

Contents

<i>Preface to the second edition</i>	xi
<i>Preface to the first edition</i>	xiii
<i>Acknowledgements</i>	xvi
<i>Symbols</i>	xvii
1 Introduction	1
1.1 Graph of a network	2
1.2 Eigenvalues and eigenvectors of a graph	2
1.3 Interpretation and contemplation	3
1.4 Outline of the book	7
1.5 Classes of graphs	8
1.6 Outlook	11
Part I Spectra of graphs	13
2 Algebraic graph theory	15
2.1 Graph related matrices	15
2.2 The incidence matrix B	20
2.3 Connectivity, walks and paths	28
2.4 The line graph	35
2.5 Permutations, partitions and the quotient graph	42
2.6 Cospectral graphs	47
3 Eigenvalues of the adjacency matrix	51
3.1 General properties	51
3.2 Characteristic polynomial $c_A(\lambda)$ of the adjacency matrix A	52

viii	<i>Contents</i>	
3.3	Regular graphs	57
3.4	Powers of the adjacency matrix	60
3.5	The number of walks	63
3.6	Diameter of a graph	75
3.7	The spectral radius λ_1	78
3.8	Eigenvalue spacings	88
3.9	Adding or removing nodes or links	91
3.10	Additional properties	101
3.11	The stochastic matrix $P = \Delta^{-1}A$	108
4	Eigenvalues of the Laplacian Q	111
4.1	General properties	111
4.2	The pseudoinverse matrix Q^\dagger of the weighted Laplacian \tilde{Q}	128
4.3	Second smallest eigenvalue of the Laplacian Q	134
4.4	Partitioning of a graph	146
4.5	The modularity and the modularity matrix M	153
4.6	Bounds for the diameter	163
4.7	Eigenvalues of graphs and subgraphs	169
5	Effective resistance matrix	175
5.1	Effective resistance matrix Ω	175
5.2	Effective graph resistance	177
5.3	Properties of the effective resistance	179
5.4	The pseudoinverse Q^\dagger and the effective resistance matrix Ω	184
5.5	The spectrum of the effective resistance matrix Ω	184
5.6	The effective resistance and spanning trees	187
5.7	Bounds for the effective resistance matrix Ω	190
5.8	Lower and upper bound for the effective graph resistance R_G	191
6	Spectra of special types of graphs	193
6.1	The complete graph	193
6.2	A small-world graph	193
6.3	A cycle on N nodes	201
6.4	A path of $N - 1$ hops	203
6.5	A path of h hops	211
6.6	The wheel W_{N+1}	211
6.7	The complete bipartite graph $K_{m,n}$	212
6.8	A general bipartite graph	214
6.9	Complete multipartite graph	219
6.10	An m -fully meshed star topology	222
6.11	Uniform degree graph	230
6.12	A link joining two disconnected graphs	235
6.13	A chain of cliques	237

<i>Contents</i>		ix
6.14	The lattice	244
7	Density function of the eigenvalues	247
7.1	Definitions	247
7.2	The density when $N \rightarrow \infty$	251
7.3	Examples of spectral density functions	252
7.4	Density of a sparse regular graph	257
7.5	Random matrix theory	260
8	Spectra of complex networks	271
8.1	Simple observations	271
8.2	Distribution of the Laplacian eigenvalues and of the degree	273
8.3	Functional brain network	276
8.4	Rewiring Watts-Strogatz small-world graphs	277
8.5	Assortativity	279
8.6	Reconstructability of complex networks	287
8.7	Spectral graph metrics	290
8.8	Laplacian spectrum of interdependent networks	293
8.9	Graph sparsification	296
8.10	Machine learning: Assigning labels to nodes	301
8.11	Graph neural networks	302
Part II Eigensystem		305
9	Topics in linear algebra	307
9.1	Matrix transformations	307
9.2	Vector and matrix norms	315
9.3	Formulae of determinants	319
9.4	Function of a matrix	336
10	Eigensystem of a matrix	345
10.1	Eigenvalues and eigenvectors	345
10.2	Locations of eigenvalues	355
10.3	Hermitian and real symmetric matrices	358
10.4	Recursive eigenvalue equation of a symmetric matrix	368
10.5	Interlacing	373
10.6	Non-negative matrices	379
10.7	Doubly stochastic matrices	384
10.8	Positive (semi) definiteness	387
10.9	Eigenstructure of the matrix product AB	391
10.10	Perturbation theory	394

Part III	Polynomials	399
11	Polynomials with real coefficients	401
	11.1 General properties	401
	11.2 Transforming polynomials	410
	11.3 Interpolation	415
	11.4 The Euclidean algorithm	424
	11.5 Descartes' rule of signs	432
	11.6 The number of real zeros in an interval	442
	11.7 Real zeros and the sequence of coefficients	445
	11.8 Locations of zeros in the complex plane	447
	11.9 Iterative algorithms for the zeros	455
	11.10 Zeros of complex functions	456
	11.11 Bounds on values of a polynomial	459
	11.12 Bounds for the spacing between zeros	460
	11.13 Bounds on the zeros of a polynomial	462
12	Orthogonal polynomials	467
	12.1 Definitions	467
	12.2 Properties	469
	12.3 The three-term recursion	473
	12.4 Zeros of orthogonal polynomials	479
	12.5 Gaussian quadrature	482
	12.6 The Jacobi matrix	487
	12.7 Chebyshev polynomials	494
	<i>Bibliography</i>	503
	<i>Index</i>	513