
Index

- adaptive estimation, 176
 adjoint, 267, 268
 affine transformation, 245
 algebra, 258
 Apianus, Petrus, ix
 arrowhead matrix, 408, 409
 associative algebra, 437
 axiom of total probability, 9

 BA, *see* bundle adjustment
 backward substitution, 449
 Baker, Henry Frederick, 272
 Baker–Campbell–Hausdorff, 272–275, 278, 288, 289, 320, 328, 380, 458
 Bayes filter, ix, 4, 71, 97, 102, 104–108, 112, 119, 120, 129, 130, 144
 Bayes’ rule, 4, 11, 36, 42, 43, 52, 100, 103
 Bayes, Thomas, 11
 Bayesian, 9
 Bayesian inference, 11, 22, 41, 46, 48, 71–73, 97, 98, 137, 139, 148, 151
 BCH, *see* Baker–Campbell–Hausdorff
 belief function, 103
 Bernoulli, Jakob, 273, 284
 Bernoulli numbers, 273
 Bessel’s correction, 14
 Bessel, Friedrich Wilhelm, 14
 best linear unbiased estimate, 73
 biased, 108, 141
 BLUE, *see* best linear unbiased estimate
 Bucy, Richard Snowden, 90
 bundle adjustment, 401, 411, 414, 416, 417

 camera, 242
 Campbell, John Edward, 272
 Cauchy cost function, 173
 Cauchy product, 284
 Cauchy, Baron Augustin-Louis, 284
 causal, 62
 Cayley, Arthur, 435
 Cayley–Hamilton theorem, 51, 446
 Cayley–Rodrigues parameters, 226
 CDL, *see* cumulative distribution function
 characteristic equation, 445
 chi-squared distribution, 26
 Cholesky decomposition, 54–58, 89, 93, 115, 122, 124, 127, 131, 322, 335, 382, 383, 408–410, 416, 427, 448
 Cholesky smoother, 54, 90
 Cholesky, André-Louis, 448
 consistent, 74, 158, 163
 continuous symmetry, 316, 338
 continuous time, x, 4, 9, 35, 36, 40, 76, 77, 89, 97, 102, 148, 151, 369, 418, 423, 424, 426, 432
 coordinates, 440
 covariance, 12
 covariance matrix, 12
 Cramér, Harold, 15
 Cramér–Rao lower bound, 15, 16, 30, 31, 73, 75, 121
 CRLB, *see* Cramér–Rao lower bound
 cross product, 219
 cubic Hermite polynomial, 88
 cumulative distribution function, 9, 16
 curvature, 239

 DARCES, *see* data-aligned rigidity-constrained exhaustive search
 data association, 157, 169
 data-aligned rigidity-constrained exhaustive search, 170
 definite matrix, 446
 determinant, 443
 dimension, 436
 Dirac, Paul Adrien Maurice, 36
 directional derivative, 288, 458
 discrete time, 25, 35, 40, 53, 62, 76, 77, 83, 89, 91, 101, 102, 130, 148, 151, 153, 323, 368, 369, 371, 374, 418, 424, 426, 427
 disparity, 249
 dot product, 218, 220, 439
 Draper, Charles Stark, 385
 duplication matrix, 453

 early estimation milestones, 4
 eigenvalue, 445
 eigenvector, 445
 EKF, *see* extended Kalman filter
 ELBO, *see* evidence lower bound
 EM, *see* expectation minimization
 epipolar constraint, 245
 epipolar line, 245
 equivariance, 339
 ergodic hypothesis, 158, 160

- error function, 17
- ESGVI, *see* exactly sparse Gaussian variational inference
- essential matrix (of computer vision), 243
- estimate, 41
- estimation, *see* state estimation
- Euler parameters, *see* unit-length quaternions
- Euler's rotation theorem, 223, 257
- Euler, Leonhard, 222
- evidence lower bound, 185
- expectation operator, 11
- exponential map, 260
- extended Kalman filter, 74, 97, 105, 106, 108, 109, 111–113, 119, 122, 124, 125, 127–130, 137, 144, 153, 347, 368, 369, 371–373
- extended pose, 385
- exteroceptive, 3
- extrinsic sensor parameters, 241

- factor graph, 417
- Faulhaber, Johann, 284
- Faulhaber's formula, 284
- filter, 62
- FIM, *see* Fisher information matrix
- Fisher information matrix, 471
- Fisher, Ronald Aylmer, 16, 471
- fixed-internal smoother, 45, 53
- focal length, 242
- forward substitution, 448
- Frenet, Jean Frédéric, 239
- Frenet–Serret frame, 238–240, 253
- frequentist, 9
- Frobenius inner product, 439
- Frobenius norm, 325, 337
- Frobenius, Ferdinand Georg, 439
- frontal projection model, 242
- fundamental matrix (of computer vision), 244
- fundamental matrix (of control theory), 150, 310

- Gauss, Carl Friedrich, 2, 4, 73
- Gauss–Newton optimization, 132–137, 140, 141, 144, 290–293, 329, 367, 368, 375–378, 381, 405–408
- Gaussian estimator, 52, 66, 112
- Gaussian filter, 119
- Gaussian inference, 19
- Gaussian noise, 1, 2, 73, 91, 98, 105, 106, 157, 163, 165, 370, 402
- Gaussian probability density function, 9, 12, 13, 16, 18, 19, 22, 24, 26–28, 30, 33, 36, 63, 66, 99, 105–110, 112–119, 122, 123, 127, 129, 138, 151, 314, 334, 379
- Gaussian process, x , 4, 9, 35, 36, 77–79, 83, 86, 89–91, 148, 150–152, 424, 427, 478

- Gaussian random variable, 9, 20, 22, 33, 40, 312, 313, 316
- Gaussian variational inference, 182
- Geman–McClure cost function, 173
- generalized mass matrix, 367
- generalized velocity, 304
- Gibbs vector, 226
- Gibbs, Josiah Willard, 226
- global positioning system, 3, 169–171
- GN, *see* Gauss–Newton
- GP, *see* Gaussian process
- GPS, *see* global positioning system
- group, 257
- GVI, *see* Gaussian variational inference

- Hadamard product, 453
- half-vectorization operator, 453
- Hamilton, Sir William Rowan, 224
- Hausdorff, Felix, 272
- Heaviside, Oliver, 226
- Heaviside step function, 81
- Hermite basis function, 88
- Hermite, Charles, 88
- Hilbert, David, 439
- HMM, *see* hidden Markov model
- homogeneous coordinates, 236, 286, 331
- homography matrix, 247

- ICP, *see* iterative closest point
- idempotent matrix, 442
- identity matrix, 219
- IEKF, *see* iterated extended Kalman filter
- improper rotation, 256
- IMU, *see* inertial measurement unit
- inconsistent, 108
- inertial measurement unit, 250–252, 255
- inertial navigation, 385
- information form, 19, 58, 59, 69
- information matrix, 19, 55
- information vector, 19, 52
- injection, 314
- injective, 21
- inner product, 438
- inner-product space, 439
- interoceptive, 3
- interpolation matrix, 413
- intrinsic parameter matrix, 244
- invariance, 338
- invariant EKF, 373, 480
- inverse covariance form, *see* information form
- inverse-Wishart distribution, 179
- invertible, 443
- IRLS, *see* iterated reweighted least squares
- ISPKF, *see* iterated sigmapoint Kalman filter
- Isserlis, Leon, 33
- Isserlis' theorem, 33, 334
- iterated extended Kalman filter, 109,

- 111–113, 127–129, 138, 139, 144, 151, 153, 213
 iterated sigmapoint Kalman filter, 126, 128–130, 213
 iterative closest point, 347, 348
 iteratively reweighted least squares, 174
 Itô calculus, 79
 Itô, Kiyoshi, 79
- Jacobi's formula, 263
 Jacobi, Gustav Jacob, 259
 Jacobian, 265, 274, 276, 289
 John Harrison, 2
 joint probability density function, 10
- Kálmán, Rudolf Emil, 2
 Kalman filter, ix, 2, 4, 40, 61, 62, 67, 71–73, 75, 90, 164, 165, 169
 Kalman gain, 70
 Kalman–Bucy filter, 90, 91
 kernel matrix, 78, 82
 KF, *see* Kalman filter
 kinematics, 227, 239–241, 300, 301, 304, 305, 307, 308, 310–312, 320, 369, 371
 KL, *see* Kullback–Leibler
 Kōwa, Seki, 273
 Kronecker product, 452
 Kronecker, Leopold, 452
 Kullback–Leibler divergence, 13, 29
 kurtosis, 12, 118
- Laplace approximation, 135, 210, 429
 Laplace, Pierre-Simon, 135, 443
 Laplace expansion, 443
 law of large numbers, 112
 law of the unconscious statistician, 11
 LDU, *see* lower-diagonal-upper
 Levenberg–Marquardt, 134, 135, 293
 LG, *see* linear-Gaussian
 Lie algebra, 258
 Lie derivative, 289
 Lie group, *see* matrix Lie group
 Lie product formula, 274
 Lie, Marius Sophus, 256
 lifted form, 42, 46, 80
 line search, 134, 293
 linear time-invariant, 85
 linear time-varying, 40, 79, 84, 90, 149, 310, 312, 478
 linear-Gaussian, 41, 42, 46, 48, 62, 65, 67, 76, 103, 169
 linearly dependent, 440
 linearly independent, 440
 LOTUS, *see* law of the unconscious statistician
 Lovelace, Ada, 273
 lower-triangular matrix, 448
 LTI, *see* linear time-invariant
 LTV, *see* linear time-varying
- M-estimation, 173
 Magnus expansion, 302, 306
 Mahalanobis, 328
 Mahalanobis distance, 27, 44, 160
 Mahalanobis, Prasanta Chandra, 27
 main diagonal of a matrix, 435
 many-to-one, 261
 MAP, *see* maximum a posteriori
 marginal, 11, 158
 marginalization, 11
 marginals, 20
 Markov property, 67, 73, 102
 Markov, Andrey Andreyevich, 73
 matrix addition, 436
 matrix inversion lemma, *see* Sherman–Morrison–Woodbury
 matrix Lie group, 256, 257
 matrix multiplication, 437
 maximum a posteriori, 42, 66, 67, 72, 91, 92, 97, 99–101, 111, 112, 128–130, 139, 140, 152, 153, 157, 213, 369, 371, 374, 375, 414, 425
 maximum likelihood, 139, 140, 155, 157, 379, 380, 403, 405, 413
 mean, 12, 16
 mean rotation, 315
 minimal polynomial, 262, 266, 271, 460, 462
 minimum mean-squared error, 73
 ML, *see* maximum likelihood
 MMSE, *see* minimum mean-squared error
 Möbius, Augustus Ferdinand, 236
 Monte Carlo, 105, 112, 325, 336
 Moore–Penrose pseudoinverse, *see* pseudoinverse
 mutual information, 13, 28
- NASA, *see* National Aeronautics and Space Administration
 National Aeronautics and Space Administration, 3, 105
 natural gradient descent, 183, 187, 471
 NEES, *see* normalized estimation error squared
 negative-definite matrix, 447
 negative-semidefinite matrix, 447
 Newton's method, 132
 NGD, *see* natural gradient descent
 NIS, *see* normalized innovation squared
 NLNG, *see* nonlinear, non-Gaussian
 non-commutative group, 225, 231, 256
 nonlinear, non-Gaussian, 97, 102
 norm, 439
 normalized image coordinates, 242
 normalized innovation error squared, 160
 normalized product, 15, 26
 nullity, 442
 nullspace, 442
- observability, 2, 51, 167

- observability matrix, 51
- onto, 261
- optical axis, 242
- orthogonal basis, 440
- orthogonal complement, 442
- orthogonal matrix, 450
- orthonormal basis, 440
- outlier, 157, 171

- particle filter, 97, 119–121
- partitioned matrix, 444
- PDF, *see* probability density function
- point-cloud alignment, 347
- point-clouds, 347
- Poisson's equation, 230
- Poisson, Siméon Denis, 230
- pose-graph relaxation, 378
- poses, 217, 235, 257, 259, 263, 275, 280, 285, 291, 304, 311, 316, 319, 327
- positive-definite matrix, 447
- positive-semidefinite matrix, 447
- posterior, 11, 41
- power spectral density matrix, 36, 79, 90, 91, 478
- prior, 11, 41
- probability, 9
- probability density function, 9–11, 13, 14, 16–18, 22–24, 26–28, 30, 37, 101, 103, 105, 106, 108, 109, 112–120, 152, 314, 315, 322, 329
- projection, 441
- projection matrix, 442
- proper rotation, 256
- pseudoinverse, 45

- Q-Q plot, 13
- quantile function, 13, 29, 159, 161
- quaternion, 348

- RAE, *see* range-azimuth-elevation
- random sample consensus, 171, 172, 180, 181, 347, 355
- random sampling, 14
- random variable, 9
- range-azimuth-elevation, 249, 250
- rank, 442
- RANSAC, *see* random sample consensus
- Rao, Calyampudi Radhakrishna, 15
- Rauch, Herbert E., 58
- Rauch–Tung–Striebel smoother, 4, 53, 57, 58, 61, 90
- Rayleigh quotient, 447
- realization, 14, 16, 41
- reference frame, 217
- retraction, 293
- Riemann, Georg Friedrich Bernhard, 293
- Riemannian gradient, 294
- Riemannian manifold, 293, 294
- robust cost, 173

- rotary reflection, 256
- rotation matrix, 220, *see also* rotations
- rotations, 217, 256, 261, 274, 279, 282, 283, 287, 300, 307, 313
- RTS, *see* Rauch–Tung–Striebel

- sample covariance, 14
- sample mean, 14
- scalar multiplication of matrix, 436
- Schmidt, Stanley F., 105
- Schur complement, 18, 20, 68, 408–410, 416, 427, 444
- Schur, Issai, 18
- SDE, *see* stochastic differential equation
- Serret, Joseph Alfred, 239
- Shannon information, 13, 27, 28, 36
- Shannon, Claude Elwood, 13
- Sherman–Morrison–Woodbury, 31, 47, 58, 59, 78, 126, 139, 151
- sigmapoint, 114, 119, 123
- sigmapoint Kalman filter, 97, 122, 124, 125, 127–129
- sigmapoint transformation, 114, 117, 122, 123, 152, 318, 322, 325, 331, 334
- simultaneous localization and mapping, 405, 414, 417, 424–427
- simultaneous trajectory estimation and mapping, 423
- single-entry matrix, 440
- singular values, 450
- singular-value decomposition, 355, 449
- skew-symmetric, 438
- skewness, 12, 118
- SLAM, *see* simultaneous localization and mapping
- sliding-window filter, 144
- smooth manifold, 293
- smoother, 62
- SMW, *see* Sherman–Morrison–Woodbury
- SP, *see* sigmapoint
- span, 440
- sparse bundle adjustment, 408
- sparse Cholesky decomposition, 449
- sparse matrices, 451
- special Euclidean group, *see also* poses, 257
- special orthogonal group, *see also* rotations, 256
- SPKF, *see* sigmapoint Kalman filter
- state, 1, 41
- state estimation, 1, 4, 41
- state transition matrix, 310
- statistical moments, 12
- statistically independent, 10, 12, 19, 30
- STEAM, *see* simultaneous trajectory estimation and mapping
- Stein, Charles, 32
- Stein's lemma, 32
- stereo baseline, 247
- stereo camera, 97

Index

513

- stochastic differential equation, 149, 312, 427
 Striebel, Charlotte T., 58
 subspace, 441
 surjective-only, 261
 SVD, *see* singular-value decomposition
 SWF, *see* sliding-window filter
 Sylvester, James Joseph, 29, 435
 Sylvester's determinant theorem, 29
 symmetric, 438
 symmetric matrix, 446
 symmetricize operator, 453
- tangent space, 259, 293
 taxonomy of filtering methods, 130
 time machines, 392
 torsion, 239
 trace, 438
 transformation matrix, 236, *see also* poses
 transition function, 80, 478
 transition matrix, 41, 81
 transport theorem, 304, 307
 transpose, 438
- tridiagonal matrix, 449
 Tung, Frank F., 58
- UDL, *see* upper-diagonal-lower
 UKF, *see* sigmapoint Kalman filter
 unbiased, 16, 74, 158, 163
 uncertainty ellipsoid, 28
 uncorrelated, 12, 19
 unimodular, 280
 unit-length quaternions, 224
 unscented Kalman filter, *see* sigmapoint Kalman filter
 upper-triangular matrix, 448
- variance, 16
 variational inference, 182
 vector, 217
 vectorization operator, 451
 vector space, 436
 vectrix, 218
- Wahba, Grace, 351
 Wahba's problem, 351
 white noise, 36