1 Newton's Laws

Sir Isaac Newton (Figure 1) was born in 1642 and died in 1726 or 1727. What? How can there be any ambiguity over something so straightforward as the year of Newton's death? In his time, two calendars were in use in Europe: the Julian 'old style' calendar (introduced by Julius Caesar in 46 BC), and the Gregorian 'new style' calendar (introduced by Pope Gregory XIII in October 1582). While the Julian calendar counts the length of a year as exactly 365.25 days long, meaning a leap year should occur every four years, the Gregorian calendar has the following more sophisticated prescription:

> Every year that is exactly divisible by four is a leap year, except for years that are exactly divisible by 100, but these centurial years are leap years if they are exactly divisible by 400. For example, the years 1700, 1800, and 1900 are not leap years, but the year 2000 is. (US Naval Observatory, 2022)

The Gregorian calendar is now the calendar most widely used across the globe. Unlike the Julian calendar, it makes the average calendar year 365.2425 days long, thereby more closely approximating the 365.2422-day 'solar' year that is determined by the Earth's revolution around the Sun. The merit of the Gregorian over the Julian calendar is that the latter 'drifts' with respect to the solar year (because the Julian calendar does not as accurately line up with the solar year): given enough time, Christmas in the northern hemisphere would occur in summer according to the Julian calendar! One does not face these issues with the Gregorian calendar: in a sense, it is better 'adapted' to salient physical events (in this case, the Earth's going around the Sun); in turn, this often renders its descriptions of physical goings-on simpler (for example, the Earth will be at the same point in its orbit around the sun every year according to the Gregorian calendar, but not according to the Julian calendar). To anticipate some terminology which I will use later in this section: there is a sense in which the Gregorian calendar better approximates an 'inertial frame' – a coordinatisation of the world such that our description of physical dynamics is simplest – than does the Julian calendar.[1]

In fact, a central question in the philosophy of spacetime physics has to do with precisely these issues: What does it *mean* for our physical descriptions to be 'well-adapted' to nature? Is it indeed appropriate (as assumed so far in this Element) to regard 'inertial frames' as those in which physical dynamics simplifies maximally, or is there some other, superior way of understanding such structures – perhaps in terms of the structures of space and time themselves?

[1] Of course the Gregorian calendar is not perfect either: this is why we must introduce 'leap seconds' and other gadgetry in order to forestall 'drift' against the solar year.

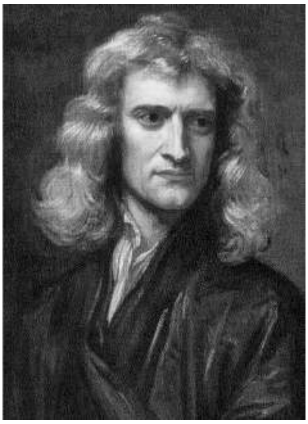

Figure 1 Sir Isaac Newton, 1642–1726/7

These are pressing questions, to which I will return throughout this Element – but they are also tangible questions: the entire set of ideas underlying them is encapsulated in the ambiguity over Newton's death year.

My purpose in this section is to expand upon these central themes in the foundations of spacetime theories, as they constitute the essential bedrock upon which I will build my philosophical analysis of special relativity in later sections. In order to proceed, I will turn again to Newton: this time not to his death date, but rather to his *laws*. These turn out to be a conceptual minefield – but grappling with how to understand the content of these laws will afford exactly the right toolkit with which to address the philosophy of special relativity in later sections.[2]

1.1 Newton's Laws

Let me begin by stating Newton's laws. These should be familiar to anyone who has studied high school physics:

N1L: Force-free bodies travel with uniform velocity.

N2L: The total force on a body is equal to the product of that body's mass and its acceleration. ($\mathbf{F} = m\mathbf{a}$.)

N3L: Action and reaction are equal in magnitude and opposite in direction – that is, if one body exerts a force $\mathbf{F}$ on a second body, then the second exerts a force $-\mathbf{F}$ on the first.

[2] In many respects, this first section will be the hardest of the Element, because I will introduce a large number of concepts and issues in quite short succession. But readers should not be deterred: I will go into all such concepts and issues in much greater depth in the remaining sections.

Stare at these laws for just a minute, and inevitably a range of conceptual questions will arise. For example:

1. What does ‘force-free’ mean?
2. Is not **N1L** a special case of **N2L**? So why state it as a separate law?
3. (Relatedly:) Is **N1L** supposed to be a definition, or something else?
4. In which frames of reference are these laws supposed to hold?
5. Does **N1L** presuppose **N3L**?

Only by answering such questions can we secure a full and clear understanding of the content of Newton’s laws. But doing so has long been recognised to be no easy business. Here is Hertz in 1894:

> It is quite difficult to present the introduction to mechanics to an intelligent audience without some embarrassment, without the feeling that one should apologize here and there, without the wish to pass quickly over the beginnings. (Hertz, 1894)

And here is the physicist Rigden, writing in 1987:

> The first law . . . is a logician’s nightmare. . . . To teach Newton’s laws so that we prompt no questions of substance is to be unfaithful to the discipline itself. (Rigden, 1987)

As foreboding as the challenge of making sense of Newton’s laws might seem, an honest philosopher of physics must try to make progress here – and, indeed, philosophers have engaged with these questions in a surprisingly diverse range of manners. In my view, in order to appreciate the range of options which are available in answering the aforementioned questions, it is helpful to present two approaches which, in many respects, are polar opposites: the ‘dynamics first’ approach of Brown (2005), and the ‘geometry first’ approach of Friedman (1983). Indeed, I will use these two authors (and their respective allies) as poles for navigation not just through this section, but over the course of the entirety of this Element.

1.2 Inertial Frames

I will begin with the fourth question in the preceding list: in which frames of reference are Newton’s laws supposed to hold?[3] Focusing on **N1L**, it is transparent that this law cannot hold in *all* frames of reference, for envisage a

[3] For the time being, I make no distinction between a frame of reference and a coordinate system. Some authors regard the former as consisting in ‘extra structure’ – I will return later to this idea of ‘extra structure’, but here I set it aside. (For more on the difference between frames and coordinate systems, see Earman and Friedman (1973).)

force-free body moving with uniform velocity according to some temporal and spatial coordinates, then move to a coordinate system accelerating with respect to the first. In this new coordinate system, the force-free body no longer moves with uniform velocity! Thus, Newton's laws obtain only in particular frames of reference.

We can make these points quantitative as follows. In a given coordinate system x^μ ($\mu = 0, \ldots, 3$),[4] suppose the path of any free particle can be expressed as

$$\frac{d^2 x^\mu}{d\tau^2} = 0, \tag{1}$$

where τ is a monotonic parameter on the path in question. Integration yields

$$x^\mu(\tau) = x^\mu(0) + \tau v^\mu(0), \tag{2}$$

where $v^\mu(0) = \frac{dx^\mu}{d\tau}$ at $\tau = 0$, so we obtain straight-line motion in the four-dimensional manifold. *This* is the property which **N1L** tells us holds of force-free particles – so in the frames in which **N1L** holds, we have $\frac{d^2x^\mu}{d\tau^2} = 0$.

Now perform an arbitrary coordinate transformation $x^\mu \rightarrow x'^\mu(x^\nu)$, along with an arbitrary parameter transformation $\tau \rightarrow \lambda(\tau)$. Our simple force law $\frac{d^2x^\mu}{d\tau^2} = 0$ becomes, in the new frame (Brown, 2005, p. 17),

$$\frac{d^2 x'^\mu}{d\lambda^2} - \frac{\partial^2 x'^\mu}{\partial x^\rho \partial x^\gamma} \frac{\partial x^\rho}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x^\sigma} \frac{dx'^\nu}{d\lambda} \frac{dx'^\sigma}{d\lambda} = \frac{d^2\tau}{d\lambda^2} \frac{d\lambda}{d\tau} \frac{dx'^\mu}{d\lambda}. \tag{3}$$

So force-free particles *accelerate* in arbitrary frames (the acceleration is quantified by the two extra terms which have been introduced in this frame: sometimes, these are called 'fictitious force' terms) – they only move on straight lines in the inertial frames.

It is crucial to note at this point that the frames in which **N1L** holds are those in which the very same dynamics takes a particularly simple form.[5] Recalling our discussion of the calendar systems, let us call the frames of reference in which Newton's laws hold the *inertial frames* of reference. Knox, indeed, gives the following very sensible definition of inertial frames:

> In Newtonian theories, and in special relativity, inertial frames have at least the following three features:

[4] It is standard practice in physics to use Greek indices ($\mu, \nu, \ldots$) to range over the four coordinates of space *and* time (where the 0 coordinate is the time coordinate), and to use Latin indices $i, j, \ldots$ to range over the three spatial coordinates. I will follow suit in this Element.

[5] Throughout this Element, by dynamical equations taking their 'simplest form' in some coordinate system, I mean something like those equations exhibiting the fewest number of terms in that coordinate system. Although somewhat vague, this notion of simplicity is perfectly clear in practice. For further discussion, see Read, Brown, and Lehmkuhl (2018); Weatherall (2021).

> 1. Inertial frames are frames with respect to which force-free bodies move with constant velocities.
> 2. The laws of physics take the same form (a particularly simple one) in all inertial frames.
> 3. All bodies and physical laws pick out the same equivalence class of inertial frames (universality). (Knox, 2013, p. 348)

So, Newton's laws hold in the inertial frames of reference, which are those coordinate systems in which the dynamics simplify maximally and in which force-free bodies move with uniform velocities. It is important to note, though, that this definition of an inertial frame is what is known as a *functional* definition: it tells us the properties which we expect (or, indeed, demand) that the objects in question (here, inertial frames) possess, but it does not (as yet) afford us any independent means of identifying those objects (again, here frames), or knowing whether they exist. Indeed, it is exactly at this juncture that authors such as Brown and Friedman begin to follow different courses. Beginning with the existence question, Brown maintains that inertial frames *do* exist in nature:

> A kind of highly non-trivial pre-established harmony is being postulated, and it takes the form of the claim that there exists a coordinate system x^μ and parameters τ such that [$\frac{d^2x^\mu}{d\tau^2} = 0$] holds for each and every free particle in the universe. (Brown, 2005, p. 17)

On the other hand, Friedman denies the existence of inertial frames:

> Newtonian physics is (would be) true even if there are (were) no inertial frames. The First Law deals with the existence of inertial frames only counterfactually: if there were inertial frames (for example, if there were no gravitational forces), free particles would satisfy [$\frac{d^2x^\mu}{d\tau^2} = 0$] in them. (Friedman, 1983, p. 118)

The difference between our two authors amounts to this. Friedman's point is that no particle is *actually* force-free, so inertial frames in the strict sense do not *actually* exist. Brown, on the other hand, would reply that inertial frames at least *approximately* exist. In fact, though, Friedman anticipates this response on behalf of Brown when he writes:

> This reply is inadequate. Newtonian physics is only approximately true, but not because of the existence of *gravity* [i.e., some universal physical force]. (Friedman, 1983, p. 118)

The reader would be forgiven for finding this passage from Friedman puzzling at this stage. It will make more sense once we understand in more detail the differing theoretical commitments of the parties involved – for this reason, I will defer a detailed discussion of this response until the end of the following subsection. For the time being, we need only note this: for Brown, **N1L** is a claim about the existence of (approximate) inertial frames in the real world; for Friedman, by contrast, **N1L** is a counterfactual statement, since in fact there are no inertial frames in the actual world. So much for the existence question. But the question of what the inertial frames *are* remains. To make progress here, we must turn now to the first of the questions in our list: what is the meaning of 'force-free'?

1.3 Force-Free Bodies

To get a better handle on what it means for a particle to be force-free, we must turn to **N2L**, which (recall) says that the total force on a body is equal to the product of that body's (inertial) mass and its acceleration. With **N2L** in mind, a natural further conceptual puzzle arises: is not **N1L** just a special case of **N2L**, given that the former (it seems) reduces to the latter in the case $\mathbf{F} = 0$? Friedman straightforwardly gives an affirmative answer to this question. On the other hand, Brown gives a negative answer:

> It will be recalled that the acceleration $\ddot{\mathbf{x}}$ of the body is defined relative to the inertial frame arising out of the first law of motion. It is for this reason that the first law is not a special case of the second for $\mathbf{F} = 0$. (Brown, 2005, p. 37, fn. 9)

In other words, for Brown, **N1L** plays the crucial role of telling us *what the inertial frames are*; for this reason, and in this sense, **N1L** is not merely a special case of **N2L**. I will come back to this, but before doing so let me explain why Friedman *does* think that **N2L** is a special case of **N1L**.

For Friedman, notions of acceleration and force are to be defined in terms of a background spatio-temporal structure. (For the time being, I will not address the question of the metaphysical status of this spatio-temporal structure, and its relation to material bodies – that is, I will not address the substantivalism/relationalism debate (on which see Pooley, 2013); I will have more to say on this in later sections, in particular Section 7.) In Newtonian mechanics, for Friedman, a particle is genuinely accelerating just in case it follows a curved path with respect to the standard of straightness of paths across time given by (neo-)Newtonian spacetime.[6] A particle is force-free just in case it follows a straight path with respect to that standard of straightness.[7] This gives us a

[6] I will explain the 'neo-' prefix here, as well as the general notion of spacetime in Newtonian mechanics, in Section 5 and 6.

[7] More on what this standard of straightness amounts to in Sections 5 and 6.

definition of force-freeness *and* makes clear that **N1L** is just a special case of **N2L**. Thus, helping oneself to a background spatio-temporal structure as does Friedman affords elegant and simple answers to the questions of what it means for a body to genuinely accelerate and what it means for a particle to be force-free. Indeed, this approach also affords a very straightforward independent definition of an inertial frame: the inertial frames are those at rest or moving uniformly *with respect to Newtonian absolute space*.[8]

Brown rejects Friedman's spacetime-based answers to these questions, for in his view such explanations are either opaque (what exactly is the relation between spacetime structure and the motions of material bodies?) or not explanations at all (if spacetime – as is the case for Brown, as we will see – is to be reduced to the motions of material bodies and the dynamical laws governing them, then ultimately I need a way of understanding notions of, for example, force-freeness with reference to material bodies only). In a sense, Brown's philosophical attitude is more *empiricist* than that of Friedman: he seeks an understanding of the notion of an inertial frame (say) directly in terms of material entities, rather than in terms of the (for him) more ethereal notion of spacetime. In fact, there is a long tradition, going back to Lange, Thomson, Tait, and others, of attempting to *empirically ground* the notions of inertial motion, force-freeness, and so forth (Barbour, 1989, ch. 12); Brown certainly can be situated as an ally of this tradition.

There are, indeed, a few different ways in which one might seek to define notions of force-freeness and so forth in an empiricist manner. The approach Brown favours is to take force-free bodies to be those which are sufficiently isolated from all other bodies in the universe; one *defines* such bodies to be force-free and defines inertial frames as those in which such bodies move with uniform velocities (recalling the quote from Brown, we can now see why the fact that a single frame exists in which all such bodies move with uniform velocities is '[a] kind of highly non-trivial pre-established harmony' (Brown, 2005, pp. 16–17)). Brown takes **N1L** to offer this prescription implicitly; any particle accelerating in such a frame is then regarded as subject to a genuine force, as per **N2L**. Note that, if such an approach is successful, no appeal to spacetime structure was needed to afford meaning to the relevant terms under consideration.

Brown's own preferred approach is, however, not the only means by which one might seek an empiricist grounding of the notions of inertial frame,

[8] I do not mean to suggest this definition is devoid of problems: open questions remain regarding why such frames are those in which the motions of *material* bodies should simplify maximally. I will return to this issue in later sections.

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force-freeness, and so forth. Another option is found in what is known as the 'regularity relationalism' of Huggett (2006). I do not need to get into the details of this view here; rather, a sanitised presentation of the prescription will suffice:[9]

1. Find the frame in which the dynamical equations governing the greatest number of bodies simplify across the total history of the universe.
2. By definition, these are the inertial frames.
3. Any body which follows a straight trajectory in these frames is force-free, by definition.
4. (It is a *conspiracy* – the *conspiracy of inertia* – that these force-free bodies all follow straight-line trajectories in these frames.)
5. Any body which does *not* follow a straight-line trajectory in these frames is subject to a genuine force.
6. **N1L** is not a special case of **N2L** because the accelerations in the latter are with respect to the inertial structure picked out in the former.
7. Extra forces in non-inertial frames are classified as 'fictitious'.

What are the merits of the 'Brown-style' prescription over the 'Huggett-style' prescription, or vice versa? One advantage of the latter is that it makes no initial assumption about the nature of forces in the universe – by contrast, Brown assumes that forces fall off with distances. On the other hand, Huggett's approach assumes that one must have a 'God's-eye view' of the entire material content of the universe – Brown, by contrast, does not do this.

For my purposes, it does not matter which of these approaches one prefers. (To anticipate, there are also other empiricist approaches to the meaning of 'force-free': for example, Torretti (1983) seeks to identify the inertial frames with those frames of reference in which **N3L** holds: I will get back to this shortly.) The central point is that none of these approaches (seem to) require recourse to spatio-temporal structure in order to afford meaning to the terms under consideration.

> **Question:** Which empiricist approach to the content of Newton's laws do you think is superior, and why?

Having now better understood the differences between Brown and Friedman with respect to the notions of inertial frames and force-free bodies, return now to the quote from Friedman presented at the end of the previous subsection.

[9] I should be clear that the following is only *inspired* by Huggett's work; I do not mean to claim he would actually endorse it.

This, I claim, is best understood as follows. Friedman supposes initially that Newton's laws are true, where the relevant terms are to be cashed out in terms of the structure of (neo-)Newtonian spacetime, as we have already seen. He also supposes material bodies interact with one another via the gravitational force. In a universe of sufficient complexity (such as the actual world, at least when appropriately idealised), the nature of the gravitational interaction will mean *no* body is truly force-free, in the sense of moving on a uniform trajectory with respect to the standard of straightness given by the background spacetime. For Friedman, the nature of the gravitational force does not mean Newtonian mechanics is in fact false (which would render the theory, in a certain sense, self-undermining), but rather that there simply are no inertial frames embodied as the rest frames of observers in the actual world.

Brown's perspective is very different: he does *not* begin by countenancing entire universes in which such-and-such laws (in this case, Newtonian gravity) obtain; rather, his concern is to afford meaning to notions and certain terms (in this case, for example, 'inertial frame') such that one may then proceed to *build up* one's theoretical commitments. For Brown, a definition of inertial frames (say) which obtains only approximately is still sufficient to build up, in a useful way, the machinery of Newton's laws. In this sense, while Friedman's critique makes sense in the context of his own theoretical commitments, it misfires against the very different methodology of Brown, who has not even constructed the notion of the gravitational interaction at the point when he seeks to define an operationalised notion of inertial frames.

There are various ways of putting the differences between the two parties here. For 'geometrical' authors such as Friedman, it is quite common to take a 'transcendent' conception of physics (in the Kantian sense of 'stepping outside of the world'), and to account for physical phenomena from that perspective, with all of the metaphysics it entails (in particular, the metaphysics of particular physical theories, e.g., Newtonian gravity) as inputs. For 'dynamical' authors such as Brown, by contrast, it is more common to take an 'immanent' conception of physics (in the Kantian sense of being 'embedded in the world'), and to construct the relevant metaphysical and physical notions on the basis of empirical studies in the world. This is vague, but I think useful to keep in mind when one reads debates between the relevant authors: failure to keep track of these different attitudes can often lead to individuals talking past one another, as the passage from Friedman indicates.[10]

[10] When put in this way, it is not completely obvious that the two views are incompatible: one begins with empirical data, 'ascends' (via the 'dynamical' approach) to a set of metaphysical commitments, which one then uses to 'descend' (via the 'geometrical' approach) to explain

Question: Do you think Brown's 'dynamics first' approach to the content of Newton's laws is to be preferred over Friedman's 'geometry first' approach, or vice versa? Why?

1.4 Summary of the Views

Let us return to our list of conceptual questions regarding Newton's laws, and consider how both Brown and Friedman would answer these questions. (For the time being, I omit the fifth question; I will discuss that in the following subsection.) First Brown:

1. Bodies are to be designated 'force-free' on the basis of some to-be-articulated operational procedure.
2. **N1L** is not a case of **N2L** because **N1L** allows us to identify the inertial frames (those in which force-free bodies move with uniform velocities); having fixed such frames, **N2L** then allows us to identify the particles subject to genuine forces (and the magnitudes of those forces).
3. **N1L** is not a definition – force-free particles are not *defined* to be those moving with uniform velocity.
4. Newton's laws are supposed to hold in the inertial frames of reference.

As we know by now, the answers Friedman would give to these four questions are very different:

1. 'Force-free' means moving uniformly with respect to the standard of straightness given by (neo-)Newtonian spacetime.
2. **N1L** is a special case of **N2L**.
3. **N1L** is not a definition – in fact, it is redundant.
4. As stated in a coordinate-based description, Newton's laws are supposed to hold in the inertial frames, which are the frames 'adapted' to (neo-)Newtonian spacetime (i.e., are the frames at rest or moving uniformly with respect to Newtonian absolute spacetime). Insofar as a world (e.g., an idealised version of the actual world) may in fact contain no bodies which are truly force-free, **N1L** cannot be operationalised in that world (in this sense, **N1L** obtains only counterfactually).

The reader will notice that, up to this point, I have not mentioned **N3L**, and I have not addressed the associated question (5), of whether **N1L** is a special

further data. This tale of ascent and descent is a familiar one in philosophy, going back to Plato's cave. (My thanks to Niels Linnemann for discussions here.)