
Index

- ∂_x , = $x \cdot (\partial/\partial x)$, 245
 ∇ , divergence, 242
 \sim , linear equivalence, xv
 $\sim_{\mathbb{Q}}$, \mathbb{Q} -linear equivalence, xv
 $\sim_{\mathbb{R}}$, \mathbb{R} -linear equivalence, 434
 \equiv , numerical equivalence, xv
 \equiv , identity of sequences or polynomials, 187
 \otimes , tensor product for divisorial sheaves, 129
 \dashrightarrow , rational map, xiv
 \rightarrow , morphism, xiv
 $\{ \}$, fractional part, xv
 $\lfloor \rfloor, \lceil \rceil$, rounding down or up, xv
 $\widehat{}$, completion, xvii, 397
 \leq , for sequences, 187
 \leq , for polynomials, 187
- $a(, ,)$, discrepancy of divisor, 420
 $a_{\ell}(, ,) := a(, ,) + 1$, log discrepancy, 420
 \mathbb{A}_k^n or $\mathbb{A}_{\mathbf{x}}^n$, affine n -space, xiv
 $\mathbb{A}^n / \frac{1}{m}(a_1, \dots, a_n)$, quotient singularity, 27
 two-dimensional, 248
 Adjunction, 424
 inversion of, 424
 $\text{AEnv}()$, affine envelope, 435
 AFI , Alexeev–Filipazzi–Inchiostro, 320
 functor, $\mathcal{AFI}(,)$, 321
 Alexeev stable, 234
 good moduli theory, 235
 Ample
 fiber-wise, 193
 \mathbb{R} -divisor, 439
 relatively, xv
 strongly, 326
 Approximation of \mathbb{R} -divisors, 436, 437
 $\text{Ass}()$, associated points or subschemes, 370
 Asymptotic Riemann–Roch, 439
 $\text{Aut}_S()$, $\mathbf{Aut}_S()$, xvii, 337
 finite for stable families, 338
- Base change
 dualizing sheaf, 100, 102
 K -flat, 281
 notation, xvii
 pluricanonical, 112
 Bertini theorem
 C -flatness, 290
 divisorial support, 278
 flatness, 400
 generically Cartier family, 153
 hulls, 377
 husks, 355
 inverse, local stability, 64, 184
 K -flatness, 263
 local stability, 64
 relative hull, 351
 S_m , 377
 Big, divisor, xvi
 Birational
 transform, xv
 fiberwise, 11
 map, xiv
 Boundary, xvi
 Boundedness, 16
 marked pairs, 332

- set of sheaves, 360
- strong, 224
- weak, 224
- $C_a(X, L)$, affine cone, 81
- Calabi–Yau pair, xvi
- Canonical
 - algebra, 427
 - class, 17, 419
 - class, relative, 61, 133
 - divisor, 17
 - line bundle, 17
 - model, xvi, 23, 427
 - model of resolutions, xvi
 - model, existence, 428
 - model, nef slc case, 318
 - model, simult. numerical criterion, 186
 - model, simult. of resolutions, 185
 - model, simultaneous, 185
 - modification, xvi, 428
 - modification, simultaneous, 188
 - ring, 23, 26
 - ring, not finitely generated, 36
 - sheaf, 17, 61
 - sheaf, absolute, 419
 - sheaf, relative, 419
 - singularity, 24, 421
 - surface singularity, list, 68
- Canonically
 - embedded family, 333
 - polarized family, 332
- Cartier
 - divisor, 146
 - divisor, relative, 150
 - divisor, valuative criterion, 156
 - generically \sim pull-back, 140, 154
 - generically \sim , relative, 152
 - index, xv
 - non- \sim locus, 152
- Categorical
 - moduli as \sim quotient, 334, 335
 - quotient by group action, 334
- Cayley–Chow
 - correspondence, 176
 - correspondence, Mumford divisors, 176
 - correspondence, over fields, 171
 - family, 120
 - form, 171
 - hypersurface, 171, 284
 - hypersurface, flag, Grassmann, incidence, product versions, 284, 285
 - inverse, scheme-theoretic, 177
 - type hypersurface, 172
- $C^m\mathcal{E}^s\mathcal{M}\mathcal{S}ch()$, $C^m\mathcal{E}^s\mathcal{M}\mathcal{S}ch()$, functor and moduli of canonically embedded, marked schemes, 333
- Center
 - $\log \sim$, 422
 - \log canonical \sim , 422
 - of a divisor, xv
- $C^m\mathcal{E}\mathcal{S}\mathcal{P}()$, $C^m\mathcal{E}\mathcal{S}\mathcal{P}()$, functor and moduli of m -canonically embedded, stable pairs, 222, 333, 337
- C-flat, 281
 - Bertini theorem, 290
 - locally \sim , 288
 - stably \sim , 281
- $\text{Ch}()$, Cayley–Chow hypersurface, 171
- Ch_{gr} , Ch_{in} , Ch_{fl} , Ch_{pr} , versions of Cayley–Chow hypersurface, 284
- $\text{Ch}_{sch}^{-1}()$, Cayley–Chow inverse, 177
- Chow
 - equations, ideal of, 173
 - hull of Mumford divisor, 175
 - hull, of a cycle, 174
 - variety, 5
- CM, Cohen–Macaulay, xv, 372
- $\text{coeff}()$, set of coefficients, 418
- $\text{coeff}_E()$, coefficient of E in, 418
- Coefficient
 - of a prime divisor, 418
 - vector of marking, 310
- floating, 317
- generic, 216
- major, 216
- standard, 216
- Cohen–Macaulay, CM, xv, 372
- Completion, $\widehat{}$, xvii, 397
- Component-wise dominant, 155
- Cone
 - affine, 81
 - deformation to, 39, 82
- Continuous choice, 221
- Contraction, xiv
 - crepant, 318
 - simultaneous, 323
 - simultaneous, crepant, 323
- $\text{CoSp}()$, \mathbb{Q} -vector space spanned by coefficients, 434
- $C^m\mathcal{P}^s\mathcal{M}\mathcal{S}ch()$, functor of canonically polarized, marked schemes, 333
- Crepant, contraction, 318

- Curve
stable, 10
stable extension, 11
Curvilinear scheme, 402
- Cycle
degree of, 171
effective, 170
fundamental \sim , [], 170
geometrically reduced, 171
on a scheme, 170
width of, 174
Cyclic cover, 426
- $\Delta^{>c}$, $\Delta^=c$, $\Delta^{<c}$, 418
 Δ_c or Δ_c^{div} , divisorial fiber, 108
 \mathbb{D} , unit disc, 32
- Decomposition
locally closed, 415
partial, 415
- Deformation, 237
 \mathbb{W} - \sim , 247
 \lesssim - \sim , 247
hypersurface singularities, 76
KSB- \sim , 247
locally trivial, 237
nonalgebraic, 31
of quotients, 73
- Demi-normal, 430
open condition, 390
- Depth, 371
along a subscheme, 371
and flatness, 374
and push forward, 372
of a sheaf, 371
of slc scheme, 425
semicontinuity, 380
- Descent, 338
and functorial polarization, 339
for flat, projective morphism, 340
for rigid, projective morphism, 341
- Dévissage, 381
- Diff(), different, 423
- Different, 423
properties of \sim , 424
- Differentiation, 238
cohomological, 238
- Discrepancy
log \sim , 420
of a divisor, 420
- Discrete choice, 220
- Divergence, ∇ , 242
- Divisor
big, xvi
canonical, 17
Cartier, 146
Cartier, relative, 150
Cartier, valuative criterion, 156
generically \mathbb{Q} - or \mathbb{R} -Cartier, relative, 141
generically Cartier, 146
generically Cartier, relative, 152
Mumford, xv, 146
Mumford, relative, 175
Mumford, universal family, 179
on a scheme, xv
over a scheme, xv
 \mathbb{Q} - \sim , 147
 \mathbb{R} - \sim , 147
reduced, xv
Weil, 147
- Divisorial
fiber, Δ_c or Δ_c^{div} , 108
log terminal, 421
pull-back, 141
restriction, D_t or D_t^{div} , xvii, 106
sheaf, 129
sheaf, flat family, 129
sheaf, generically flat, 130
sheaf, mostly flat, 130
sheaf, valuative criterion, 155
subscheme, 146
subscheme, family of \sim , 149
support and Fitting ideal, 275
support, $\text{DSupp}(\)$, 274
support, Bertini theorem, 278
support, final definition, 276
- dlt, divisorial log terminal, 421
is CM, 425
- $\text{DSupp}(\)$, divisorial support, 274
- Du Bois singularity, 97
cohomology and base change, 98
- Du Val singularity, 68
- Dual graph, 67
- Dualizing
sheaf, 17, 61
sheaf, base change, 102
sheaf, construction, 101
sheaf, other deformations, 65
sheaf, relative, 419
- Elementary étale, 66
- $\text{emb}(\)$, embedded subsheaf, 370
- Embedded point, 370

- Embedding, locally closed, 414
 $\mathcal{E}^s\mathcal{M}\mathcal{S}ch(\)$, $\mathcal{E}^s\mathcal{M}\mathcal{S}ch(\)$ functor and moduli of
 embedded, marked schemes, 331
 Enough one-parameter families, 416
 Envelope, affine or linear, 435
 Etale, elementary, 66
 $\text{Ex}(\)$, exceptional set, xv

 F^{**} , reflexive hull, 348
 $F^{[*]}$, hull or S_2 -hull, 348
 F^H , relative hull, 350
 Family
 algebraic, 120
 canonically embedded, 333
 canonically polarized, 332
 Cartier, normal base, 141
 Cayley–Chow, 120
 divisorial sheaves, flat, 129
 divisorial sheaves, mostly flat, 130
 divisorial subschemes, 149
 generically Cartier divisors, 152
 Hilbert–Grothendieck, 121
 locally stable, 59, 118
 marked pairs, 311
 mostly flat of line bundles, 193
 non-projective, 22
 of polarized schemes, 327
 one-parameter, 59
 pairs, 59, 138
 polarized with K-flat divisors, 330
 stable, 89
 stable over smooth base, 167
 stable, extension of, 169
 universal, 121
 varieties, 58
 well-defined, 140
 well-defined, reduced base, 141
 Fano pair, xvi
 Fiber, divisorial, Δ_c or Δ_c^{div} , 108
 Fiber-wise ample, 193
 Field of moduli, 52
 hyperelliptic curve, 53
 Fine moduli space, 7
 universal family, 341
 Fitting ideal, 273
 Flat
 family, divisorial sheaves, 129
 generically, 130
 mostly, 130
 FlatCM($\)$, flat and CM locus, 276

 Flatness
 associated points, 398
 Bertini theorem, 400
 curvilinear fibers, 403
 Hironaka’s theorem, 409
 is open, 374
 nodal fibers, 406
 relative codimension ≥ 3 , 410
 relative codimension 0, 199, 401
 relative codimension 1, 405
 relative codimension 2, 408
 residue field extension, 400
 with reduced fibers, 198
 Flattening decomposition, 127
 Floating coefficient, 317
 Formally K-flat, 281
 Framing, projective, 329
 Free group action, 335
 Frobenius power, $l^{[q]}$, 173
 Full subscheme, 294
 Functorial polarization, 339
 Fundamental cycle [$\]$, 170

 General type, xiv, 23
 Generically
 Cartier divisor, 146
 flat, 130
 flat and pure, 276
 \mathbb{Q} - or \mathbb{R} -Cartier divisor, relative, 141
 Genus, sectional, 123
 Geometric quotient
 by free group action, 337
 by group action, 336
 existence, 336
 Geometrically injective, 414
 Grothendieck–Lefschetz theorem, 116

 Henselisation, 66
 strict, 67
 Hilbert
 function, of ω_X , 132
 function, of divisorial sheaf, 132
 functor, 6
 \sim -Grothendieck family, 121
 scheme, 6
 \sim -to-Chow map, 124
 Hilb($\)$, Hilbert scheme, 122
 Hilb^{str} , strongly embedded part of Hilb, 329
 $\text{Hom}_X(\ , \)$, $\mathcal{H}om_X(\ , \)$, $\mathbf{Hom}_S(\ , \)$, xvii, 360
 Homeomorphism, universal, 412

- $\mathcal{H}ull(\)$, $Hull(\)$, functor and moduli space of
 universal hulls, 363
- Hull, 130
 $S_2 \sim$, 348
 algebraic spaces, 368
 Bertini theorem, 351, 377
 of a sheaf, 348
 pull-back, 130
 pure, 348
 reflexive, 348
 relative, 350
 universal, 353
 universal, characterization, 353
 universal, fine moduli space, 363
- Hurwitz formula, 425
- $\mathcal{H}usk(\)$, $Husk(\)$, functor and moduli of husks,
 357
- Husk, 355
 algebraic spaces, 368
 Bertini theorem, 355
 quotient, 355
 quotient, relative, 356
 relative, 356
 tight, 355
- Hypersurface
 Cayley–Chow, 171, 284
 Cayley–Chow, flag, Grassmann, incidence,
 product versions, 284, 285
 K-flatness, 295
- $\mathcal{I}(\ , \)$, intersection form, 321
- $\mathcal{I}(\ , \)$ intersection numbers, 187
- $\mathcal{I}^{[q]}$, Frobenius power, 173
- Index
 Cartier \sim , xv
 of ω_X , 247
 of a variety, xv
- Intersection number, xv
- Inversion of adjunction, 424
- $\text{Isom}_S(\ , \)$, $\text{Isom}_S(\ , \)$, xvii, 337
 finiteness of, 90, 338
- Isotrivial family, 52
- K_X , canonical class, 17, 419
- $K_{X/C}$, relative canonical class, 61
- $\kappa(X)$, Kodaira dimension, xiv, 23
- $\mathcal{K}Div(\)$, $KDiv(\)$, functor and moduli of
 K-flat divisors, 261
- K-flat, 259
 additive, 262
 base change, 281
- Bertini theorem, 263
 equals stable C-flat, 283
 family, polarized, 330
 flat implies \sim , 262
 formal nature of, 283
 formally \sim , 281
 functor of \sim pull-backs, 293
 hypersurface singularities, 295
 implies C-flat, 282
 linear equivalence, 262
 locally \sim , 281
 multiplicative, 262
 over reduced base, 262
 push forward of, 262
 reasons for definition, 260
 seminormal curves, 301
- Kodaira dimension, xiv, 23
 jump of, 29–31
- Kodaira lemma, 440
- KSB, Kollár–Shepherd-Barron, 229
 good moduli theory, 230–232
 stable, 229, 230
 stable, major coefficients, 232
- KSB-deformation, 247
- KSBA, Kollár–Shepherd-Barron–Alexeev
 good moduli theory, 308, 313, 315, 316
 stable strong form, 316
 stable, general coefficients, 315
 stable, rational coefficients, 313
- $L^{[m]}$, reflexive power of divisorial sheaf, 129
- lc, log canonical, 421
 center, 422
- $L\text{Env}(\)$, linear envelope, 435
- $L\text{Env}_{\mathbb{Z}}(\)$, integral points of $L\text{Env}$, 314
- Lexicographic order, \leq , 187
- Lie derivative, 239
- Linear
 equivalence, \sim , xv
 \mathbb{Q} - \sim equivalence, $\sim_{\mathbb{Q}}$, xv
 \mathbb{R} - \sim equivalence, $\sim_{\mathbb{R}}$, 434
 system, 4
- Link, 158
- Local
 morphism, 155
 numerical criterion of \sim stability, 184
 Picard group, 116
 stability, Bertini, 64
 stability, reduced base, 118
 stability, representable, reduced base, 143

- Locally
 C-flat, 288
 closed decomposition, 415
 closed embedding, 414
 closed partial decomposition, 415
 K-flat, 281
 stable, 28
 stable morphism, 59, 118
 stable pair, 58
 stable, equivalent conditions for, 142
 stable, KSB version, 135
 stable, reduced base, 118
- Locus
 flat or flat-CM, 276
 non-Cartier, 152
- Log big, 429
- Log canonical, 27
 lc, 421
 center, 422
 modification, 428
- Log center, 422
 mld bounds, 422
- Log discrepancy, 420
- Log resolution, xvi
- Major coefficient, 216
- Map, rational, xiv
 birational, xiv
- Marked
 family of \sim pairs, 311
 pair, with divisors, 310
 reasons for, 309
- Marking, of pair or family, 310, 311
- Matsusaka inequality, 440
- $M\mathcal{D}iv(\)$, $M\mathcal{D}iv(\)$, functor and moduli of
 Mumford divisors, 175
- Minimal log discrepancy, mld, 421
 mld, minimal log discrepancy, 421
- Model, canonical, xvi, 23
- Modification
 canonical, xvi, 428
 finite, 410
 log canonical, 428
 semi-log-canonical, slc, 211
 simultaneous, canonical, 188
- Moduli
 boundary, 217
 embedded pairs, 221
 embedded varieties, 221
 enough one-parameter families, 416
 field of, 52
 interior, 217
 representable, 13
 separated, 15
- Moduli space
 categorical, 8
 categorical quotient, 334, 335
 coarse, 8
 fine, 7
 fine for universal hull, 363
 genus 2 curves, 45
 husks, 357
 hypersurfaces, 38, 40
 irreducible components proper, 169
 KSBA, exists, 308
 non-separated, 18, 21, 41, 48
 projectivity of, 225
 quotient by group action, 333
 quotient husks, 357
 reduced version, 138
 stable varieties, 120
- Moduli theory
 Alexeev, 235
 good, 226
 KSB, 230
 KSB, standard coeffs, 231
 KSBA, rational coeffs, 313
 KSBA, real coeffs, 315
 KSBA, strong form, 316
 V^+ , 234
- Monomorphism, 414
- $\text{Mor}_S(\ , \)$, $\mathbf{Mor}_S(\ , \)$, xvii, 337
- Morphism, xiv
 dominant, 155
 Hilbert-to-Chow, 124
 locally stable, 59
 locally stable, reduced base, 118
 pure dimensional, 104
 scheme, \mathbf{Mor} , 337
 small, xv
 stable, 13
- Morse lemma, 390
- Mostly flat, 130
 divisorial sheaf, 130
 family of line bundles, 193
 S_2 sheaf, 130
- Mumford divisor, xv, 146
 along subscheme, 146
 Cayley–Chow correspondence, 176
 Chow hull of, 175
 flat, 227
 functor and moduli, 175

- relative, 175
- relative class group, 265
- universal family, 179
- Nagata openness criterion, 375
- Nakai–Moishezon criterion, 441
- Nef, xv
- Negativity lemma, 444
- Node, 430
 - deformation of, 407
- Noether normalization
 - étale version, 395
 - fails for affine morphism, 394
 - local version, 396
- Norm, 117
- Normal, 411
 - crossing, simple, xvi
 - pair, 411
- Numerical
 - criterion for relative line bundles, 184
 - criterion of local stability, 184
 - criterion of stability, 181, 183
 - criterion of simult. canonical model, 186
 - equivalence, \equiv , xv
 - pull-back, 442, 443
- Numerically
 - log canonical, 163
 - polarized, 328
 - \mathbb{Q} -Cartier, 163
 - \mathbb{R} -Cartier, 163
 - \mathbb{R} -Cartier, lc modification of, 428
 - relatively trivial, 163
 - semi-log-canonical, 163
 - slc is slc, 164
- $\omega_{X/S}$, relative dualizing sheaf, 419
- ω_X , canonical or dualizing sheaf, 17
- $\omega_X^{[m]}$, reflexive power of canonical sheaf, 18
- Obstruction theory, 218
- \mathbb{P}_k^n or \mathbb{P}_k^n , projective n -space, xiv
- Pair, xvi, 418
 - Calabi–Yau, xvi
 - family of, 59, 138
 - family, marked, 311
 - Fano, xvi
 - locally stable, 58
 - marked with divisors, 310
 - normal, 411
 - rigid and universal family, 341
 - seminormal, 411
 - stable, 58
 - weakly normal, 411
 - well-defined family, 140
- Partial decomposition, 415
- PGL, group scheme, 333
- $\text{Pic}()$, $\mathbf{Pic}()$, Picard group and scheme, xvii
- $\text{Pic}^{\text{loc}}()$, $\mathbf{Pic}^{\text{loc}}()$, local Picard group and scheme, 116, 164
- Picard group, xvii
 - for smooth morphisms, 159
 - local, 116, 164
- $\mathcal{P}^s\mathcal{MSch}()$, functor of polarized, marked schemes, 331
 - stack version, 332
- Poincaré residue map, 423
- Pointed scheme, 330
- Polarization
 - family of schemes, 327
 - functorial, 339
 - numerical, 328
 - scheme, 327
 - strong, 327
- Potentially, slc or ..., 99
- Pre-polarization, 327
- Preserve residue fields, 414
- Projection
 - approximation of, 279
 - various versions, 280
- Projective
 - framing, 329
 - moduli space, 225
- Proper
 - group action, 335
 - valuative-~, 15
- $\mathcal{P}^s\mathcal{Sch}()$, functor of polarized schemes, 327
 - étale sheafification of, 328
 - stack version, 330
- Pull-back
 - C-flat, 294
 - Cartier, 154
 - divisorial, 141
 - generically Cartier, 140
 - hull-~, 130
 - K-flat, 293
 - locally stable, representable, 160
 - numerical, 442, 443
 - \mathbb{Q} - or \mathbb{R} -Cartier divisors, 141
 - stable, representable, 160
 - Weil-divisor, 139
- Pure
 - quotient, 147, 371

- relatively, 365
- scheme, 371
- sheaf, 370
- vertically, 352
- Purely log terminal, plt, 421
- $q^{(*)}(D)$, pull-back (generically Cartier), 140
- $q_{\text{Wdiv}}^*(D)$, Weil-divisor pull-back, 139
- \mathbb{Q} -Cartier divisor, xv
 - valuative criterion, 156
- \mathbb{Q} -divisor, xv, 147
- $QHusk()$, $QHusk()$, functor and moduli of quotient husks, 357
- Quasi-étale, 62
- $Quot()$, $Quot()$, functor and scheme, 359
- Quot-scheme, 359
- Quotient
 - categorical, by group action, 334
 - geometric, by free group action, 337
 - geometric, by group action, 336
 - geometric, existence, 336
 - husk, 355
 - singularity, 27
- $R(\ , \)$, canonical ring, 23, 26
- \mathcal{R} , restriction or Poincaré residue map, 423
- Rational
 - double point, 68
 - map, xiv
 - singularity, xv
- \mathbb{R} -Cartier divisor, 434
 - class, 440
 - valuative criterion, 156
- \mathbb{R} -divisor, xv, 147, 434
 - ample, 439
 - convex approximation, 436, 437
 - depth of pluricanonical sheaf, 438
- \mathbb{R} -line bundle, 440
- Reduced
 - normal form, 230
 - divisor, xv
- Regular sequence, 371
- Relatively
 - ample, xv
 - generically Cartier, 141
 - isomorphic, 364
 - pure, 365
- Representable
 - C-flat pull-back, 294
 - Cartier pull-back, 154
 - flat, divisorial pull-back, 154
 - flatness, 127
 - functor, 126
 - hull of divisorial sheaves, 131
 - invertible hull of sheaves, 131
 - K-flatness, 261, 293
 - local stability, 119, 160
 - local stability, reduced base, 143
 - moduli theory, 13
 - pull-backs, 160
 - stability, 119, 160
 - stability, over reduced base, 143
- Residue map, 423
- Resolution, xv
 - dual graph of \sim , 67
 - $\log \sim$, xvi
 - of Du Val singularities, 68
- Restriction
 - divisorial, D_t or D_t^{div} , xvii, 106
 - map, 107, 353
- Riemann-Roch, asymptotic, 439
- Rigid, scheme or pair, 337
 - and universal family, 341
- Ring, canonical, 23, 26
- S_2 , Serre's condition, 371
 - divisorial sheaf, 129
 - family of varieties, 118
 - for families, 107
 - for restriction, 106
 - Hilbert-to-Chow map, 124
 - mostly flat, 130
- Semi-log-canonical, slc, 28, 431
 - and depth, 425
 - modification, 211
- Seminormal, 411
 - K-flatness for \sim curves, 301
 - pair, 411
- Separatedness
 - for husks, 358
 - for stable maps, 90, 338
 - moduli spaces, 15
 - valuative criterion, 15
- Serre's condition S_m , 371
 - along a subset, 105
- Seshadri criterion, 441
- Sheaf
 - canonical, 17
 - divisorial, 129
 - dualizing, 17
- Simple normal crossing, snc, xvi

- Singularity
 canonical, 24, 421
 cyclic quotient $\mathbb{A}^2/\frac{1}{n}(1, q)$, 27, 248
 dlt, 421
 Du Bois, 97
 Du Val, 68
 klt, 421
 lc, log canonical, 27, 421
 log terminal, 421
 plt, 421
 potentially slc or ..., 99
 quotient, 27
 slc, semi-log-canonical, 28
 terminal, 421
 slc, semi-log-canonical, 28, 431
 characterization using normalization, 431
 depth, 425
 potentially \sim , 99
 S_m , Serre's condition, 371
 along a subset, 105
 Bertini theorem, 377
 is open, 374
 Small morphism, xv
 snc, simple normal crossing, xvi
 $\mathcal{SP}()$, $\text{SP}()$, functor and moduli of stable,
 marked pairs, 137, 223, 308
 $\mathcal{SP}^{\text{rigid}}()$, $\text{SP}^{\text{rigid}}()$, functor and moduli of
 rigid pairs, 341
 SSupp, scheme-theoretic support, 274
 Stability
 automatic in codimension ≥ 3 , 62, 115, 184
 local, representable, 119
 numerical criterion, 181, 183
 representable, 119
 representable over reduced base, 143
 Stabilization functor, 34, 35
 over nodal curves, 36
 Stable
 Alexeev \sim , 234
 Alexeev–Filipazzi–Inchiostro, 320
 curve, 10
 equivalent conditions for, 142
 extension for curves, 11
 extension, weak, 96
 family over smooth base, 167
 family, 1 parameter, 89
 family, extension of, 169
 KSB, 135, 229, 230
 KSB, major coefficients, 232
 KSBA, general coefficients, 315
 KSBA, rational coefficients, 313
 KSBA, strong form, 316
 locally \sim , 28
 morphism, 13
 one parameter family, 59
 pair, 58
 $V^+ \sim$, 233
 variety, 28
 Stably C-flat, 281
 equals K-flat, 283
 independence of embedding, 293
 Standard coefficient, 216
 Stratum, of an snc pair, 422
 Strongly ample, 326
 Subscheme, divisorial, 146
 Support
 divisorial, 274
 divisorial, final definition, 276
 scheme-theoretic, 274
 $\mathcal{SV}()$, $\text{SV}()$, functor and moduli of stable
 varieties, 120
 Tight husk, 355
 tors(), torsion subsheaf, 370
 Trace map, 101
 Transform, birational, xv
 Tree, 67
 Twig, 67
 Univ(), universal family, 121
 Universal
 family, 121
 family for rigid pairs, 341
 family of flat Mumford divisors, 227
 homeomorphism, 412
 hull, 353
 hull, characterization, 353
 Universally flat, 353
 V^+ , strict Viehweg stability, 233
 V-deformation, 247
 Valuative criterion
 for \mathbb{Q} -Cartier divisor, 156
 for flat, divisorial sheaf, 155
 for relative Cartier divisor, 156
 locally closed embedding, 415
 morphism, 413
 section, 413
 Valuative-proper, 15
 stable map, 90
 Vanishing theorem
 Ambro–Fujino version, 429
 Fujita version, 440

- Variety, xiv
 - general type, xiv, 23
 - stable, 28
- Vertically pure, `vpure()`, 352
- Volume, 16, 385
 - and push-forward, 385
 - birational models, 387, 388
 - finite maps, 388
 - perturbations, 389
- `vpure()`, vertically pure, 352
- W-deformation, 247
- Weakly normal, 411
 - pair, 411
- Weil divisor, 147
 - pull-back, 139
 - relative, 148
- Well-defined
 - family, 140
 - family, reduced base, 141
- Width of a cycle, 174
- \mathbb{Z} -divisor, xv