

Contents

	<i>page</i>
<i>Preface</i>	ix
<i>Acknowledgements</i>	xii
<i>Nomenclature</i>	xiii
1 An Invitation	1
1.1 Origins	2
1.2 Dramatis Personæ	5
1.3 Navigating This Book	5
2 Algebraic Preliminaries	7
2.1 Vector Spaces	7
2.2 Polytopes	9
2.3 Groups	11
2.4 Matrices and Groups	20
2.5 Equivalence Relations and Conjugacy Classes	21
2.6 Normal Subgroups and Homomorphisms	26
2.7 Representations	27
Exercises	36
3 ADE Classifications	40
3.1 Polytopes	40
3.2 Root Systems and Polyhedral Groups	44
3.3 Graphs and Spectra	62
3.4 Binary Polyhedral Groups and SU(2)	69
3.5 Lie Groups and Algebras	83
Exercises	89
4 ADE Correspondences	93
4.1 Trinity Revisited	93
4.2 The Platonic Symmetries	95
4.3 ADE and Affine ADE	95

4.4	3D to 4D Root Systems	99
4.5	The McKay Correspondence	109
4.6	A Trinity of Correspondences?	115
	Exercises	120
5	Advanced Miscellany	122
5.1	Monstrous Connections	122
5.2	The Triangle Property	131
5.3	Graphs with Least Eigenvalue -2 or Greater	137
5.4	Quiver Representations	142
5.5	Cluster Algebras	145
5.6	von Neumann Algebras and Subfactors	148
5.7	Catastrophes: Arnold Singularities	149
5.8	Calabi–Yau: du Val Singularities	152
5.9	Elliptic Fibrations	158
5.10	Back to Arnold’s Trinities	160
5.11	Summarising Outlook: A Rogues’ Gallery	163
	Exercises	163
	<i>References</i>	165
	<i>Index</i>	175