
Contents

<i>Preface</i>	xi
<i>Notation and Conventions</i>	xv
0 Appetizer: Triangles and Equations	1
0.1 Schur's Theorem	1
0.2 Progressions	5
0.3 What's Next in the Book?	9
1 Forbidding a Subgraph	11
1.1 Forbidding a Triangle: Mantel's Theorem	12
1.2 Forbidding a Clique: Turán's Theorem	14
1.3 Turán Density and Supersaturation	19
1.4 Forbidding a Complete Bipartite Graph: Kővári–Sós–Turán Theorem	22
1.5 Forbidding a General Subgraph: Erdős–Stone–Simonovits Theorem	27
1.6 Forbidding a Cycle	31
1.7 Forbidding a Sparse Bipartite Graph: Dependent Random Choice	33
1.8 Lower Bound Constructions: Overview	37
1.9 Randomized Constructions	38
1.10 Algebraic Constructions	39
1.11 Randomized Algebraic Constructions	46
2 Graph Regularity Method	52
2.1 Szemerédi's Graph Regularity Lemma	53
2.2 Triangle Counting Lemma	61
2.3 Triangle Removal Lemma	63
2.4 Graph Theoretic Proof of Roth's Theorem	66
2.5 Large 3-AP-Free Sets: Behrend's Construction	69
2.6 Graph Counting and Removal Lemmas	70
2.7 Exercises on Applying Graph Regularity	75
2.8 Induced Graph Removal and Strong Regularity	76
2.9 Graph Property Testing	83
2.10 Hypergraph Removal and Szemerédi's Theorem	85
2.11 Hypergraph Regularity	86
3 Pseudorandom Graphs	89
3.1 Quasirandom Graphs	90
3.2 Expander Mixing Lemma	101

viii	<i>Contents</i>	
3.3	Abelian Cayley Graphs and Eigenvalues	104
3.4	Quasirandom Groups	109
3.5	Quasirandom Cayley Graphs and Grothendieck's Inequality	116
3.6	Second Eigenvalue: Alon–Boppana Bound	119
4	Graph Limits	127
4.1	Graphons	128
4.2	Cut Distance	131
4.3	Homomorphism Density	135
4.4	W -Random Graphs	137
4.5	Counting Lemma	140
4.6	Weak Regularity Lemma	142
4.7	Martingale Convergence Theorem	146
4.8	Compactness of the Graphon Space	148
4.9	Equivalence of Convergence	152
5	Graph Homomorphism Inequalities	158
5.1	Edge versus Triangle Densities	161
5.2	Cauchy–Schwarz	166
5.3	Hölder	174
5.4	Lagrangian	182
5.5	Entropy	186
6	Forbidding 3-Term Arithmetic Progressions	197
6.1	Fourier Analysis in Finite Field Vector Spaces	197
6.2	Roth's Theorem in the Finite Field Model	202
6.3	Fourier Analysis in the Integers	209
6.4	Roth's Theorem in the Integers	210
6.5	Polynomial Method	216
6.6	Arithmetic Regularity	220
6.7	Popular Common Difference	226
7	Structure of Set Addition	230
7.1	Sets of Small Doubling: Freiman's Theorem	231
7.2	Sumset Calculus I: Ruzsa Triangle Inequality	233
7.3	Sumset Calculus II: Plünnecke's Inequality	234
7.4	Covering Lemma	237
7.5	Freiman's Theorem in Groups with Bounded Exponent	239
7.6	Freiman Homomorphisms	240
7.7	Modeling Lemma	242
7.8	Iterated Sumsets: Bogolyubov's Lemma	244
7.9	Geometry of Numbers	248
7.10	Finding a GAP in a Bohr Set	251
7.11	Proof of Freiman's Theorem	253
7.12	Polynomial Freiman–Ruzsa Conjecture	254
7.13	Additive Energy and the Balog–Szemerédi–Gowers Theorem	257

<i>Contents</i>		ix
8	Sum-Product Problem	264
8.1	Multiplication Table Problem	265
8.2	Crossing Number Inequality and Point-Line Incidences	266
8.3	Sum-Product via Multiplicative Energy	270
9	Progressions in Sparse Pseudorandom Sets	273
9.1	Green–Tao Theorem	273
9.2	Relative Szemerédi Theorem	275
9.3	Transference Principle	279
9.4	Dense Model Theorem	280
9.5	Sparse Counting Lemma	286
9.6	Proof of the Relative Roth Theorem	293
	<i>References</i>	297
	<i>Index</i>	313