A First Course in Magnetohydrodynamics

This text introduces readers to magnetohydrodynamics (MHD), the physics of ionised fluids. Traditionally, MHD is taught as part of a graduate curriculum in plasma physics. By contrast, this text – one of a very few – teaches MHD exclusively from a fluid dynamics perspective, making it uniquely accessible to senior undergraduate students. Part I of the text uses the MHD Riemann problem as a focus to introduce the fundamentals of MHD: Alfvén's theorem, waves, shocks, rarefaction fans, and so on. Part II builds upon this with presentations of broader areas of MHD: fluid instabilities, viscid hydrodynamics, steady-state MHD, and non-ideal MHD. Throughout the text, more than 125 problems and several projects (with solutions available to instructors) reinforce the main ideas. In addition, large-font lesson plans for a "flipped-style" class are available free of charge to instructors who use this text as required reading for their course. This book is suitable for advanced undergraduate and beginning graduate students of physics, requiring no previous knowledge of fluid dynamics or plasma physics.

David Clarke is a retired professor of astronomy and physics from Saint Mary's University in Halifax, Nova Scotia. Over his thirty-year career, he has taught numerous courses in physics and astronomy at the undergraduate and graduate levels, including courses in fluid dynamics and MHD that inspired this text. He is co-developer of the original *ZEUS* MHD code and currently the primary developer of *ZEUS-3D* that he uses for his research in astrophysical jets and has made available open source to hundreds of investigators worldwide.

Professor Hannes Olof Gösta Alfvén (1908–1995), father of magnetohydrodynamics, 1970 Nobel Prize laureate for physics. Portrait created in 1972 by Benno Movin-Hermes (1902–1977) using a *three-colour foil method* developed by the artist. Reproduced with permission from I. Movin and the Moderna Museet, Stockholm.

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For Jodi, my life-long love.

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Preface

WHENEVER a university professor stares down the barrel of a new course preparation, the first question invariably asked is *Is there a text?* For the most part in undergraduate physics, the answer is usually *Yes, and plenty to choose from.* But for a senior undergraduate or beginning graduate course in *magnetohydrodynamics* (MHD), the selection is much narrower.

MHD is a relatively new branch of physics. Developed by Hannes Alfvén during the 1940s, it didn't gain wide acceptance among physicists writ large until the late 1950s culminating in Alfvén's Nobel Prize in 1970.[1] As such, MHD is often touted as a "classical afterthought", the only branch of classical physics introduced *after* quantum mechanics with many of its fundamentals – the Riemann problem, magneto-rotational instability, and non-ideal effects – still being worked out in the 1990s and early aughts.

Thus, MHD has not had as long a history as other branches of physics in which textbooks could accumulate, particularly at the undergraduate level. Indeed, MHD has largely been considered a graduate-level subject and, because of this, the vast majority of existing texts specialise in areas such as fusion physics, solar physics, and planetary discs, many written for students already with some familiarity of plasma physics or fluid dynamics.

Another extenuating circumstance is MHD is a divided field whose practitioners – largely plasma physicists and fluid dynamicists – approach the subject in two very different ways. From a plasma physicist's point of view (PoV), an MHD system is the isotropic limit of an ensemble of charged particles – a plasma – governed by velocity moments of the *Vlasov–Boltzmann equation*, a 6-D inhomogeneous partial differential equation (PDE) at the heart of plasma physics. From a fluid dynamicist's PoV, an MHD system is never considered as an ensemble of particles, but rather as a continuous medium governed by simple conservation rules that *any* undergraduate physics student can understand. This leads to a hyperbolic set of equations that can be analysed entirely in terms of *waves*. The two approaches couldn't be more different.

After thirty years of teaching graduate and undergraduate (astro)physics, it is my considered opinion that for MHD to be approachable by undergraduates, it needs to be taught from the fluids PoV. Wave mechanics – so fundamental in classical mechanics, electrodynamics, and quantum mechanics – is already ingrained in the mind of a fourth-year student. On the other hand, velocity moments of a six-

[1] On pages 127–128, there's an amusing anecdote on what – or who – changed the physics community's collective mind on MHD, and a link to Anthony Peratt's short biography on Alfvén.

dimensional PDE are not. Further, most texts taking the plasma PoV are focused on laboratory plasmas and fusion physics, and the wave nature of MHD is often overlooked. To my taste, MHD is *the* prototype for teaching and reinforcing *wave mechanics*, and this is precisely the approach I take in this text.

While there are plenty of textbooks on MHD from the plasma PoV, precious few exist from the fluids PoV. In the survey of texts I did as part of my proposal for this text, I found more than 100 books written over the past six decades from the plasma PoV focused on plasma physics with a substantial portion devoted to the "MHD limit". Indeed, a dozen or so of these textbooks include MHD in their titles.[2] By contrast, I found just *two* texts on MHD written entirely from the fluids PoV and directed to senior undergraduates: Kendall & Plumpton's *Magnetohydrodynamics with Hydrodynamics* (1964); and Galtier's *Introduction to Modern Magnetohydrodynamics* (2016).

This text offers a third. My approach focuses on the *fundamentals* of the subject and teaches MHD for its own sake rather than dwelling on directed applications and current areas of research; these, I argue, are better suited for graduate texts of which there are plenty. I do provide numerous examples from the literature, but these are selected to emphasise certain ideas (*e.g.*, planetary discs with non-ideal MHD, stellar winds with steady-state MHD, astrophysical jets with Bernoulli's principle, *etc.*) and none should distract the reader from the current discussion. Once endowed with the fundamentals, I contend, students can carry these forward to further their study at the graduate level, should they choose.

In keeping with the undergraduate theme, the first part – *1-D MHD in Ten Weeks* – is designed around a single goal: solving the 1-D MHD Riemann problem. I also assert that to understand MHD, one first has to understand ordinary hydrodynamics (HD) which is, after all, just the zero-field limit of MHD. To these ends, Chap. 1 introduces the student to the fundamentals of HD that includes a novel and simple derivation of the three ideal HD equations. Chapter 2 focuses on 1-D applications of HD including sound waves, shocks, bores, and Bernoulli's principle while Chap. 3 develops a semi-analytic solution to the hydrodynamical Riemann problem. In so doing, students learn how the equations of HD lead to a wave equation, and are shown three ways to extract information about hydrodynamical waves: direct solution of the wave equation; normal mode analysis using linear algebra; and via Riemann invariants and their characteristic paths. In my experience, introducing students to these methods – particularly the latter two – for the relatively simple case of HD is critical for them to understand how they apply to the much more complicated MHD case.

The magnetic induction, $\vec{B}$, doesn't appear until Chap. 4 where the ideal induction equation and the Lorentz force are introduced, along with Alfvén's theorem, magnetic helicity, and flux linking. Chapter 5 examines the MHD equations in 1-D to uncover all three types of waves (slow, Alfvén, fast) and all discontinuities (tan-

[2]The most ambitious and a very recent example of this is Goedbloed, Keppens, & Poedts' *Magnetohydrodynamics of Laboratory and Astrophysical Plasmas* (2019). This is a comprehensive tour de force which could support at least three graduate-level courses.

gential, rotational, shocks) including all three shock subtypes (slow, intermediate, fast). Chapter 6 introduces slow and fast rarefaction fans, and then brings it all together to show students how an exact MHD Riemann solver can be assembled. As this is a semi-analytical solution, students learn or have reinforced semi-analytical techniques including Runge–Kutta methods, multivariate secant root finders, methods for maintaining machine accuracy, and the list goes on.

Part I is designed to be completed in twenty-five hours of instruction (ten weeks at most Canadian universities). The four chapters in Part II, *Additional Topics in (M)HD*, are independent from each other, depend only on material from Part I, and give the instructor options to complete the semester. These include (M)HD instabilities in Chap. 7, viscid HD (Navier–Stokes equation) in Chap. 8, steady-state MHD in Chap. 9, and non-ideal MHD in Chap. 10. In the interest of expediency, sections designated as "optional" can be omitted without loss of continuity.

Parts of the text may come across as "mathematically dense"; this is deliberate. As an undergraduate, I always found it frustrating and distracting when I was unable to fill in the large gaps left between lines of logic in the texts my professors chose, and I was not going to produce a text that did the same. That said, the densest parts of the mathematics can largely be skimmed on first read and certainly don't need to be covered in detail in class, as the main results from which the physics is extracted are always boxed within each development. For the student like I was who needs to know how the derivations are done, the gaps between the mathematical steps are small enough that a careful second read should suffice.

More than 125 problems – many exploiting "teaching moments" – and several computer projects are distributed amongst the ten chapters' problem sections, each generally digestible by a senior undergraduate or first-year graduate student. Problems without an asterisk can and should be done in a page or less (and often a few lines), one asterisk indicates a two-page solution, two asterisks indicate a three- to four-page solution, while three asterisks indicate a more involved problem, generally requiring more than five pages and/or a substantive computer program to solve. A complete solution set including the computer projects is available to the instructor upon request.

The eight appendices are designed to remind students of particularly critical material prerequisite to this text. Students who do not recognise, recall, or know how to use any of this material are encouraged to review the relevant material from previous courses. Following the appendices is a glossary of symbols used throughout the text, a list of references, and finally an extensive index.

While this text assumes no previous knowledge of (M)HD, students should have had second- and third-year courses in mechanics, electrodynamics, and thermodynamics. On the math side, students should be fluent in vector calculus (at the level of App. A), adept at solving differential equations including PDEs such as the wave equation, and thoroughly familiar with linear algebra and, in particular, *eigen*algebra. In addition, some experience in scientific computing (algorithm and code development) would be beneficial.

Finally, an acknowledgement of the biases of the author is in order. While this text includes numerous astrophysical applications, astronomy is by no means a prerequisite, nor is this text designed just for budding astrophysicists. I would like to think *any* physicist interested in learning about fundamental MHD will find this book useful. As for units, I follow the bulk of the physics community (but not astronomy!) and use `mks` exclusively. Lastly, all program listings in this text are in *FORTRAN 77*, the *only* computer language – this old programmer would assert – a computational scientist really needs to know!

Like many of my contemporaries, I learned MHD "at the knee of my advisors", by reading select chapters in certain texts, by going through journal articles, and talking to experts. As a student and post-doctoral fellow, it always struck me as a bit unfair that all other branches of physics seemed to be taught in more systematic and accessible ways – dedicated courses, self-contained textbooks, problem sets at appropriate levels – but somehow not MHD. Granted, MHD doesn't enjoy the same "critical mass" of students as other areas of physics, and perhaps this inaccessibility is part of the reason why.

This text – almost two decades in the making – is the textbook I wish I had access to forty years ago when I started out in this game. And now as I enter my retirement, it is my profound hope that within these pages, new students of MHD will find a self-contained introduction to the subject that will help launch them into a fascinating, life-long adventure as I have enjoyed!

Each chapter in Part II benefitted from written projects or theses submitted by Saint Mary's graduate and undergraduate students taking my (M)HD course and/or working with me as a research student in years past. For these efforts, I thank Joel Tanner, Patrick Rogers, Jonathan Ramsey, Nicholas MacDonald, Michael Power, and Christopher MacMackin.

I thank my editors Nicholas Gibbons, Sarah Armstrong, Stephanie Windows, and Jane Chan at Cambridge University Press for their capable and patient guidance of this first-time author. It definitely made my job a lot less daunting! A big thank-you goes to Patricia Langille at Saint Mary's Patrick Power Library for doing *all* the heavy lifting in getting permissions for the copyrighted material used in this text; she gets the first signed copy! I would also like to acknowledge the academic freedom afforded to me over the past three decades by Saint Mary's University that made long-term projects like this possible. Thank you all.

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And then there are the magnificent villages of Ménerbes and Saint-Pierre-

Toirac, France. The most difficult sections of this text for me to write were inspired and completed during my long *séjours* there and, other than my home province of Nova Scotia, I can't think of anywhere else I'd rather spend months at a time than in a small French village. So much about France charms me including, of all things, their speed-limit signs! It's not enough to tell you what the speed limit is, the French also feel the need to tell you you're being *reminded* of what the speed limit is! So in homage to my second country, all footnotes throughout the text serving as a "reminder to the reader" are heralded by the French translation *Rappel*.

And finally to my wife Jodi (MEL) to whom this text is dedicated. Words can't express my love and gratitude for sticking by me these nearly forty years and for helping create such a wonderful and supportive home for our family here in Halifax. You're the best!

Any constructive feedback on what works and doesn't work in this text, as well as any error reports, omissions, redundancies, *etc.* are welcome and can be sent to me directly at `AfciMHD@gmail.com`. Instructors of courses based on this text may download a solution set to the problems and a fully-developed set of course lecture notes from CUP's website, `www.cambridge.org/9781009381475`.

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