1 Introduction

In this Element, I will argue that Plato was *not* a mathematical Platonist.[1] My arguments will be based primarily on the evidence found in the *Republic*'s Divided Line analogy and in Book 7.[2] I will present Plato's view as it develops in the text, which, while perhaps not as reader-friendly as one might like,[3] emphasizes the significant changes that Plato intends to be surprising, even shocking, to his reader – changes that are often missed in the interpretative literature. First, I will offer what I take to be an accurate translation of the text,[4] before critically considering which claims remain the same and which change, especially as we transition from the Divided Line to Book 7. Finally, I will bring these claims together into a consistent picture of Plato's view of mathematics, demonstrating that he was *not* a mathematical Platonist.

Typically,[5] the mathematical Platonist story is told on the basis of three realist components: (a) that mathematical objects, as Platonic forms, exist independently of us in the metaphysical realm of forms; (b) that the way things are in this metaphysical realm fixes the truth of mathematical statements; and (c) that we come to know such truths by, somehow or other, "recollecting" the way things are in the metaphysical realm.[6] This Platonist story, by confusing the hypothetical method of mathematics with the dialectical method of philosophy, conflates the two types of realism at play in Plato: *methodological realism* and *metaphysical realism*. My aim is to show that while Plato is a *philosophical Platonist* – that is, he adopts metaphysical realism for philosophical inquiry – he is a *mathematical as-ifist* – that is, he adopts methodological realism for mathematical inquiry. Thus, it is by keeping these methods distinct that we will see that, as regards (c), we come to know mathematical objects by treating our hypotheses

[1] Just as Whitehead (1929, p. 39), claimed that the history of philosophy consists of a series of footnotes to Plato, this Element, for the most part, consists of a series of footnotes (literal and figurative) to Burnyeat (2000); but, as we will see, with important differences. Most significant among these is that Burnyeat holds that Plato leaves open the question of the existence of mathematical objects. I disagree. Plato is clear: mathematicals, or as I will call them mathematical objects, are *not* forms.

[2] As we will see, there are also assertions found in the *Meno* and the *Theaetetus* that further witness my arguments, but my primary focus is the *Republic*.

[3] To provide the reader with a consistently flowing interpretation that also follows the order of Plato's arguments, I have opted to place some of the critical discussions and analyses of the interpretative literature in footnotes.

[4] All translations are from Reeve (2004), unless otherwise indicated.

[5] A notable exception to the standard story is found in the historically rich and philosophically robust book by Panza and Sereni (2013).

[6] I have demonstrated that recollection in the *Meno* is *not* offered as a *method* for mathematical knowledge (Landry 2012). As we will see, what is offered as the mathematician's method for attaining knowledge, in both the *Meno* and the *Republic*, is the *hypothetical method*.

as if they were true first principles for the purpose of using these to solve mathematical problems.

As regards (a), I will show that mathematical objects depend on the mathematical problem that we are attempting to solve. It is the problem at hand that gives rise to the needed hypotheses that themselves are taken *as if* they were true, and it is these hypotheses that give rise to the needed objects of thought that we take *as if* they exist for the purpose of solving the problem. Mathematical objects, then, exist in a methodological sense but *not* in a metaphysical sense. Against (b), what fixes the truth of a mathematical statement is its method, not its metaphysics – that is, mathematical truth is fixed by a demonstration that shows that the answer to our problem can be deduced from our hypothesis; it is not fixed by the way things are in the metaphysical realm of forms. As we will see, in mathematics, existence is a consequence of truth – that is, is a consequence of taking our hypotheses *as if* they were true for the purpose of solving a problem. In philosophy, by contrast, truth is a consequence of existence, that is, is a consequence of our tethering our hypotheses to independently existing forms. It is these considerations, which arise by keeping distinct the mathematician's and the philosopher's methods, that allow us to see that Plato was *not* a mathematical metaphysical realist; rather, he was a mathematical *methodological realist*.

My aim is to argue that since the *method* used by the mathematician is distinct from that of the philosopher, then so too must be their *objects*. From a methodological standpoint, I will show that the mathematician uses the *hypothetical method* and travels downward from a hypothesis, taken *as if* it were a true first principle, toward a conclusion. The philosopher, on the other hand, uses the *dialectical method* to first travel upward from a hypothesis, taken as a hypothesis, toward a first principle, the truth of which is fixed by a form, and they then travels downward from a form-tethered or true first principle to a conclusion. I will further show that, as a result of these methodological differences, the mathematician, in their goal of solving mathematical problems, needs only take their objects *as if* they exist. This is why, now from an epistemological standpoint, mathematical objects are to be taken as objects of *thought*, whereas philosophical objects are to be taken as objects of *understanding* (or, at the end of Book 7, as objects of *knowledge*). Bringing these two standpoints together, I will argue that mathematical objects, as things that arise from "images," or from drawn or constructed diagrams, are nonetheless to be taken as distinct from such "images" and so are to be taken as "things themselves." However, even as "things themselves," mathematical objects are distinct from "forms themselves"; they are *methodologically real* – that is, we treat them *as if* they exist to solve a mathematical problem – but they are *not* metaphysically real.

Indeed, this is why, at the end of Book 7, Plato likens the faculty of thought to that of imagination and, as a consequence, comes to reserve the term "knowledge" for philosophical knowledge *only.* Thus, taking my evidence primarily from the Divided Line analogy and Book 7, I will argue that Plato was not a mathematical Platonist; mathematical objects are not forms, they do not either exist in some metaphysical realm or fix the truth of mathematical statements, and we do not come to know them via recollection.

2 The Interpretive Lay of the Land

The number of interpretations of Plato's views of mathematics is vast. Some consider the whole of Plato's works, others focus on specific dialogues. My interpretation will focus primarily on what Plato says in the *Republic*'s Divided Line and Book 7. The reason for this is twofold; except for the *Meno*, these are the only places where Plato presents a sustained account of mathematics, and there seems little debate that this dialogue was written by Plato.[7] In a broad stroke, my interpretation is intended to cut a midpoint between the two prevailing and competing views. The first is the view of Cornford (1932), White (1976), Tait (2002), and Benson (2006; 2010; 2012) that the hypothetical method is *part of* the dialectical method so that mathematical objects must, in some sense, be *part of* the realm of forms. The second is the view of Burnyeat (2000) and Broadie (2020) that the mathematician's hypothetical method is *distinct from* the philosopher's dialectical method, but that Plato adopts a quietist stance on the ontological status of mathematical objects – that is, on the question of whether mathematical objects are to be taken as *distinct from* forms.

Benson's *part of* view has a long history and is well captured by Cornford's argument that Plato has two types of dialectic at play, each with its own methodology: one mathematical and having as its objects mathematical forms, the other philosophical, or ethical, and having as its objects forms like Justice, Virtue, and Good. Likewise, Benson (2012) sees both types as part of the same method, but further distinguishes between the mathematician's *dianoetic* method and the philosopher's *dialectic* method, arguing that

> the distinction is less a distinction between two different methods, than one between *two different applications* of the *same method*. Both the dianoetician and the dialectician apply or use the method of hypothesis, but the former does so *inadequately and incorrectly.* The dianoetician [as exemplified by

[7] I have analyzed what Plato tells us of the benefits and limits of the mathematician's method in the *Meno* (Landry 2012). While there is a discussion of mathematics in the *Seventh Letter*, it is far from clear whether this work is Plato's.

> "current practitioners" of mathematics], unlike the dialectician, … *mistakes* her hypothesis for *archai* [for unhypothetical first principles].
>
> (pp. 1–2; italics added)

Most *part of* interpreters hold that these unhypothetical first principles are unhypothetical because they are tethered to, or fixed by, a stable *metaphysical* domain (i.e., by a realm of mathematical objects taken as *philosophical* forms, or, like Tait, by a realm of foundational mathematical objects taken as *geometric* forms).

Burnyeat (2000), in contrast, uses his *distinct from* interpretation to point to two stances that one may adopt as regards the ontological status of mathematical objects: the "internal" stance taken by practicing mathematicians and the "external" metaphysical stance taken by the philosopher of mathematics. He remains oddly silent on what the practicing mathematician's internal stance comes to but, as regards the latter, holds that Plato "leaves the external question tantalisingly open" (p. 22). Likewise, Broadie (2020) holds that "Plato shows no interest in this metaphysical question" (p. 15).

Benson (2000) similarly holds that "Plato is less concerned to offer a fourfold ontology associated with the four sections of the Line, than he is to describe the correct method of the greatest *mathēma* – the knowledge of the Form of the Good" (p. 1). But, as we have noted, Benson, as many other interpreters who believe that the philosopher's method *must* be adopted by the mathematician, holds that this external question *must* be answered. The various, what I will call, *metaphysical* interpretations that seek to answer this external question agree that one must adopt a mathematical Platonist position but bifurcate over whether this should be answered at a philosophical or at a metamathematical level – that is, whether one must adopt the view that mathematical objects are philosophical forms themselves or are to be founded on an ontologically preferred metamathematical theory of forms, such as foundational theory of geometric forms.

Another option, however, is to argue that mathematical objects are "intermediates" between philosophical forms and sensible objects. Indeed, forgoing his internal/external distinction for the moment, Burnyeat's (2000) position seems to purposefully leave open the possibility of an interpretation of mathematical objects as intermediates:[8]

[8] As too does Broadie: "Plato also postulates two correspondingly different levels of intelligible reality, the forms proper and the distinct 'intermediate' or mathematicals which we know from Aristotle came to be posited in Plato's school" (p. 15). McLarty (2005) also argues for an "intermediates" position: "Glaucon in Plato's *Republic* fails to grasp intermediates. He confused pursuing a goal (of searching for first principles) with achieving it, and so he (mistakenly) adopts 'mathematical platonism'" (p. 115). See also Foley's (2008) article, for an illuminating discussion of how the ratios and the proportions of the line can be used to partition debates about the

> That is the main result of the Divided Line passage (511c–d): the introduction of a new intermediate epistemic state, which turns out to have an intermediate degree of clarity when it is compared, on the one side with the ordinary person's opinion about sensibles, and on the other with the dialectician's understanding of Forms. Socrates can then correlate this intermediate degree of cognitive clarity with the intermediate degree of truth or reality which belongs to the non-sensible objects that mathematicians talk about (511d–e). (p. 42)

However, even if he leaves himself open to an "intermediates" interpretation, Burnyeat does forestall those aforementioned Platonist interpretations, like Benson's, that require that mathematicians or philosophers of mathematics adopt the dialectical method on the basis of a supposed criticism that the mathematicians problematically mistake their hypotheses for unhypothetical first principles and, in so doing, leave their hypotheses unaccounted for. I agree with Burnyeat, and, as I will show, it is this criticism that itself is *the* mistake of all *part of* interpretations. As Burnyeat rightly notes, 'mathematics is not criticised but *placed*. Its intermediate placing in the larger epistemological and ontological scheme of the *Republic* will enable it to play a pivotal, and highly positive, role in the education of future rulers' (p. 42).

Next we must ask: What is this important role? We are told that an education in mathematics will enable the philosopher to grasp the Good, but how does that work? Why does this education take ten years?[9] What is Plato's criticism of mathematics as currently practiced? Why are the branches of mathematics so ordered? Finally, what is the relationship between mathematical reasoning and philosophical/moral reasoning? With respect to the last question, I begin by noting my agreement with Broadie's (2020) claim that 'the text of the *Republic* offers virtually *no evidence* that his [Plato's] problem lies in meta-mathematical ambition for dialectic or in the theory that ethical reality itself is mathematically structured' (p. 29; italics added).

I fully agree with the first of these disjuncts but disagree somewhat with the second. As regards the first disjunct, when we focus on what the text says, we will see that Plato's problem is set at making space for the beneficial role that mathematics plays in preparing the mind for philosophical dialectic, by turning us away from a reliance on beliefs and opinions founded on sense experience. He is also showing us the limits of mathematical inquiry, namely, that it is

ontological status of mathematical objects, and his critical analysis of how these considerations impact upon the various "intermediates" interpretations.

[9] We are told in the *Republic* (537b–e) that our philosopher in training is to spend ten years, between the ages of twenty and thirty, studying the mathematical subjects that, as children, "they learned in no particular order," now aiming to "bring [them] together into a unified vision of their kinship with one another and with the nature of what is."

conjectural and so cannot yield the kind of fixity demanded of philosophical knowledge.[10]

This conjectural aspect of mathematics, against all *part of* interpretations, is no criticism of its current practitioners – that is, it is no problem of the method of mathematics that needs fixing by some metaphysical or metamathematical account of its hypothesis as unhypothetical first principles. Again, as Broadie notes:

> [T]he claim [of the superiority of dialectic over mathematics] is not based on any intrinsic contempt on Plato's part for mathematics, for he is going to make mathematics, in its fullest development across all its known branches, the basis of future rulers' training in dialectic … Yet emphasizing the greatness of mathematics only serves to bring out the surpassing importance of dialectic. (p. 19)

What I will show is that, to appreciate both the benefits and the limits of the method of mathematics and measure these against benefits of the method of philosophy, these methods must be kept distinct, and, consequently, so too must both their epistemology and their ontology.

So, against all of Cornford, White, Tait, Benson, Burnyeat, and Broadie, I will show that Plato *is* concerned to offer a fourfold ontology associated with the four sections of the Divided Line. As I will argue, only then can we understand why mathematical inquiry, while distinct from philosophical inquiry, is "good for the soul." Thus, it is a mistake to claim, as Broadie does, that Plato, in so separating the method of mathematics from that of philosophy, "went well beyond what was needed for making it clear that philosophical thinking, in particular the sort of ethical thinking that would be engaged in by philosophers-rules like those of Plato's ideal state, is not mathematical in character and is not to be modelled on mathematics" (p. 22).

With Burnyeat,[11] I will disagree with Broadie; there is certainly textual evidence for the claim that philosophical or moral reasoning *is* to be modeled on mathematics. By placing the mathematical theory of proportion as the

[10] As Broadie notes, "the cognitive superiority – of dialectic to mathematics – in fact the *huge* cognitive superiority of dialectic to mathematics – is the main thing that Plato wants to convey by means of the image of the Divided Line" (p. 13). I am not convinced, however, that it's the *main thing* that Plato wants to convey; on my interpretation, Plato wants to convey, in the Divided Line and in Book 7, both the benefits and the limits of the method of mathematics. I have gone into more detail on the benefits and limits of the method of mathematics in the *Meno* elsewhere (Landry 2012).

[11] See Burnyeat's (2000) claim: "The mathematics and meta-mathematics prescribed for further rulers is much more that instrumental training for the mind. They are somehow supposed to bring an enlargement of ethical understanding" (p. 46). I disagree with him, however, that "*dialectical* debate about the conceptual foundations of mathematics is itself, as a very abstract level, a debate about values like justice" (p. 46; italics added). Here, I will agree with Broadie that making the

highest theory and the Good as the *highest form*, Plato is showing us that even though their methods are distinct, moral reasoning is to be taken as "akin to" mathematical reasoning.

Indeed, I will argue that not only does this supposition answer why the mathematical branches are so ordered, again with the metamathematical theory of proportion itself as the highest theory, but it also answers *the* question of why the study of mathematics is good for the soul, namely, because the "concord" and "harmony," or the *good order*, of both the objects of the branches of mathematics and the objects of philosophy are to be accounted for by proportional reasoning. So, while the ethical realm is not structured by mathematics per se, the proportional structure of the realm of forms is to be taken as "akin to" the proportional structure of the realm of mathematical objects in the sense that the good order of the forms themselves is to be accounted for by the mathematical notion of proportion. Thus, Plato's criticism of mathematics as currently practiced is not, as *part of* interpretations build their case on, that it makes use of hypotheses. Rather, it is that its arguments are taken to rely on sense experience (e.g., "counted units" in the case of arithmetic, "constructed diagrams and figures" in the case of theories of geometry, "ornaments of the heavens" in the case of theories of astronomy, and "audible concordances" in the case of theories of cosmology).

In our investigation into the order of the branches of mathematics, what we will further see, however, is that the geometrical theory of proportion plays a double role: as Plato's preferred mathematical theory of cosmology and as the highest, or metamathematical, theory that provides a *good ordering* of the branches of mathematics. In this metamathematical use, the mathematician qua philosopher-in-training will come to see that the notion of proportion itself is to be taken as a measure of harmony and concord itself. It is this use, when next applied to the philosopher's forms, that will lend itself to the philosopher's inquiry into moral matters and, in so doing, get them closer to grasping the Good as the highest form – that is, it will allow the philosopher to see, via the use of proportional reasoning, the sense in which the Good itself provides a *good ordering* of the forms.

Finally, I will appeal to this account of the metamathematical use of a geometric theory of proportion to conclude that Plato does take a stand on the "external" question, answering clearly that mathematical objects are not forms, either philosophical or foundational. Keeping in mind what Plato shows by placing mathematical objects within the realm of Being, and what the *Republic*

metamathematical debate a *dialectical* debate will lead to "the obverse idiocy of demanding that mathematics should model itself on ethical philosophizing" (p. 23).

says – namely, that "philosophic natures always love the sort of learning that makes clear to them *some feature of being that always is* and does not wander around between coming to be and decaying" (485b; italics added) – we will see that mathematical objects must have some "feature of being" but, again, only hypothetically so. Thus, and against both quietest and Platonist interpretations, I will argue that, as regards mathematical objects, Plato is a methodological realist – that is, he is a realist on the basis of what objects of thought are needed to solve both mathematical and metamathematical problems.

More pointedly, as regards the latter problems, what the geometrical theory of proportion brings to the mathematician's table is an answer to the *internal* metamathematical question: What branch of mathematics *accounts for* or *good orders* the other branches of mathematics? What the geometric theory of proportion brings to both the mathematician's and the philosopher's table is talk of harmony and concord itself (i.e., talk of *good order* itself, as expressed by reasoning in terms of proportions). Indeed, as we will see, this is why the Divided Line is so divided into the geometric ratios that it is! I will show that the use of proportional reasoning itself plays an overarching and essential role in three ways: (a) in the overall argument scheme of the Divided Line and in Book 7, the notion of clarity, as a measure of truth and reality, is accounted for by the proportion of ratios between the lines themselves;[12] (b) in his account of the good order of the branches of mathematics; and (c) in his account of the good order of the forms. This last explains why the study of mathematics is needed to grasp the Good.

So, against the *quietist* interpretations of Burnyeat and Broadie, Plato does answer the "external" questions of the ontological status of mathematical objects and the metamathematical sense in which mathematical objects are "akin." However, against the *part of* interpretations of Cornford, White, Benson, and Tait, he does this by reducing both questions to *internal* questions – that is, to problems that themselves can be answered via the mathematician's hypothetical method. Simply, then, the philosopher's dialectical method and its need to appeal to an external philosophical or foundational realm is made mute. Plato's mathematician is a methodological realist, they are *not* a metaphysical realist; they take mathematical hypothesis *as if* they were true first principles for the purpose of solving a problem and, in virtue of this, they take mathematical objects *as if* they exist. Thus, to require of mathematics that its objects are forms, be these philosophical or foundational forms, is to mistakenly confuse both the method and the epistemology of mathematics with that of philosophy. Finally, and now in hand with Plato, my counsel, as regards current practitioners of philosophy of mathematics, is as follows: We too would do well to keep the

[12] Smith (2018) provides a more detailed analysis of Plato's use of the notion of clarity.

methodological requirements for mathematics distinct from those of philosophy – that is, we would do well to place more focus on the mathematician's method and so on mathematical practice than we do on philosophical metaphysics or mathematical foundations.

3 The Divided Line

In Book 6 of the *Republic*, in attempting to explain the nature of the Good itself, Socrates first uses the Sun analogy to show the way in which the Sun is an "offspring" (506e) of the Good, and thereby comes to separate the visible and the intelligible realms. Next, Socrates uses the Divided Line analogy to further explain the epistemic and ontic distinctions that result from the distinctions between the visible and intelligible. Following Glaucon's claim that he has, through Socrates' use of the Sun analogy, understood "these two kinds" (the visible and the intelligible) (509d), Socrates introduces the Divided Line analogy to further explain his claim that "what the latter (the Good) is in the intelligible realm in relation to understanding and intelligible things, the former (the Sun) is in the visible realm in relation to sight and visible things" (508c). Bringing the two analogies together, Socrates begins with the assumption that the Sun is "sovereign" over the visible realm and the Good is "sovereign" over the intelligible realm (509e).

Socrates then subdivides each realm, according to the *clarity* of its objects:

> Represent them, then, by a line *divided into two unequal sections*. Then divide each section – that of the visible and that of the intelligible – *in the same ratio* as the line. In terms now of *relative clarity and opacity*, you will have as one subsection of the visible, *images*. By images I mean, first, shadows, then reflections in bodies of water and in all close-packed, smooth, and shiny materials, and everything of that sort. Do you understand? (509d–510a; italics added)

It is important to pause here to note that the notion of clarity and the *ratios* of clarity as set by the various divisions and subdivisions are here intended to do both epistemic and ontological work. As Plato himself remarks,

> when it [the soul] focuses on something that is *illuminated both* by *truth and what is*, it *understands*, *knows*, and manifestly possesses understanding. But when it focuses on what is mixed with obscurity, on what comes to be and passes away, it *believes* and is dimmed … and seems *bereft of understanding*. (508d; italics added)

Given Glaucon's assent that he has understood both the distinction between the intelligible and the visible realm and the nature of the objects of the first, opaque, subsection of the visible realm, Socrates next considers the objects of the clear subsection, and moves to consider the ontic and epistemic consequences of these distinctions made within this realm:

> [I]n the other subsection of the visible, put the *originals of these images* – that is, the animals around us, every plant, and the whole class of manufactured things … . Would you be willing to say, then, that, *as regards truth and untruth,* the *division is in this ratio:* as what is *believed* is to what is *known*, so *the likeness is to the thing it is like*? (510a; italics added)

Thus, physical objects themselves and their images respectively relate, on the basis of the ratio of their clarity or opacity (which is illuminated by the Sun [508b]), *ontologically* to existence and nonexistence, and *epistemically* to truth and untruth, and so to knowledge and opinion.

We subsequently come to the subdivisions of the intelligible realm:

> Next, consider how the section of the intelligible is to be divided … As follows: in one subsection, the soul *using as images the things that were imitated before*, is *forced to* base its inquiry on *hypothesis*, proceeding not to a first principle, but to a conclusion. In the other subsection, *by contrast*, it makes its way to an *unhypothetical first principle*, proceeding from a hypothesis, but without the images used in the previous subsection, using *forms themselves* and making its investigation through them. (510b)

In the first subsection of the intelligible realm, then, the soul uses "images"[13] and its method is such that it is *forced to*[14] base its inquiry on hypotheses, reasoning from a hypothesis down to a conclusion.

In the other subsection, the soul reasons from a hypothesis up to an unhypothetical first principle and then down to a conclusion,[15] making no use of images but only of forms themselves. Glaucon is here confused, and so Socrates begins anew, now making mention of the mathematicians' method:

> Let's try again. You see, you will understand it more easily after the explanation. I think you know that students of geometry, calculation, and the like *hypothesize the odd* and *the even*, *the various figures*, *the three kinds of angles*, and other things akin to these in each of their investigations,

[13] As we will see, it is best to think of a diagram or figure as an example of what is meant here by "image."

[14] What explains the *fundamental* difference between my interpretation and Benson's (and many others; see, for example, works by Tait [2002], Robinson [1953], and Annas [1981]) is that I, like Burnyeat (2000), do not take the fact that mathematicians are "forced to" use hypotheses as the criticism made by Plato of current practitioners and then use this to argue that the mathematician, like the philosopher, must take up the dialectical method. Here I agree with Burnyeat (and McLarty [2005]) that hypotheses are taken by Plato as "intrinsic to the nature of mathematical thought … To demand that the mathematicians give an account of their initial hypotheses … would be to make them stop doing mathematics and do something else instead … . It is thus no criticism to say that mathematicians give no account of their hypotheses. It is simply to say that mathematics is what they are doing, not dialectic" (pp. 37–38).

[15] This is yet another reason why, against Cornford, White, and Benson's view, the hypothetical method cannot be taken as *part of* the dialectical method; for the first method, the soul reasons *down* from a hypothesis, for the second it reasons *up* from a hypothesis to an unhypothetical first principle.