

Contents

	<i>Preface</i>	<i>page xi</i>
1	Preview	1
	1.1 The Borel construction	1
	1.2 Fiber bundles	3
	1.3 The localization package	4
	1.4 Schubert calculus and Schubert polynomials	6
2	Defining Equivariant Cohomology	9
	2.1 Principal bundles	9
	2.2 Definitions	12
	2.3 Chern classes and fundamental classes	16
	2.4 The general linear group	18
	2.5 Some other groups	20
	2.6 Projective space	23
3	Basic Properties	26
	3.1 Tori	26
	3.2 Functoriality	28
	3.3 Invariance	29
	3.4 Free and trivial actions	32
	3.5 Exact sequences	33
	3.6 Gysin homomorphisms	34
	3.7 Poincaré duality	37
4	Grassmannians and flag varieties	40
	4.1 Schur polynomials	40

vi	<i>Contents</i>	
	4.2	Flag bundles 44
	4.3	Projective space 47
	4.4	Complete flags 48
	4.5	Grassmannians and partial flag varieties 50
	4.6	Poincaré dual bases 55
	4.7	Bases and duality from subvarieties 57
5	Localization I	62
	5.1	The main localization theorem (first approach) 62
	5.2	Integration formula 69
	5.3	Equivariant formality 75
6	Conics	79
	6.1	Steiner's problem 79
	6.2	Cohomology of a blowup 80
	6.3	Complete conics 83
7	Localization II	93
	7.1	The general localization theorem 93
	7.2	Invariant curves 97
	7.3	Image of the restriction map 100
	7.4	The image theorem for nonsingular varieties 105
8	Toric Varieties	113
	8.1	Equivariant geometry of toric varieties 113
	8.2	Cohomology rings 115
	8.3	The Stanley–Reisner ring 117
	8.4	Other presentations 122
9	Schubert Calculus on Grassmannians	126
	9.1	Schubert cells and Schubert varieties 126
	9.2	Schubert classes and the Kempf–Laksov formula 128
	9.3	Tangent spaces and normal spaces 131
	9.4	Double Schur polynomials 132
	9.5	Poincaré duality 135
	9.6	Multiplication 137
	9.7	Grassmann duality 141
	9.8	Littlewood–Richardson rules 145
10	Flag Varieties and Schubert Polynomials	152
	10.1	Rank functions and Schubert varieties 152
	10.2	Neighborhoods and tangent weights 155

<i>Contents</i>		vii
10.3	Invariant curves in the flag variety	157
10.4	Bruhat order for the symmetric group	158
10.5	Opposite Schubert varieties and Poincaré duality	161
10.6	Schubert polynomials	164
10.7	Multiplying Schubert classes	169
10.8	Partial flag varieties	171
10.9	Stability	173
10.10	Properties of Schubert polynomials	175
11	Degeneracy Loci	182
11.1	The Cayley–Giambelli–Thom–Porteous formula	182
11.2	Flagged degeneracy loci	185
11.3	Irreducibility	186
11.4	The class of a degeneracy locus	189
11.5	Essential sets	193
11.6	Degeneracy loci for maps of vector bundles	194
11.7	Universal properties of Schubert polynomials	199
11.8	Further properties of Schubert polynomials	201
12	Infinite-Dimensional Flag Varieties	208
12.1	Stability revisited	208
12.2	Infinite Grassmannians and flag varieties	211
12.3	Schubert varieties and Schubert polynomials	214
12.4	Degeneracy loci	219
13	Symplectic Flag Varieties	226
13.1	Degeneracy loci for symmetric maps	226
13.2	Isotropic subspaces	228
13.3	Symplectic flag bundles	230
13.4	Lagrangian Grassmannians	232
13.5	Cohomology rings	236
14	Symplectic Schubert Polynomials	240
14.1	Schubert varieties	240
14.2	Double Q -polynomials and Lagrangian Schubert classes	249
14.3	Symplectic degeneracy loci	252
14.4	Type C Schubert polynomials	254
14.5	Properties of type C Schubert polynomials	259

15	Homogeneous Varieties	265
15.1	Linear algebraic groups	265
15.2	Flag varieties	270
15.3	Parabolic subgroups and partial flag varieties	274
15.4	Invariant curves	279
15.5	Compact groups	281
15.6	Borel presentation and equivariant line bundles	283
16	The Algebra of Divided Difference Operators	290
16.1	Push-Pull operators	290
16.2	Restriction to fixed points	294
16.3	Difference operators and line bundles	299
16.4	The right W -action	301
16.5	Left-Handed actions and operators	303
16.6	The convolution algebra	305
17	Equivariant Homology	311
17.1	Equivariant Borel–Moore homology and Chow groups	311
17.2	Segre classes	315
17.3	Localization	318
17.4	Equivariant multiplicities	320
18	Bott–Samelson Varieties and Schubert Varieties	326
18.1	Definitions, fixed points, and tangent spaces	326
18.2	Desingularizations of Schubert varieties	329
18.3	Poincaré duality and restriction to fixed points	333
18.4	A presentation for the cohomology ring	337
18.5	A restriction formula for Schubert varieties	339
18.6	Duality	343
18.7	A nonsingularity criterion	344
19	Structure Constants	350
19.1	Chevalley’s formula	350
19.2	Characterization of structure constants	353
19.3	Positivity via transversality	356
19.4	Positivity via degeneration	362
Appendix A Algebraic Topology		370
A.1	Homology and cohomology	370
A.2	Borel–Moore homology	373

<i>Contents</i>		ix
A.3	Class of a subvariety	374
A.4	Leray–Hirsch theorem	377
A.5	Chern classes	379
A.6	Gysin homomorphisms	381
A.7	The complement of a variety in affine space	382
A.8	Limits	383
<i>Appendix B</i> Specialization in Equivariant Borel–Moore Homology		388
<i>Appendix C</i> Pfaffians and Q-polynomials		392
C.1	Pfaffians	392
C.2	Schur Q -polynomials	393
C.3	Double Q -polynomials and interpolation	398
<i>Appendix D</i> Conventions for Schubert Varieties		406
D.1	Grassmannians	406
D.2	Flag varieties	410
D.3	General G/P	414
<i>Appendix E</i> Characteristic Classes and Equivariant Cohomology		415
	<i>References</i>	420
	<i>Notation Index</i>	433
	<i>Subject Index</i>	435