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Peter Blythe , Emma Low , Andrew Manning , Karen Morrison , Raju Taniparti , Jasmine S. M. Teo
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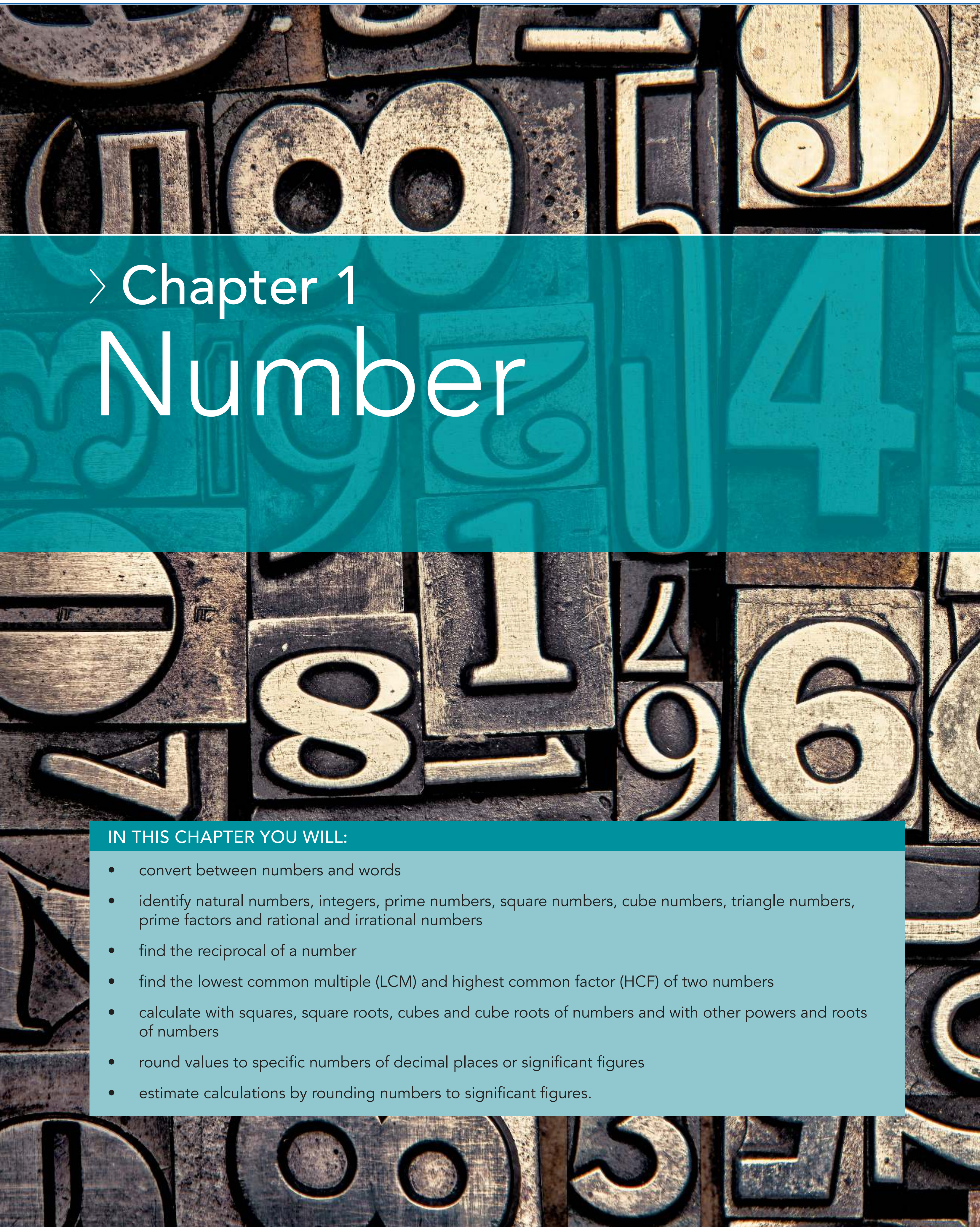

Chapter 1

Number

IN THIS CHAPTER YOU WILL:

- convert between numbers and words
- identify natural numbers, integers, prime numbers, square numbers, cube numbers, triangle numbers, prime factors and rational and irrational numbers
- find the reciprocal of a number
- find the lowest common multiple (LCM) and highest common factor (HCF) of two numbers
- calculate with squares, square roots, cubes and cube roots of numbers and with other powers and roots of numbers
- round values to specific numbers of decimal places or significant figures
- estimate calculations by rounding numbers to significant figures.

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GETTING STARTED

1 Write down the whole numbers:
   a between 10 and 20
   b greater than 25 but less than or equal to 30.

2 In the list of numbers:

   25, $\frac{1}{2}$, 0.125

   write down a:
   a decimal b fraction c whole number.

3 List the factors of:
   a 12 b 30 c 42

4 Write down the first five multiples of:
   a 3 b 5 c 11

KEY WORDS

- cube number
- highest common factor (HCF)
- integer
- irrational number
- lowest common multiple (LCM)
- natural numbers
- prime number
- reciprocal
- root
- square number
- triangle or triangular number

1.1 Types of number

Being able to identify different types of number will help you to understand how number operations work and to evaluate problems involving numbers.

Natural numbers

The **natural numbers** are: 0, 1, 2, 3, 4, 5, 6, …

Natural numbers have no decimal or fractional parts.

TIP

Some books define the natural numbers starting from 1, but in this course the natural numbers start from zero.

Converting between numbers and words

Natural numbers have different place values. These place values allow you to convert natural numbers into words.

This table summarises the place values for the number 1 234 567 890.

Digit	Place value
1 234 567 89**0**	ones
1 234 567 8**9**0	tens
1 234 567 **8**90	hundreds
1 234 56**7** 890	thousands
1 234 5**6**7 890	ten thousands
1 234 **5**67 890	hundred thousands
1 23**4** 567 890	millions
1 2**3**4 567 890	ten millions
1 **2**34 567 890	hundred millions
1 234 567 890	billions

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You write 1 234 567 890 in words as one billion, two hundred and thirty-four million, five hundred and sixty-seven thousand, eight hundred and ninety.

WORKED EXAMPLE 1

1 Write the following numbers in words.

a 20 316

b 7 000 000 009

2 Write the following in numbers.

a One hundred and twenty-four thousand

b Fifteen million and seventy-eight

Answers

1 a First look at the place value of each digit in the number 20 316.

Digit	Place value
20 31**6**	six ones
20 3**1**6	one ten
20 **3**16	three hundreds
2**0** 316	zero thousands
20 316	twenty thousands

Starting from the highest place value, you have twenty thousand, three hundred and sixteen.

b In the number 7 000 000 009, only the digits 7 and 9 are non-zero.

Digit	Place value
7 000 000 00**9**	nine ones
7 000 000 009	seven billions

Starting from the highest place value, you have seven billion and nine.

2 a Consider the place value of the numbers in words.

Words	Numbers	Remarks
One hundred and twenty-four **thousand**	100 000	A thousand is 3 zeros, so a hundred thousand is 100 with 3 zeros.
One hundred and **twenty-four thousand**	24 000	Twenty-four thousand is 24 with 3 zeros.

Add up the numbers: 100 000 + 24 000 = 124 000

So, one hundred and twenty-four thousand in numbers is 124 000.

TIP

When expressing a number in words, omit any place value with a zero digit.

CONTINUED

b Consider the place value of the numbers in words.

Words	Numbers	Remarks
Fifteen million and seventy-eight	15 000 000	A million has 6 zeros, so fifteen million is 15 with 6 zeros.
Fifteen million and **seventy-eight**	78	

Add up the numbers: 15 000 000 + 78 = 15 000 078

So fifteen million and seventy-eight in numbers is 15 000 078.

Categorising natural numbers

There are many different ways to categorise natural numbers. For example, natural numbers can be even or odd.

Even numbers are divisible by two.

The even numbers are: 0, 2, 4, 6, 8, 10, …

In general, the last digit of an even number is a multiple of 2.

Numbers that are not even are odd numbers.

The odd numbers are: 1, 3, 5, 7, 9, 11, …

Odd numbers are **not** divisible by two.

In general, the last digit of an odd number is 1, 3, 5, 7, or 9.

DISCUSSION 1

1 Work with a partner to determine if the following statements are true or false.

- **a** The sum of two even numbers is always even.
- **b** The sum of two odd numbers is always odd.
- **c** The sum of an even and an odd number is always odd.
- **d** The product of two even numbers is always even.
- **e** The product of two odd numbers is always odd.
- **f** The product of an even and an odd number is always even.

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CONTINUED

You can also classify natural numbers using their factors.

2 Copy and complete this table. Write down all the factors of the given numbers. The factors of 12 have been written for you.

Number	Factors	Working
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12	1, 2, 3, 4, 6, 12	$12 = 1 \times 12$ $= 2 \times 6$ $= 3 \times 4$
13		
14		

3 Copy and complete this table. Sort the numbers from the table in question 2 into the following groups:

Group	Number of different factors	Numbers
I	Number(s) with exactly one factor	
II	Number(s) with exactly two different factors	

The numbers in group II are known as **prime numbers**.

Zero and one are not prime numbers.

REFLECTION

Why are zero and one not prime numbers?

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Prime numbers

A **prime number** is a natural number that has exactly two different factors, one and itself.

INVESTIGATION 1

Sieve of Eratosthenes

The Sieve of Eratosthenes is a method to find prime numbers.

In this investigation, you will use the Sieve of Eratosthenes to find all the prime numbers from 2 to 100.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Start from 2 because 2 is the smallest prime number.

Follow the given instructions.

Step 1: circle 2, the smallest prime number, and cross out all the multiples of 2 less than or equal to 100.

Step 2: circle the next smallest number that is not crossed out, in this case 3. Cross out all the multiples of 3 less than or equal to 100.

Step 3: circle the next smallest number that is not crossed out, in this case 5. Cross out all the multiples of 5 less than or equal to 100.

Step 4: circle the next smallest number that is not crossed out and cross out all multiples of that number less than or equal to 100.

Step 5: repeat the same process until all the numbers are either crossed out or circled.

Answer the following.

1 Write down all the numbers you circled. These numbers are prime numbers less than 100.

2 How many prime numbers less than 100 are there?

3 How many **even** prime numbers less than 100 are there?

4 How many **odd** prime numbers less than 100 are there?

You can use the same process to find prime numbers less than any number.

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A number is prime if it is not divisible by any of the prime numbers less than or equal to the square **root** of the number.

You can use this result to determine whether a given number is a prime number.

WORKED EXAMPLE 2

Determine whether the following numbers are prime numbers.

a 291 **b** 269

Answers

a **Step 1:** use a calculator to find the square root of 291: $\sqrt{291} = 17.058\,72\ldots$

Step 2: identify all the prime numbers less than or equal to the square root of 291.

The prime numbers less than or equal to 17.058 72… are:
2, 3, 5, 7, 11, 13, 17.

Step 3: try dividing 291 by these prime numbers. If 291 is not divisible by any of the prime numbers identified in step 2, then it is a prime number.

Consider 2: Since 291 is not even, it is not divisible by 2.

Try 3: 291 ÷ 3 = 97, so 291 is divisible by 3.

Since 291 is divisible by 3, it is not a prime number.

b $\sqrt{269} = 16.401\,21\ldots$

The prime numbers less than or equal to $\sqrt{269} = 16.401\,21\ldots$ are:
2, 3, 5, 7, 11, 13.

Consider 2: 269 is odd, so it is not divisible by 2.	Only even numbers are divisible by 2.
Try 3: 269 ÷ 3 = 89.666 6…, so 269 is not divisible by 3.	A number is divisible by 3 if the sum of the digits of the number is divisible by 3. The digits of 269 are 2, 6 and 9. 2 + 6 + 9 = 17, which is not divisible by 3, so 269 is not divisible by 3.
Try 5: The last digit of 269 is 9, which is neither 0 nor 5, so 269 is not divisible by 5.	Only numbers with a last digit of 0 or 5 are divisible by 5.
Try 7: 269 ÷ 7 = 38.428 57…, so 269 is not divisible by 7.	A number is divisible by 7 if twice the last digit of the number subtracted from the remaining number is divisible by 7. The last digit of 269 is 9. 9 × 2 = 18. The remaining number is 26. 26 − 18 = 8. Since 8 is not divisible by 7, 269 is also not divisible by 7.

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Try 11: 269 ÷ 11 = 24.454 5…, so 269 is not divisible by 11.	A number is divisible by 11 if the alternating sum of the digits (alternate sum and difference) is divisible by 11. For 269, the alternating sum is 2 − 6 + 9 = 5. Since 5 is not divisible by 11, 269 is not divisible by 11.
Try 13: 269 ÷ 13 = 20.692 3…, so 269 is not divisible by 13.	A number is divisible by 13 if the difference between four times the last digit of the number and the remaining number is divisible by 13. The last digit of 269 is 9. 9 × 4 = 36. The remaining number is 26. 36 − 26 = 10. Since 10 is not divisible by 13, 269 is also not divisible by 13.

Since 269 is not divisible by 2, 3, 5, 7, 11, or 13, which are all the primes less than or equal to $\sqrt{269}$ = 16.401 21…, then 269 must be a prime number.

WORKED EXAMPLE 3

If a and b are natural numbers such that $a \times b = 19$, find the value of $a + b$.

Answer

Since 19 is a prime number, the only factors of 19 are 1 and 19.

So a and b must be 1 and 19 in whichever order. Then $a + b = 1 + 19 = 20$

Integers (positive, negative and zero)

The natural numbers are part of the **integers**. Integers are numbers that do not have fractional or decimal parts. Integers can be positive or negative or zero.

The positive integers are: 1, 2, 3, 4, 5, …, 100, …

The negative integers are: −1, −2, −3, …, −45, …

0 is an integer but it is neither positive nor negative.

MATHEMATICAL CONNECTIONS

You will learn about the four operations for calculations with integers in Chapter 2.

Exercise 1.1

1 Write the following numbers in words.

a 540 018 **b** 9 000 342 **c** 41 020 679
d 3 000 000 853 **e** 9 000 231 038 **f** 60 582
g 6 500 453 684

2 Write the following in numbers.

a Two million, six hundred and eighteen thousand, four hundred and twenty-two.

b Five billion, four hundred and sixty-one.

c Seven hundred and four thousand and thirty-seven.

d Eighteen million, one hundred and fifty-three thousand and six.

3 Write down all the prime numbers that are less than 100.

4 Determine if each of these numbers is a prime number.

a 173 b 129 c 237 d 281 e 383

5 If a and b are natural numbers such that $a \times b = 23$, find the value of $a + b$.

6 If a and b are natural numbers such that $a \times b = 89$, find the value of $a + b$.

1.2 Other types of number

Index notation, square numbers and cube numbers

Index notation is a way of writing numbers when you multiply a number by itself one or more times.

For example, you can write 3×3 as 3^2.

You read 3^2 as 'three to the power of two', where the number three is the base and the number two is the power or exponent.

The base represents the number that you multiplied by itself. The power is the number of times you multiplied the base by itself. So:

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$ reads as 'two to the power of five'

$3 \times 3 \times 3 \times 3 = 3^4$ reads as 'three to the power of four'

$5 \times 5 \times 5 = 5^3$ reads as 'five to the power of three'.

When the power is two, you can also say that the base is 'squared'.

For example, you read 3^2 as 'three squared'.

Numbers with the power of two are called **square numbers**.

Examples of square numbers are:

$2^2 = 2 \times 2 = 4$ $3^2 = 3 \times 3 = 9$ $4^2 = 4 \times 4 = 16$ $5^2 = 5 \times 5 = 25$

When the power is three, you can also say that the base is 'cubed'.

For example, you read 5^3 as 'five cubed'.

Numbers with the power of three are called **cube numbers**.

Examples of cube numbers are:

$2^3 = 2 \times 2 \times 2 = 8$ $3^3 = 3 \times 3 \times 3 = 27$ $4^3 = 4 \times 4 \times 4 = 64$

MATHEMATICAL CONNECTIONS

Indices are discussed further in Chapter 6.

WORKED EXAMPLE 4

Write the following in index notation.

a $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **b** $a \times a \times a \times a \times a$

Answers

a $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

The number two is multiplied by itself, so the base is two. The base two is multiplied by itself seven times, so the power is seven.

b $a \times a \times a \times a \times a = a^5$

The number a is multiplied by itself, so the base is a. The base a is multiplied by itself five times, so the power is five.

WORKED EXAMPLE 5

Evaluate.

a 17^2 **b** 6^3 **c** 11^4

Answers

a $17^2 = 17 \times 17 = 289$

You can use a calculator to evaluate the product of such a big number.
To evaluate a square number, key in:

[1] [7] [x^2] [EXE]

b $6^3 = 6 \times 6 \times 6 = 216$

To use your calculator to evaluate 6^3, key in:

[6] [^] [3] [EXE]

c $11^4 = 11 \times 11 \times 11 \times 11 = 14\,641$

To use your calculator to evaluate 11^4, key in:

[1] [1] [^] [4] [EXE]

Rational numbers and irrational numbers

A rational number is a number that can be expressed in the form $\frac{a}{b}$ where a and b are natural numbers and $b \neq 0$.

Examples of rational numbers are: $2, 3, \frac{1}{2}, -4\frac{2}{5}, -2.5$

All natural numbers and fractions are rational. Some decimals are rational.

An **irrational number** is a number that **cannot** be expressed in the form $\frac{a}{b}$ where a and b are natural numbers and $b \neq 0$.

Examples of irrational numbers are: $\sqrt{2}, \sqrt{5}, \sqrt[3]{7}, \pi$

In general, square roots and cube roots that are not exact are irrational.

MATHEMATICAL CONNECTIONS

You will look closer at square roots and cube roots in Section 1.5.