Introduction to Electrodynamics

Fifth Edition

David J. Griffiths is Emeritus Professor of Physics at Reed College, Oregon, where he has taught for over 30 years. He received his BA and Ph.D. from Harvard University, where he studied elementary particle theory. He has published three widely-used textbooks on the topics of quantum mechanics, electrodynamics, and particle physics, and a lower-level text titled *Revolutions in Twentieth-Century Physics* (Cambridge, 2012).

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DAVID J. GRIFFITHS
Reed College, Oregon

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Preface

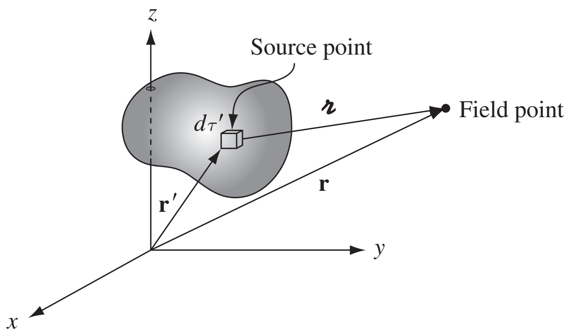
This is a textbook on classical electricity and magnetism, designed for an undergraduate course at the junior or senior level. It can be covered comfortably in two semesters, maybe even with room to spare for special topics (AC circuits, numerical methods, plasma physics, transmission lines, antenna theory, etc.). A one-semester course could reasonably stop after Chapter 7. Unlike quantum mechanics or thermal physics (for example), there is a fairly general consensus with respect to the teaching of electrodynamics; the subjects to be included, and even their order of presentation, are not particularly controversial, and textbooks differ mainly in style and tone. My approach is perhaps less formal than most; I think this makes difficult ideas more interesting and accessible.

For this new edition I have made a number of small changes, in the interests of clarity and grace. In a few places I have corrected errors or expanded the discussion. I have added some new problems, including a few (flagged by a mouse in the margin) that will require a computer. It seemed worthwhile to provide at least some minimal guidance to those wanting to plot a given field (in the form of representative arrows), to draw a field line (starting from a particular point), or to graph the trajectory of a charged particle moving through a specified electric and/or magnetic field (starting at a particular point with a given velocity). These problems are keyed to Mathematica (specifically, version 13.1), but instructors should have no trouble converting them to other languages, if they prefer. I have added some new commentary on subtle issues: ambiguities in the definition of polarization in crystals, problematic aspects of electric and (especially) magnetic field lines, the awkward role of intrinsic spin (a strictly quantum phenomenon) in a classical discussion of magnetic materials.

And I have included more references to the accessible literature (particularly the *American Journal of Physics*). I realize, of course, that most readers will not have the time or inclination to consult these resources, but I think it is worthwhile anyway, if only to emphasize that electrodynamics, notwithstanding its venerable age, is very much alive, and intriguing new discoveries are being made all the time. I hope that occasionally a problem will pique your curiosity, and you will be inspired to look up the references – some of them are real gems.

I have maintained three items of unorthodox notation:

- The Cartesian unit vectors are written $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ (and, in general, all unit vectors inherit the letter of the corresponding coordinate).



- The distance from the z-axis in cylindrical coordinates[1] is designated by s, to avoid confusion with r (the distance from the *origin*, and the radial coordinate in the spherical system).
- The script letter $\boldsymbol{\mathscr{r}}$ denotes the vector from a source point $\mathbf{r}'$ to the field point $\mathbf{r}$, as in the figure above. Some authors prefer the more explicit $(\mathbf{r} - \mathbf{r}')$. But this makes many equations distractingly cumbersome, especially when the unit vector $\hat{\boldsymbol{\mathscr{r}}}$ is involved. I realize that unwary readers are tempted to interpret $\boldsymbol{\mathscr{r}}$ as $\mathbf{r}$ – it certainly makes the integrals easier! *Please take note:* $\boldsymbol{\mathscr{r}} \equiv (\mathbf{r} - \mathbf{r}')$, which is *not* the same as $\mathbf{r}$. I think it's good notation, but it does have to be handled with care.[2]

As in previous editions, I distinguish two kinds of problems. Some have a specific pedagogical purpose, and should be worked immediately after reading the section to which they pertain; these I have placed at the pertinent point within the chapter. (In a few cases the solution to a problem is used later in the text; these are indicated by a bullet (•) in the left margin.) Longer problems, or those of a more general nature, will be found at the end of each chapter. When I teach the subject, I assign some of these, and work a few of them in class. Unusually challenging problems are flagged by an exclamation point (**!**) in the margin. Many readers have asked that the answers to problems be provided at the back of the book; unfortunately, just as many are strenuously opposed. I have compromised, supplying answers when this seems particularly appropriate. A complete solution manual is available (to instructors) from the publisher; go to the Cambridge University Press website (www.cambridge.org/electrodynamics5e) to order a copy.

I have benefitted from the comments of many colleagues. I cannot list them all here, but I would like to thank the following people for especially useful contributions to this edition: Catherine Crouch (Swarthmore), Ted Jacobson (Maryland), Don Koks (Adelaide), T. R. Lemberger (Ohio State),

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[1] D. Pontius (*Phys. Teach.* **55**, 388, 2017) suggests we call it the **abaxial coordinate**.

[2] In MS Word, $\boldsymbol{\mathscr{r}}$ is "Kaufmann font," but this can be very difficult to install in TEX. TEX users can download a pretty good facsimile from my website: www.reed.edu/physics/faculty/griffiths/.

Kirk McDonald[3] (Princeton), Jim McTavish (Liverpool), Mahdiyar Noorbala (Tehran), Darrell Schroeter (Reed), Herschel Snodgrass (Lewis and Clark), Larry Tankersley (Naval Academy), and above all Joel Franklin (Reed). Practically everything I know about electrodynamics – certainly about teaching electrodynamics – I owe to Edward Purcell.

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[3] Kirk's website, http://kirkmcd.princeton.edu/examples/, is a fantastic resource, with clever explanations, nifty problems, and useful references.

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WHAT IS ELECTRODYNAMICS, AND HOW DOES IT FIT INTO THE GENERAL SCHEME OF PHYSICS?

Four Realms of Mechanics

In the diagram below, I have sketched out the four great realms of mechanics:

Classical Mechanics	**Quantum Mechanics**
(Newton)	(Bohr, Heisenberg, Schrödinger, et al.)
Special Relativity	**Quantum Field Theory**
(Einstein)	(Dirac, Pauli, Feynman, Schwinger, et al.)

Newtonian mechanics is adequate for most purposes in "everyday life," but for objects moving at high speeds (near the velocity of light) it is incorrect, and must be replaced by special relativity (introduced by Einstein in 1905); for objects that are extremely small (near the size of atoms) it fails for different reasons, and is superseded by quantum mechanics (developed by Bohr, Heisenberg, Schrödinger, and many others, in the 1920s, mostly). For objects that are both very fast *and* very small (as is common in modern particle physics), a mechanics that combines relativity and quantum principles is in order; this relativistic quantum mechanics is known as quantum field theory – it was worked out in the thirties and forties, but even today it cannot claim to be a completely satisfactory system. In this book, save for the last chapter, we shall work exclusively in the domain of classical mechanics, although electrodynamics extends with unique simplicity to the other three realms. (In fact, the theory is in most respects *automatically* consistent with special relativity, for which it was, historically, the main stimulus.) But please remember, this is a book about *classical* (and in Chapter 12 relativistic) electrodynamics; there is no quantum mechanics here, and you can't expect the theory to explain everything. As we'll see in Chapter 6, the classical theory struggles to provide even a *qualitative* account of magnetic materials. And there is no mention of the photon (well … that's not quite true: the photon does make a brief appearance in Chapter 12, as an example of a massless particle, but never in its more significant role as the quantum of the electromagnetic field). Classical electrodynamics describes a *possible* world, but it is not in every detail the world we actually inhabit; there's more to the story.

Four Kinds of Forces

Mechanics tells us how a system will behave when subjected to a given **force**. There are just *four* fundamental forces known (presently) to physics; I list them in the order of decreasing strength:

(1) Strong
(2) Electromagnetic
(3) Weak
(4) Gravitational

The brevity of this list may surprise you. Where is friction? Where is the "normal" force that keeps you from falling through the floor? Where are the chemical forces that bind molecules together? Where is the force of impact between two colliding billiard balls? The answer is that *all* these forces are *electromagnetic*. Indeed, it is scarcely an exaggeration to say that we live in an electromagnetic world – practically every force we experience in everyday life, with the exception of gravity, is electromagnetic at its core.

The **strong forces**, which hold protons and neutrons together in the atomic nucleus, have extremely short range, so we do not "feel" them, in spite of the fact that they are a hundred times more powerful than electrical forces. The **weak forces**, which account for certain kinds of radioactive decay, are also of short range, and they are far weaker than electromagnetic forces. As for gravity, it is so pitifully feeble (compared to all of the others) that it is only by virtue of huge mass concentrations (like the Earth and the Sun) that we ever notice it at all. The electrical repulsion between two electrons is 10^{42} times as large as their gravitational attraction, and if atoms were held together by gravitational (instead of electrical) forces, a single hydrogen atom would be much larger than the known Universe.

Not only are electromagnetic forces overwhelmingly dominant in everyday life, they are also, at present, the *only* ones that are completely understood. There is, of course, a classical theory of gravity (Newton's law of universal gravitation) and a relativistic one (Einstein's general relativity), but no entirely satisfactory quantum mechanical theory of gravity has been constructed (though many people are working on it). At the present time there is a very successful (if cumbersome) theory for the weak interactions, and a strikingly attractive candidate (called **chromodynamics**) for the strong interactions. All these theories draw their inspiration from electrodynamics; none can claim conclusive experimental verification at this stage. So electrodynamics, a beautifully complete and successful theory, has become a kind of paradigm for physicists: an ideal model that other theories emulate.

The laws of classical electrodynamics were discovered in bits and pieces by Franklin, Coulomb, Ampère, Faraday, and others, but the person who completed the job, and packaged[4] it all in the compact and consistent form it has today, was James Clerk Maxwell. The theory is now about 150 years old.

The Unification of Physical Theories

In the beginning, **electricity** and **magnetism** were entirely separate subjects. The one dealt with glass rods and cat's fur, pith balls, batteries, currents, electrolysis, and lightning; the other with bar magnets, iron filings, compass needles, and the North Pole. But in 1820, Oersted noticed that an *electric* current could deflect a *magnetic* compass needle. Soon afterward, Ampère postulated that *all* magnetic phenomena are due to electric charges in motion. Then, in 1831, Faraday discovered that a moving *magnet* generates an *electric* current. By the time Maxwell and Lorentz put the finishing touches on the theory, electricity and magnetism were inextricably intertwined. They could no longer be regarded as separate subjects, but rather as two *aspects* of a *single* subject: **electromagnetism**.

Faraday speculated that light, too, is electrical in nature. Maxwell's theory provided spectacular justification for this hypothesis, and soon **optics** – the study of lenses, mirrors, prisms, interference, and diffraction – was incorporated into electromagnetism. Hertz, who presented the decisive experimental confirmation for Maxwell's theory in 1888, put it this way: "The connection between light and electricity is now established .... In every flame, in every luminous particle, we see an electrical process .... Thus, the domain of electricity extends over the whole of nature. It even affects ourselves intimately: we perceive that we possess ... an electrical organ – the eye." By 1900, then, three great branches of physics – electricity, magnetism, and optics – had merged into a single unified theory. (And it was soon apparent that visible light represents only a tiny "window" in the vast spectrum of electromagnetic radiation, from radio through microwaves, infrared and ultraviolet, to X-rays and gamma rays.)

Einstein dreamed of a further unification, which would combine gravity and electrodynamics, in much the same way as electricity and magnetism had been combined a century earlier. His **unified field theory** was not particularly successful, but in recent years the same impulse has spawned a hierarchy of increasingly ambitious (and speculative) unification schemes, beginning in the 1960s with the **electroweak** theory of Glashow, Weinberg, and Salam (which joins the weak and electromagnetic forces), and culminating in the 1980s with the **superstring** theory (which, according to its proponents, incorporates all four forces in a single "theory of everything").

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[4] Actually, the packaging itself was perfected by Oliver Heaviside.

At each step in this hierarchy, the mathematical difficulties mount, and the gap between inspired conjecture and experimental test widens; nevertheless, it is clear that the unification of forces initiated by electromagnetism has become a major theme in the progress of physics.

The Field Formulation of Electrodynamics

The fundamental problem a theory of electromagnetism hopes to solve is this: I hold up a bunch of electric charges *here* (and maybe shake them around). What happens to some *other* charge, over *there*? The classical solution takes the form of a **field theory**: We say that the space around an electric charge is permeated by electric and magnetic **fields** (the electromagnetic "odor," as it were, of the charge). A second charge, in the presence of these fields, experiences a force; the fields, then, transmit the influence from one charge to the other – they "mediate" the interaction.

When a charge undergoes *acceleration*, a portion of the field "detaches" itself, in a sense, and travels off at the speed of light, carrying with it energy, momentum, and angular momentum. We call this **electromagnetic radiation**. Its existence invites (if not *compels*) us to regard the fields as independent dynamical entities in their own right, every bit as "real" as atoms or baseballs. Our interest accordingly shifts from the study of forces between charges to the theory of the fields themselves. But it takes a charge to *produce* an electromagnetic field, and it takes another charge to *detect* one, so we had best begin by reviewing the essential properties of electric charge.

Electric Charge

(1) **Charge comes in two varieties.** We call them "plus" and "minus," because their effects tend to *cancel* (if you have $+q$ and $-q$ at the same point, electrically it is the same as having no charge there at all). This may seem too obvious to warrant comment, but I encourage you to contemplate other possibilities: What if there were 8 or 10 different species of charge? (In chromodynamics there are, in fact, *three* quantities analogous to electric charge, each of which may be positive or negative.) Or what if the two kinds did not tend to cancel? The extraordinary fact is that plus and minus charges occur in *exactly* equal amounts, to fantastic precision, in bulk matter, so that their effects are almost completely neutralized. Were it not for this, we would be subjected to enormous forces: a potato would explode violently if the cancelation were imperfect by as little as one part in 10^{10}.

(2) Charge is conserved. It cannot be created or destroyed – what there is now has always been. (A plus charge can "annihilate" an equal minus charge, but a plus charge cannot simply disappear by itself – *something* must pick up that electric charge.) So the total charge of the Universe is fixed for all time. This is called **global** conservation of charge. Actually, I can say something much stronger: Global conservation would allow for a charge to disappear in New York and instantly reappear in San Francisco (that wouldn't affect the *total*), and yet we know this doesn't happen. If the charge *was* in New York and it *went* to San Francisco, then it must have passed along some continuous path from one to the other. This is called **local** conservation of charge. Later on we'll see how to formulate a precise mathematical law expressing local conservation of charge – it's called the **continuity equation**.

(3) Charge is quantized. Although nothing in classical electrodynamics requires that it be so, the *fact* is that electric charge comes only in discrete lumps – integer multiples of the basic unit of charge. If we call the charge on the proton $+e$, then the electron carries charge $-e$; the neutron charge zero; the pi mesons $+e$, 0, and $-e$; the carbon nucleus $+6e$; and so on (never $7.392e$, or even $1/2e$).[5] This fundamental unit of charge is extremely small, so for practical purposes it is usually appropriate to ignore quantization altogether. Water, too, "really" consists of discrete lumps (molecules), and yet, if you're taking a drink (or a bath) it's not unreasonable to think of it as a continuous fluid. This is in fact much closer to Maxwell's own view; he knew nothing of electrons and protons – he must have pictured charge as a kind of "jelly" that could be divided up into portions of any size and smeared out at will.

Units

The subject of electrodynamics is plagued by competing systems of units, which sometimes render it difficult for physicists to communicate with one another. The problem is far worse than in mechanics, where Neanderthals still speak of pounds and feet; in mechanics, at least all equations *look* the same, regardless of the units used to measure quantities. Newton's second law remains $\mathbf{F} = m\mathbf{a}$, whether it is feet–pounds–seconds, kilograms–meters–seconds, or whatever. But this is not so in electromagnetism, where Coulomb's law may appear variously as

$$\mathbf{F} = \frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \text{ (Gaussian), or } \mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \text{ (SI), or } \mathbf{F} = \frac{1}{4\pi}\frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \text{ (HL).}$$

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[5] Actually, protons and neutrons are composed of three **quarks**, which carry fractional charges ($\pm\frac{2}{3}e$ and $\pm\frac{1}{3}e$). However, *free* quarks do not appear to exist in nature, and in any event, this does not alter the fact that charge is quantized; it merely reduces the size of the basic unit.

Of the systems in common use, the two most popular are **Gaussian** (cgs) and **SI** (mks). Elementary particle theorists favor yet a third system: **Heaviside–Lorentz**. Although Gaussian units offer distinct theoretical advantages, most undergraduate instructors seem to prefer SI, I suppose because they incorporate the familiar household units (volts, amperes, and watts). In this book, therefore, I have used SI units. Appendix C provides a “dictiona” for converting the main results into Gaussian units.