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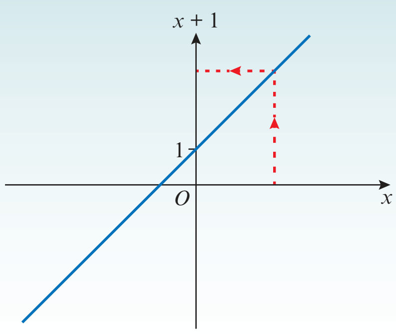
Chapter 1: Functions

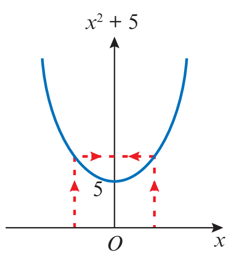
Exercise 1.1

1 $x \mapsto x + 1 \quad x \in \mathbb{R}$



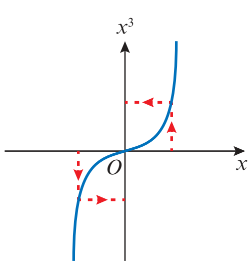
For one input value there is just one output value so this is a **one-one** mapping

2 $x \mapsto x^2 + 5 \quad x \in \mathbb{R}$



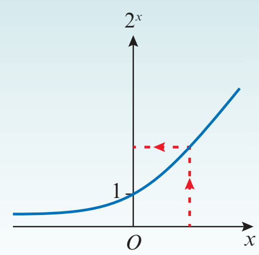
For two input values there is just one output value so this is a **many-one** mapping

3 $x \mapsto x^3 \quad x \in \mathbb{R}$



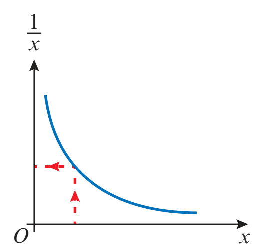
For one input value there is just one output value so this is a **one-one** mapping

4 $x \mapsto 2^x \quad x \in \mathbb{R}$



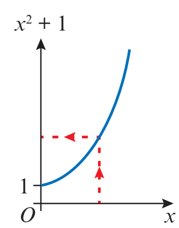
For one input value there is just one output value so this is a **one-one** mapping

5 $x \mapsto \frac{1}{x} \quad x \in \mathbb{R}, x > 0$



For one input value there is just one output value so this is a **one-one** mapping

6 $x \mapsto x^2 + 1 \quad x \in \mathbb{R}, x \geqslant 0$



For one input value there is just one output value so this is a **one-one** mapping

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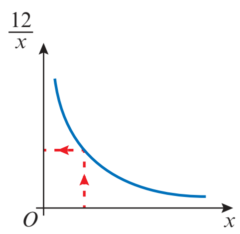
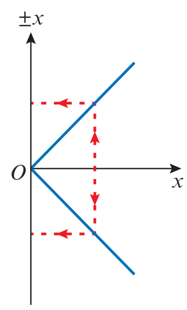
7 $x \mapsto \frac{12}{x} \quad x \in \mathbb{R}, x > 0$



For one input value there is just one output value so this is a **one-one** mapping

8 $x \mapsto \pm x \quad x \in \mathbb{R}, x \geqslant 0$



For one input value there are two output values so this is a **one-many** mapping

TIP

One-one mappings and many-one mappings are called functions.

Exercise 1.2

1 The following mappings from Exercise 1.1 are functions:

$x \mapsto x + 1 \quad x \in \mathbb{R},$

$x \mapsto x^2 + 5 \quad x \in \mathbb{R},$

$x \mapsto x^3 \quad x \in \mathbb{R},$

$x \mapsto 2^x \quad x \in \mathbb{R},$

$x \mapsto \frac{1}{x} \quad x \in \mathbb{R}, x > 0,$

$x \mapsto x^2 + 1 \quad x \in \mathbb{R}, x \geqslant 0$

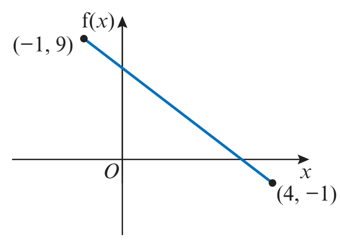
$x \mapsto \frac{12}{x} \quad x \in \mathbb{R}, x > 0$

TIP

If we draw all positive vertical lines on the graph of a mapping, the mapping is:

- a function if each line cuts the graph no more than once
- not a function if one line cuts the graph more than once.

2 **c** $f(x) = 7 - 2x, -1 \leqslant x \leqslant 4$



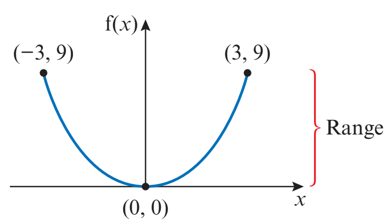
The graph of $y = 7 - 2x$ has a gradient -2 and a y-intercept 7

When $x = -1$, $y = 7 - 2(-1) = 9$

When $x = 4$, $y = 7 - 2(4) = -1$

The range of f is $-1 \leqslant f(x) \leqslant 9$

d $f(x) = x^2, \; -3 \leqslant x \leqslant 3$



The minimum value of the expression x^2 is 0 which occurs when $x = 0$

The maximum value of the expression x^2 is 9 which occurs when $x = -3$ and $x = 3$

So, the range of the function $f(x) = x^2, -3 \leqslant x \leqslant 3$ is $0 \leqslant f(x) \leqslant 9$

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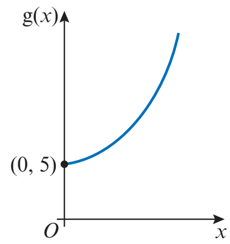
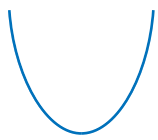
3 $g(x) = x^2 + 2$ for $x \geqslant 0$



$g(x) = x^2 + 2$ is a positive quadratic function, so the graph will be of the form



The minimum value of the expression $x^2 + 2$ is 2 which occurs when $x = 0$

When $x = 0$, $y = 0^2 + 2 = 2$

There is no maximum value of the expression $x^2 + 2$ for the domain $x \geqslant 0$

The range is $g(x) \geqslant 2$

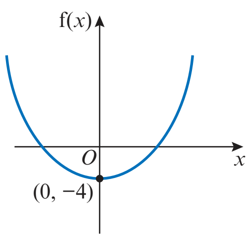
TIP

The minimum value of an expression of the form $(ax + k)^2$ is 0

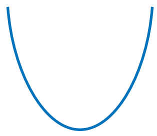
The minimum value occurs when $ax + k = 0$,

i.e., when $x = \frac{-k}{a}$

4 $f(x) = x^2 - 4 \quad x \in \mathbb{R}$



$f(x) = x^2 - 4$ is a positive quadratic function, so the graph will be of the form



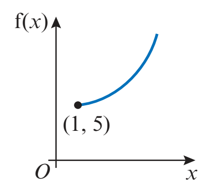
The minimum value of the expression $x^2 - 4$ is -4 which occurs when $x = 0$

When $x = 0$, $y = 0^2 - 4 = -4$

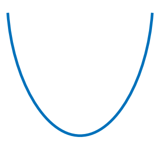
There is no maximum value of the expression $x^2 - 4$ for the domain $x \in \mathbb{R}$

The range is $f(x) \geqslant -4$

5 $f(x) = (x - 1)^2 + 5$ for $x \geqslant 1$



$f(x) = (x - 1)^2 + 5$ is a positive quadratic function, so the graph will be of the form



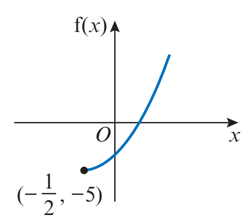
The minimum value of the expression $(x - 1)^2 + 5$ is 5 which occurs when $x = 1$

When $x = 1$, $y = (1 - 1)^2 + 5 = 5$

There is no maximum value of the expression $(x - 1)^2 + 5$ for the domain $x \geqslant 1$

The range is $f(x) \geqslant 5$

6 $f(x) = (2x + 1)^2 - 5$ for $x \geqslant -\frac{1}{2}$



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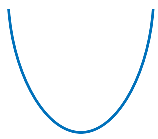
$f(x) = (2x + 1)^2 - 5$ is a positive quadratic function, so the graph will be of the form



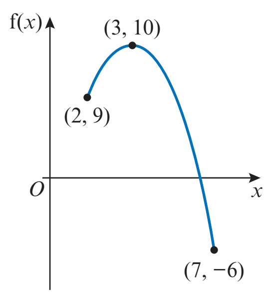
The minimum value of the expression $(2x + 1)^2 - 5$ is -5 which occurs when $x = -\frac{1}{2}$

When $x = -\frac{1}{2}$, $y = \left(2 \times -\frac{1}{2} + 1\right)^2 - 5 = -5$

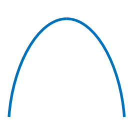
There is no maximum value of the expression $(2x + 1)^2 - 5$ for the domain $x \geqslant -\frac{1}{2}$

The range is $f(x) \geqslant -5$

7 $f : x \mapsto 10 - (x - 3)^2$, $2 \leqslant x \leqslant 7$



$f : x \mapsto 10 - (x - 3)^2$ is a negative quadratic function, so the graph will be of the form



The maximum value of the expression $10 - (x - 3)^2$ is 10, which occurs when $x = 3$

When $x = 2$, $f : x \mapsto 10 - (2 - 3)^2 = 9$

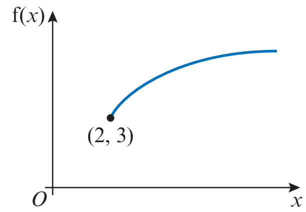
When $x = 7$, $f : x \mapsto 10 - (7 - 3)^2 = -6$

The range of $f : x \mapsto 10 - (x - 3)^2$ for $2 \leqslant x \leqslant 7$ is $-6 \leqslant f(x) \leqslant 10$

TIP

When finding the range of a function, it is helpful to be familiar with the sketches of graphs of the form $y = \frac{k}{x}$, $y = k\sqrt{x}$, $y = \sqrt{x + k}$, $y = k^x$ etc.

8 $f(x) = 3 + \sqrt{x - 2}$ for $x \geqslant 2$



The minimum value of the expression $3 + \sqrt{x - 2}$ is 3, which occurs when $x = 2$

When $x = 2$, $f(x) = 3 + \sqrt{2 - 2} = 3$

There is no maximum value of the expression $3 + \sqrt{x - 2}$ for the domain $x \geqslant 2$

The range is $f(x) \geqslant 3$

Exercise 1.3

1 fg(2) — g acts on 2 first and $g(2) = 2^2 - 1 = 3$

= f(3) — f is the function 'multiply by 2 then add 3'

= 2(3) + 3

= 9

TIP

To form a composite function, the domain of f must be chosen so that the whole of the range of f is included in the domain of g.

2 gf(5) — f acts on 5 first and $f(5) = 5^2 - 1 = 24$

= g(24) — f is the function 'multiply by 2 then add 3'

= 2(24) + 3

= 51

3 $f(x) = (x + 2)^2 - 1$ for $x \in \mathbb{R}$

$f^2(3)$ means ff(3)

ff(3) — f acts on 3 first and $f(3) = (3 + 2)^2 - 1 = 24$

= f(24) — f is the function 'add 2, square then subtract 1'

$= (24 + 2)^2 - 1$

= 675

4 gf(18) — f acts on 18 first and $f(18) = 1 + \sqrt{18 - 2} = 5$

$= g(5)$ — g is the function 'divide into 10, then subtract 1'

$= \frac{10}{5} - 1$

$= 1$

5 fg(7) — g acts on 7 first and $g(7) = \frac{2(7) + 4}{7 - 5} = 9$

$= f(9)$ — f is the function 'subtract 1, square then add 3'

$= (9 - 1)^2 + 3$

$= 67$

6 a $x \mapsto \sqrt{x} + 2$ is represented by hk

Check: $hk(x)$ — means k acts on x first and $k(x) = \sqrt{x}$

$= h(\sqrt{x})$ — h is the function 'add 2'

$= \sqrt{x} + 2$

b $x \mapsto \sqrt{x + 2}$ is represented by kh

Check: $kh(x)$ — means h acts on x first and $h(x) = x + 2$

$= k(x + 2)$ — k is the function 'square root'

$= \sqrt{x + 2}$

7 gf(x) — f acts on x first and $f(x) = 3x + 1$

$= g(3x + 1)$ — g is the function 'subtract from 2 then divide into 10'

$= \frac{10}{2 - (3x + 1)}$

But $gf(x) = 5$, so $\frac{10}{2 - (3x + 1)} = 5$

Solve $\frac{10}{2 - 3x - 1} = 5$

$\frac{10}{1 - 3x} = 5$

$10 = 5(1 - 3x)$

$10 = 5 - 15x$

$15x = -5$

$x = -\frac{1}{3}$

8 gh(x) — h acts on x first and $h(x) = 3x - 5$

$= g(3x - 5)$ — g is the function 'square then add 2'

$= (3x - 5)^2 + 2$

But $gh(x) = 51$ so $(3x - 5)^2 + 2 = 51$

Solve $(3x - 5)^2 + 2 = 51$

$(3x - 5)^2 = 49$ — square root both sides

$3x - 5 = \pm 7$ — (remember ±)

$3x - 5 = 7$ or $3x - 5 = -7$

$3x = 12$ or $3x = -2$

$x = 4$ or $x = -\frac{2}{3}$

9 fg(x) — g acts on x first and $g(x) = \frac{3}{x}$

$= f\left(\frac{3}{x}\right)$ — f is the function 'square then subtract 3'

$= \left(\frac{3}{x}\right)^2 - 3$

But $fg(x) = 13$ so $\left(\frac{3}{x}\right)^2 - 3 = 13$

Solve $\left(\frac{3}{x}\right)^2 - 3 = 13$

$\left(\frac{3}{x}\right)^2 = 16$ — square root both sides

$\frac{3}{x} = \pm 4$

$3 = \pm 4x$

$x = \pm\frac{3}{4}$

However, as $x > 0$, the only solution is $x = \frac{3}{4}$

10 gf(x) — f acts on g first and $f(x) = \frac{3x + 5}{x - 2}$

$= g\left(\frac{3x + 5}{x - 2}\right)$ — g is the function 'subtract 1 then divide by 2'

$= \frac{\left(\frac{3x + 5}{x - 2} - 1\right)}{2}$

But $gf(x) = 12$ so $\frac{\left(\frac{3x + 5}{x - 2} - 1\right)}{2} = 12$

Solve $\dfrac{\left(\dfrac{3x+5}{x-2}-1\right)}{2} = 12$

$\dfrac{3x+5}{x-2} - 1 = 24$

$\dfrac{3x+5}{x-2} = 25$

$3x + 5 = 25(x - 2)$

$3x + 5 = 25x - 50$

$-22x = -55$

$x = 2.5$

11 fg(x) — g acts on x first and $g(x) = \dfrac{10}{x}$

$= f\left(\dfrac{10}{x}\right)$ — f is the function 'add 4, square then add 3'

$= \left(\dfrac{10}{x} + 4\right)^2 + 3$

But fg(x) = 39 so $\left(\dfrac{10}{x} + 4\right)^2 + 3 = 39$

Solve $\left(\dfrac{10}{x} + 4\right)^2 + 3 = 39$

$\left(\dfrac{10}{x} + 4\right)^2 = 36$ — square root both sides

$\dfrac{10}{x} + 4 = \pm 6$

$\dfrac{10}{x} + 4 = 6$ or $\dfrac{10}{x} + 4 = -6$

$\dfrac{10}{x} = 2$ or $\dfrac{10}{x} = -10$

$x = 5$ or $x = -1$

However, $x > 0$ so the only solution is $x = 5$

12 gh(x) — h acts on x first and $h(x) = 2x - 7$

$= g(2x - 7)$ — g is the function 'square then subtract 1'

$= (2x - 7)^2 - 1$

But gh(x) = 0 so $(2x - 7)^2 - 1 = 0$

Solve $(2x - 7)^2 - 1 = 0$

$(2x - 7)^2 = 1$ — square root both sides

$2x - 7 = \pm 1$

$2x - 7 = -1$ or $2x - 7 = 1$

$2x = 6$ or $2x = 8$

$x = 3$ or $x = 4$

13 a $x \mapsto (x - 1)^3$ is the composite function fg(x)

Explanation:

fg(x) — means g acts on x first and $g(x) = x - 1$

$= f(x - 1)$ — f is the function 'cube'

$= (x - 1)^3$

c $x \mapsto x - 2$ is the composite function gg(x) or $g^2(x)$

Explanation:

gg(x) — means g acts on x first and $g(x) = x - 1$

$= g(x - 1)$ — g is the function 'subtract 1'

$= (x - 1) - 1$

$= x - 2$

14 $f(x) = \dfrac{x}{x+2}$ for $x \in \mathbb{R}$, $x \neq -2$

$g(x) = \dfrac{3}{x}$ for $x \in \mathbb{R}$, $x \neq 0$

Finding the domain of fg(x)

The domain of g(x) consists of all real numbers except $x \neq 0$ (since that input value would result in dividing by 0)

The domain of f(x) consists of all real numbers except $x \neq -2$ (since that input value would result in dividing by 0)

So, we need to exclude from the domain of g(x) the value of x for which $g(x) = -2$

Set $g(x) = -2$

$\dfrac{3}{x} = -2$

$x = -\dfrac{3}{2}$

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So the domain of fg(x) is the set of all real numbers except 0 and $-\frac{3}{2}$

This means that $x \in \mathbb{R}$, $x \neq -\frac{3}{2}$, $x \neq 0$

15 $f(x) = x^2 - 9$ for $x \in \mathbb{R}$, $x < 0$

$g(x) = 10 - \frac{x}{2}$ for $x \in \mathbb{R}$, $x > 6$

Finding the domain of fg(x)

The domain of g(x) consists of all real numbers > 6

The domain of f(x) consists of all real numbers < 0

So $x > 6$ and $g(x) < 0$

Set $10 - \frac{x}{2} < 0$

$10 < \frac{x}{2}$

$x > 20$

Overlap of $x > 6$ and $x > 20$ is $x > 20$

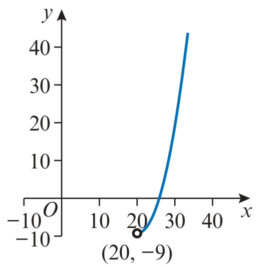
Domain of fg(x) is $x \in \mathbb{R}$, $x > 20$

Finding the range of fg(x)

$fg(x) = \left(10 - \frac{x}{2}\right)^2 - 9 \quad x \in \mathbb{R}, x > 20$

The graph of $y = fg(x) = \left(10 - \frac{x}{2}\right)^2 - 9$

$x \in \mathbb{R}$, $x > 20$ looks like:



This is a quadratic curve and the turning point occurs when $10 - \frac{x}{2} = 0$

$x = 20$

Hence the turning point is (20, − 9)

Range is $fg(x) \in \mathbb{R}$, $fg(x) > -9$

17 $f(x) = 2x - 6$ for $x \in \mathbb{R}$ $\quad g(x) = \sqrt{x}$ for $x \in \mathbb{R}$, $x \geqslant 0$

a Finding the domain of fg(x)

The domain of f(x) consists of all real numbers

The domain of g(x) consists of all real numbers $x \geqslant 0$

$x \geqslant 0$ and $g(x) \in \mathbb{R}$

So the domain of fg(x) is the set of all real numbers $\geqslant 0$

This means that the domain of fg(x) is $x \in \mathbb{R}$, $x \geqslant 0$

To find the range of fg(x) , first find fg(x)

$fg(x) = 2\sqrt{x} - 6 \quad x \in \mathbb{R}, x \geqslant 0$

The minimum value of the expression $2\sqrt{x} - 6$ is −6, which occurs when $x = 0$

When $x = 0$, $fg(x) = 2\sqrt{0} - 6 = -6$

There is no maximum value of the expression $2\sqrt{x} - 6$ for the domain $x \geqslant 0$.

The range is $fg(x) \in \mathbb{R}$, $fg(x) \geqslant -6$

b Finding the domain of gf(x)

The domain of f(x) consists of all real numbers.

The domain of g(x) consists of all real numbers $x \geqslant 0$.

$x \in \mathbb{R}$ and $f(x) \geqslant 0$

So $2x - 6 \geqslant 0$

$2x \geqslant 6$

$x \geqslant 3$

So the domain of gf(x) is the set of all real numbers $\geqslant 3$

This means that the domain of gf(x) is $x \in \mathbb{R}$, $x \geqslant 3$

To find the range of gf(x), first find gf(x)

$gf(x) = \sqrt{2x - 6}$ for $x \in \mathbb{R}$, $x \geqslant 3$

The minimum value of the expression $\sqrt{2x - 6}$ is 0, which occurs when $x = 3$

When $x = 3$, $gf(x) = \sqrt{2 \times 3 - 6} = 0$

There is no maximum value of the expression $\sqrt{2x - 6}$ for the domain $x \geqslant 3$

The range is $gf(x) \in \mathbb{R}$, $gf(x) \geqslant 0$

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19 $f(x) = 2x + 5$ for $x \in \mathbb{R}$, $x < 2$
$g(x) = (x - 3)^2$ for $x \in \mathbb{R}$, $x > 3$

a **i** The graph of $y = 2x + 5$ is a straight line with gradient 2 and a y-intercept of 5

The range of f is $f(x) \in \mathbb{R}$, $f(x) < 9$ (from substituting $x = 2$ into $f(x) = 2x + 5$)

ii The graph of $g(x) = (x - 3)^2$ is a positive quadratic function. The graph will be U shaped.

$(x - 3)^2$ is a square so it will always be greater or equal to zero. The smallest value it can be is 0. This occurs when $x = 3$ but the domain of $g(x)$ is $x > 3$ so the range of $g(x) > 0$

The range of g is $g(x) \in \mathbb{R}$, $g(x) > 0$

b Finding $gf(x)$.

f acts on x first and $f(x) = 2x + 5$

$gf(x) = g(2x + 5)$ g is the function 'minus 3 then square'

$gf(x) = (2x + 5 - 3)^2$

$gf(x) = (2x + 2)^2$

c Finding the domain of $gf(x)$

The domain of $g(x)$ consists of all real numbers > 3

The domain of $f(x)$ consists of all real numbers < 2

So $x < 2$ and $f(x) > 3$

Set $2x + 5 > 3$

$2x > -2$

$x > -1$

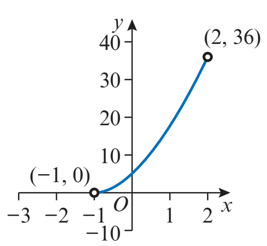
The overlap of $x > -1$ and $x < 2$ is $-1 < x < 2$

The domain of $gf(x)$ is $x \in \mathbb{R}$, $-1 < x < 2$

To find the range of $gf(x)$, first find $gf(x)$

$gf(x) = (2x + 2)^2 \quad x \in \mathbb{R}, -1 < x < 2$

The graph of $y = gf(x) = (2x + 2)^2 \quad x \in \mathbb{R}$, $-1 < x < 2$ looks like:



This is a quadratic graph and the turning point is when $2x + 2 = 0$

$x = -1$

Hence the turning point is $(-1, 0)$

Substituting $x = 2$ into $gf(x) = (2x + 2)^2$ gives 36 (which is the maximum value of $gf(x)$)

The range of $gf(x)$ is $gf(x) \in \mathbb{R}$, $0 < gf(x) < 36$

Exercise 1.4

1 **c** $|6 - 5x| = 2$

$6 - 5x = 2$ or $6 - 5x = -2$

$-5x = -4$ or $-5x = -8$

$x = \frac{4}{5}$ or $x = \frac{8}{5}$ [or as decimals 0.8 and 1.6]

CHECK: $|6 - 5(0.8)| = 2$ ✓ and $|6 - 5(1.6)| = 2$ ✓

Solution is: $x = 0.8$ or $x = 1.6$

i $|2x - 5| = x$

$2x - 5 = x$ or $2x - 5 = -x$

$x = 5$ or $3x = 5$ so $x = \frac{5}{3}$

CHECK: $|2(5) - 5| = 5$ ✓

and $\left|2\left(\frac{5}{3}\right) - 5\right| = \frac{5}{3}$ ✓

Solution is: $x = 5$ or $x = \frac{5}{3}$

2 **c** $\left|1 + \frac{x + 12}{x + 4}\right| = 3$

$1 + \frac{x + 12}{x + 4} = 3$ or $1 + \frac{x + 12}{x + 4} = -3$

$\frac{x + 12}{x + 4} = 2$ or $\frac{x + 12}{x + 4} = -4$

$x + 12 = 2x + 8$ or $x + 12 = -4x - 16$

$x = 4$ or $5x = -28$

$x = -\frac{28}{5}$

or $x = -5.6$

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CHECK: $\left|1 + \frac{4+12}{4+4}\right| = 3$ ✓

or

$\left|1 + \frac{-5.6+12}{-5.6+4}\right| = \left|1 + \frac{6.4}{-1.6}\right| = \left|1 + \frac{\frac{32}{5}}{\frac{-8}{5}}\right|$

$= |1-4| = 3$ ✓

Solution is: $x = 4$ or $x = -5.6$

f $9 - |1-x| = 2x$

$(9-2x) = |1-x|$

$(9-2x) = 1-x$ or $-(9-2x) = 1-x$

$x = 8$ or $-9+2x = 1-x$

$3x = 10$ so $x = \frac{10}{3}$

CHECK:

$9 - |1-8| = 2(8)$ and $9 - \left|1-\frac{10}{3}\right| = 2\left(\frac{10}{3}\right)$

$9 - 7 = 16$ ✗ and $9 - \frac{7}{3} = \frac{20}{3}$ ✓

Solution is: $x = \frac{10}{3}$

3 c $|4-x^2| = 2-x$

$4-x^2 = 2-x$ or $4-x^2 = -(2-x)$

$x^2-x-2 = 0$ or $4-x^2 = -2+x$

$(x-2)(x+1) = 0$ or $x^2+x-6 = 0$

$x = 2$ or $x = -1$ or $(x+3)(x-2) = 0$

$x = -3$ or $x = 2$

CHECK: If $x = 2$ and CHECK: If $x = -1$

$|4-2^2| = 2-2$ and $4-(-1)^2| = 2--1$

$0 = 0$ ✓ $3 = 3$ ✓

CHECK: If $x = -3$

$|4-(-3)^2| = 2--3$

$5 = 5$ ✓

Solution is: $x = -3$, $x = -1$ and $x = 2$

g $|2x^2+1| = 3x$

$2x^2+1 = 3x$ or $2x^2+1 = -3x$

$2x^2-3x+1 = 0$ or $2x^2+3x+1 = 0$

$(2x-1)(x-1) = 0$ or $(2x+1)(x+1) = 0$

$x = 0.5$ or $x = 1$ or $x = -0.5$ or $x = -1$

CHECK:

$|2(0.5)^2+1| = 3(0.5)$ and $|2(1)^2+1| = 3(1)$

$1.5 = 1.5$ ✓ and $3 = 3$ ✓

CHECK:

$|2(-0.5)^2+1| = 3(-0.5)$ and $|2(-1)^2+1| = 3(-1)$

$1.5 = -1.5$ ✗ and $3 = -3$ ✗

Solution is: $x = 0.5$ and $x = 1$

4 a $y = x+4$

$y = |x^2-16|$

$|x^2-16| = x+4$

$x^2-16 = x+4$ or $x^2-16 = -x-4$

$x^2-x-20 = 0$ or $x^2+x-12 = 0$

$(x-5)(x+4) = 0$ or $(x+4)(x-3) = 0$

$x = 5$ or $x = -4$ or $x = -4$ or $x = 3$

If $x = 5$, substituting into $y = x+4$

$y = 5+4$

$y = 9$

or substituting into $y = |x^2-16|$

$y = |5^2-16|$

$y = 9$ ✓

If $x = -4$, substituting into $y = x+4$

$y = -4+4$

$y = 0$

or substituting into $y = |x^2-16|$

$y = |(-4)^2-16|$

$y = 0$ ✓

If $x = 3$, substituting into $y = x+4$

$y = 3+4$

$y = 7$

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or substituting into $y = |x^2 - 16|$

$y = |3^2 - 16|$

$y = 7$ ✓

Solutions are: $x = 3, y = 7$ and $x = -4$, $y = 0$ and $x = 5, y = 9$

b $y = x$

$y = |3x - 2x^2|$

$3x - 2x^2 = x$ or $3x - 2x^2 = -x$

$2x^2 - 2x = 0$ or $2x^2 - 4x = 0$

$2x(x - 1) = 0$ or $2x(x - 2) = 0$

$x = 0$ or $x = 1$ or $x = 0$ or $x = 2$

If $x = 0$, substituting into $y = x$

$y = 0$

or substituting into $y = |3x - 2x^2|$

$y = |3(0) - 2(0)^2|$

$y = 0$ ✓

If $x = 1$, substituting into $y = x$

$y = 1$

or substituting into $y = |3x - 2x^2|$

$y = |3(1) - 2(1)^2|$

$y = 1$ ✓

If $x = 2$, substituting into $y = x$

$y = 2$

or substituting into $y = |3x - 2x^2|$

$y = |3(2) - 2(2)^2|$

$y = 2$ ✓

Solutions are: $x = 0, y = 0$ and $x = 1$, $y = 1$ and $x = 2, y = 2$

c $y = 3x$

$y = |2x^2 - 5|$

$2x^2 - 5 = 3x$ or $2x^2 - 5 = -3x$

$2x^2 - 3x - 5 = 0$ or $2x^2 + 3x - 5 = 0$

$(2x - 5)(x + 1) = 0$ or $(2x + 5)(x - 1) = 0$

$x = 2.5$ or $x = -1$ or $x = -2.5$ or $x = 1$

If $x = 2.5$, substituting into $y = 3x$

$y = 7.5$

or substituting into $y = |2x^2 - 5|$

$y = |2(2.5)^2 - 5|$

$y = 7.5$ ✓

If $x = -1$, substituting into $y = 3x$

$y = -3$

or substituting into $y = |2x^2 - 5|$

$y = |2(-1)^2 - 5|$

$y = 3$ ✗

If $x = -2.5$, substituting into $y = 3x$

$y = -7.5$

or substituting into $y = |2x^2 - 5|$

$y = |2(-2.5)^2 - 5|$

$y = 7.5$ ✗

If $x = 1$, substituting into $y = 3x$

$y = 3$

or substituting into $y = |2x^2 - 5|$

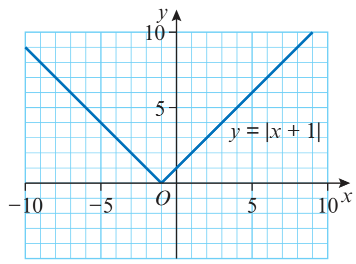
$y = |2(1)^2 - 5|$

$y = 3$ ✓

Solutions are: $x = 1, y = 3$ and $x = 2.5, y = 7.5$

Exercise 1.5

1 a



Sketch the graph $y = x + 1$

Reflect in the x-axis the part of the graph that is below the x-axis.

Intercepts at $(-1, 0)$ and $(0, 1)$