}, \frac{d\mathbf{r}}{d\tau}\right) = \gamma(\mathbf{v}_p)(1, \mathbf{v}_p) \tag{2.10}$$

will transform covariantly as a vector under the Lorentz transformations. (The third line of Eq. (2.10) is obtained from the differential equation (2.9).) Note that the corresponding invariant $uu = u^2$ has the value $u^2 = 1$. Einstein defined the *four-momentum* p of a particle as

$$p = (e, \mathbf{p}) = mu = m\gamma(\mathbf{v}_p)(1, \mathbf{v}_p) \tag{2.11}$$

The space components $\mathbf{p}$ (from now on the *momentum*) of this four-momentum (which we sometimes will call the *energy-momentum vector*)