

Contents

<i>Preface</i>	<i>page vii</i>
Part I Getting Started	
1 Introducing Quantum Groups	3
1.1 The Graph Isomorphism Game	3
1.2 The Quantum Permutation Algebra	7
1.3 Compact Matrix Quantum Groups	20
2 Representation Theory	32
2.1 Finite-Dimensional Representations	32
2.2 Interlude: Invariant Theory	52
Part II Partitions Enter the Picture	
3 Partition Quantum Groups	61
3.1 Linear Maps Associated to Partitions	61
3.2 Operations on Partitions	64
3.3 Tannaka–Krein Reconstruction	67
3.4 Examples of Partition Quantum Groups	75
4 The Representation Theory of Partition Quantum Groups	91
4.1 Projective Partitions	91
4.2 From Partitions to Representations	99
4.3 Examples	110
5 Measurable and Topological Aspects	115
5.1 Some Concepts from Non-commutative Geometry	115
5.2 The Quantum Haar Measure	123
5.3 A Glimpse of Non-commutative Probability Theory	132

Part III Further Examples and Applications		
6	A Unitary Excursion	149
6.1	The Classification of Categories of Non-crossing Partitions	149
6.2	Coloured Partitions	158
6.3	The Quantum Unitary Group	162
6.4	Making Things Complex	167
7	Further Examples	183
7.1	Quantum Reflection Groups	183
7.2	Quantum Automorphism Groups of Graphs	194
8	Back to the Game	221
8.1	Perfect Quantum Strategies	221
8.2	Finite-Dimensional Strategies	241
<i>Appendix A</i>	Two Theorems on Complex Matrix Algebras	264
<i>Appendix B</i>	Classical Compact Matrix Groups	267
<i>Appendix C</i>	General Compact Quantum Groups	271
	<i>References</i>	282
	<i>Index</i>	286