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# Advanced Concepts in Particle and Field Theory

Uniting the usually distinct areas of particle physics and quantum field theory, gravity and general relativity, this expansive and comprehensive textbook of fundamental and theoretical physics describes the quest to consolidate the basic building blocks of nature, by journeying through contemporary discoveries in the field, and analyzing elementary particles and their interactions.

Designed for advanced undergraduates and graduate students and abounding in worked examples and detailed derivations, as well as including historical anecdotes and philosophical and methodological perspectives, this textbook provides students with a unified understanding of all matter at the fundamental level. Topics range from gauge principles, particle decay and scattering cross-sections, the Higgs mechanism and mass generation, to spacetime geometries and supersymmetry. By combining historically separate areas of study and presenting them in a logically consistent manner, students will appreciate the underlying similarities and conceptual connections to be made in these fields. This title, first published in 2016, has been reissued as an Open Access publication on Cambridge Core.

**Tristan Hübsch** is a Professor of Physics at Howard University, Washington DC, where he specializes in elementary particle physics, field theory and strings.

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*To the clouds that are fuzzy,*
*to the brooks that babble,*
*and to curiosity.*

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# Preface

*I think we may yet be able to [understand atoms].*
*But in the process we may have to learn what*
*the word "understanding" really means.*
*— Niels Bohr, cited by W. Heisenberg [267, p.41]*

PHYSICS MAY BE DEFINED AS THE DISCIPLINE OF UNDERSTANDING NATURE. This definition is about as good as any other I can think of, although – or perhaps exactly because – much of the material in the following chapters is required even just to more precisely describe what it is we are to understand under *discipline*, *understanding* and *Nature*. That is, what is the nature of disciplining our understanding of something of which we ourselves are a part: Nature.

True to the meaning of the Greek original (*φύσις*), physics is indeed concerned with all aspects of Nature. Molecular phenomena are the objects of study in both chemistry and physics, which disciplines are separate but tightly related through quantum physics [477]. The science with which we study phenomena of continental proportions is called geology (but *areo*logy on Mars), whereas (planetary, stellar, galactic, cosmic) events that are at least a few orders of magnitude larger are labeled as astrophysics. Living things and events are the object of study in biology, but life itself and its characteristics quite probably derive from quantum physics [477]. Extending this point of view, phenomena of thought and feeling (commonly labeled as "psychology") may well be shown to be caused and determined by definite physical processes in the brain, so that social phenomena may be regarded as the "psychology of large ensembles of people," just as thermodynamics is the "mechanics of large ensembles of particles."[1]

Of course, a mere reduction of all phenomena to a common denominator achieves very little other than irking those who would rather keep up the appearance of separateness or those who insist on "irreducible wholeness." Hoping that this has nudged the Reader to think along (or against) such sweepingly unifying avenues of human understanding of Nature, let us turn to the real focus of this tome: to the *fundamental* physics of elementary particles.

**Subject** This book represents an attempt of a compact but comprehensive review of some of the key questions in contemporary *fundamental* physics, traditionally called both *elementary particle physics* and *high energy physics*. The correlation between these concepts is not at all accidental: The voyage towards an idealized but also pragmatically useful *fundamental* understanding of Nature really does lead through the world of ever smaller objects, the study of which requires ever larger energies in a complementary way.

The concept of "elementary particles" is in this sense a Democritean *ideal*, but it is also an evolving idea: On one hand, we follow this twenty-five-century hypothesis that the World around us may be understood as a complex system, ultimately consisting of certain basic and indivisible

[1] This paragraph was evidently meant provocatively; other fields of study do not *reduce* to physics, but emerge from it, and are "caused and determined" by it. Similarly, the babbling of a brook and the fluffiness of a cloud "derives" from hydrodynamics, but additional ideas from acoustics, nonlinear dynamics, turbulence, chaos, etc., are indispensable in a fuller (and still incomplete) understanding of these phenomena.



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objects – *elementary particles*. On the other hand, the past two centuries of the history of science warn us that concrete things (and ideas) in Nature, which we at times identify as *elementary*, not infrequently later turn out to be themselves composed of *more elementary* things (and ideas). In this sense, the list of elementary particles was very short in the first third of the twentieth century. Everything in Nature was understood to consist of either the elementary particles (matter) the electron $e^-$, the proton $p^+$, the neutron $n^0$ and (hypothetically) the neutrino $\nu_e$ – or a form of their interaction, which could also be represented in terms of exchanging elementary particles such as the photon $\gamma$. Soon enough, however, hundreds of new particles were discovered. Already their unrelentingly growing number vanquished all hope that all these particles could *really* be elementary. Indeed, even the proton and the neutron were soon shown to be consistently describable as composite systems; they both consist of more elementary *quarks*.

To date, no experiment indicates revoking "elementariness" from quarks $(u,d,c,s,t,b)$ and leptons $(e^-,\nu_e,\mu^-,\nu_\mu,\tau^-,\nu_\tau)$; see Table 2.3 on p. 67. Similarly, the electroweak, strong nuclear and gravitational fundamental interactions exhaustively describe all the known interactions of these particles. The model that includes these particles and their interactions is then rightfully called the *Standard Model*. As understood today, this model also requires the existence of the so-called Higgs particle, which has only recently been experimentally confirmed [25, 109]; see also [493, 494, 475]. Besides, the intricate structure and symmetries of the Standard Model also indicate a possible *more fundamental* description of physics.

Inspired tourist guides (see, for example, Refs. [329, 469, 162, 183, 184, 551, 404, 405, 585, 166, 267, 161, 34, 553, 164, 119, 163, 231, 263, 456, 232, 449, 233, 389, 505, 93, 234, 94, 27] but also a critique of superstring theory [489, 490, 577] and a recent response [145]), very recent lecture notes [525, 384, 539, 448, 427], textbooks (such as [407, 35, 64, 63, 306, 48, 106, 45, 218, 257, 238, 580, 241, 239, 307, 249, 240, 221, 554, 555, 159, 504, 422, 423, 538, 484, 250, 116, 588, 355, 243, 589, 7, 590] and worked out problem collections [107, 341, 446], among others) certainly provide excellent sources. In addition, internet sources such as Wikipedia are ever better organized and increasingly more complete – web-pages may be and are constantly corrected, amended and extended. No book can possibly compete with *that*. Instead, the aim of this book is fourfold:

1. a review of our subject matter and its central ideas, sharpened and re-focused by the benefit of hindsight,
2. a presentation of the structure of the theoretical description of fundamental physics and its origins within experimental results,
3. an indication of *some* recent additions in this structure, to some less traveled avenues, some shortcuts, some detours, and even some traps,
4. a general overview for novices as well as the more relaxed but valiant Readers.

This book makes it possible to present the Reader with the facts of the (fundamental) physics of elementary particles and their organization. This is accompanied by my view of the unifying philosophical *woof* that permeates not only this subject, but also the contemporary understanding of fundamental physics and science in general: Nature is one and can only so be understood. Since our goal is the description of the *fundamental* basics of understanding Nature, a discussion of this philosophical woof is an unavoidable part of the journey.

In turn, the intent of this review is to present the main factors in the challenging process of fully grokking[2] Nature. This intent stems from the Democritean idea that substance is finitely divisible, and that it has ultimately indivisible parts – elementary particles. This then provides

[2] To *fully grok* the meaning of the verb *to grok*, the Reader is kindly referred to Ref. [266], the title of which aptly summarizes the feelings of most sincere Students of the (fundamental) physics of elementary particles.

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the *warp* (to the philosophical woof from the previous paragraph) of the fabric of contemporary theoretical fundamental physics. By the end of the twentieth century it became evident that these elementary objects cannot be the "material points" used in classical physics, and we are led to fundamental *strings*. Our discussion therefore must also include the questions: what are strings, where does "stringiness" manifest, why strings and not points or something else, and how are strings woven into our incessantly and asymptotically improved understanding of Nature?

**Aperitif** The gauge principle and its consequences constitute our contemporary description of all fundamental interactions, and form the third strand – *weft* – in the triply woven fabric of our current understanding of Nature [☞ lexicon entry on p. 508, in Appendix B.1]. Gauge theories of the commutative and non-commutative (Yang–Mills) type, the corresponding conservation laws and interactions are the subject matter of Chapters 5 and 6, but are also the quoin of the Standard Model from the very description of the subject matter. Formal similarities between the gauge theories (models) of Yang–Mills type and Einstein's general theory of relativity are exhibited in Chapter 9. This clearly implies that this (gauge) principle unifies *all* symmetries, *all* conserved quantities and conservation laws of the Noether–Gauss–Ampère type, and *all* known fundamental interactions. It also gives them all a geometrical description [☞ Chapter 11].

Similarly, *quantumness* is also an indubitable feature of Nature. Students are well acquainted with this, although mostly within the non-relativistic formalism. However, the study of *quantum* and *relativistic* gauge theories discovered the phenomenon of *anomalies* as well as the unquestionable necessity of canceling these indicators of inconsistency [☞ especially Section 7.2.3]. By including finally also the only known universal mechanism for stabilizing the vacuum – supersymmetry – we arrive at the complete picture displayed in Table P.1, the business card of understanding Nature as presented herein.

**Table P.1** A telegraphic summary of the characteristics of our description of Nature; see Section 11.2

| | Characteristic | Universal property | Unifies/describes |
|---|---|---|---|
| | **Quantumness** | Stabilizes atoms | Waves and particles |
| **Gauge principle** | **Special relativity** | Links symmetries, conservation laws, forces/interactions and geometry | Spacetime, energy–momentum |
| | **General relativity** | | Acceleration–gravitation, mass–inertia |
| | **Relativity of phases** (of wave-functions) | | (Electro-magneto)+weak, and strong interactions |
| | **Supersymmetry**[a] | Stabilizes vacuum | Bosons and fermions |

[a] Supersymmetry is the only characteristic listed here that is not yet experimentally verified, but is the only (known!) universal characteristic the consequences of which include vacuum stabilization.

**Organization** This extended textbook is written for courses such as *Elementary Particle Physics* and *High Energy Physics*, and for Students near their undergraduate to graduate education transition. It aims to remind us that Nature is one; that the various courses the Student has so far mastered are only perforce separated parts of a whole, the reintegration of which into a coherent single vision remains with the Student.

The structure of this book largely follows that of the two-week block-course *Elementary Particle Physics*, as I have been teaching it annually since 2009 at the Department of Physics, University of Novi Sad. This was extended into a regular two-semester course taught in 2011/12 at the Department of Physics and Astronomy, Howard University. That plan started with D. Griffiths's

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textbook [243], but was iteratively and repeatedly modified, in response to student questions but also rooted in my own learning. A detailed map of all sources and their influences on this book is thus impossible; the unavoidably limited list of references and citations will, I hope, provide a reasonable collection of starters.

The book starts with an introductory chapter, numbered 1, where I summarize my philosophical and formal motivations for the study of the (*fundamental*) physics of elementary particles. Chapter 2 gives a historical review of the developments in this field of physics and so presents a rationale for the final structure, since the latter half of the twentieth century called the Standard Model. The technically (read: mathematically and predictively) detailed description of this subject begins with Chapter 3 and gradually introduces the elements of the Standard Model, through Chapters 4–7. Chapters 8, 9 and 10 give a basic introduction to the contemporary developments in this field and the research beyond the Standard Model. This leads towards a "unified theory of everything" for which the current favorite is described in Chapter 11. That chapter also summarizes the physical and philosophical sense of this subject, and the birth of a new subject in studying Nature: the study of complex systems and their emergent characteristics, complementing the Democritean idea. The appendices summarize various technical results and data that are useful in reading the main part of the book – and *working* through it.

The presentation and organization of the subject matter has a few formatting elements intended to help with the reading: Other than the main body, the book has a lexicon of less familiar terms and concepts in Appendix B, an index of main terms at the end of the book, as well as indicated digressions, conclusions and worked examples scattered throughout. The digressions (boxed) contain detailed computations and derivations that are not mandatory for following the main narrative. The impatient Reader is welcome to skip them on first reading. Similarly, the worked examples and comments (also boxed) serve to additionally illustrate and discuss the main narrative, and provide the derivations of results that are used later, but the mastering of which is not necessary for following through the main narrative. The in-line questions *[✎ so labeled]* prompt the Reader to pause, think through the presented argument and verify it. Frequent references and explicit citation of earlier results, conclusions, examples, etc., will hopefully help the Reader to find their way in the unavoidably multiply and nonlinearly connected presentation, and to find the information sought.

Research in contemporary fundamental physics is technically extraordinarily demanding: On one hand, the historical development and the very nature of *fundamental physics* indicate a synthesis of ideas and methods from many diverse areas in physics. On the other, one uses methods and results from many areas of mathematics such as the theory of groups and algebraic structures, differential geometry, topology, homological algebra, etc. A complete review of these areas is impossible within the confines of any one book, and the Reader is directed to the indicated references as well as earlier courses. The more ambitious Readers are directed to the textbooks [18, 457, 508, 62, 536, 287, 210, 565, 258, 581, 201, 256, 80, 260, 333, 447], with the *ominous warning*: it is impossible to first learn "all the necessary mathematics," and then turn to understanding physics. The mathematical language is best learned *en route*, as needed. In this, we are frequently limited to citing the needed results, presenting concrete examples and perhaps the motivation or basic idea behind the so-borrowed methods and techniques. In spite of this and for the sake of a minimal notion of completeness, this book contains more than enough material for a standard course, and choosing the route through the book is left to the instructor. In this, the diagram of dependencies in Figure P.1 should be useful.

Finally, most sections end with a list of problems. The serious Student is expected to work through these problems and solve them as completely as possible, first using only the material

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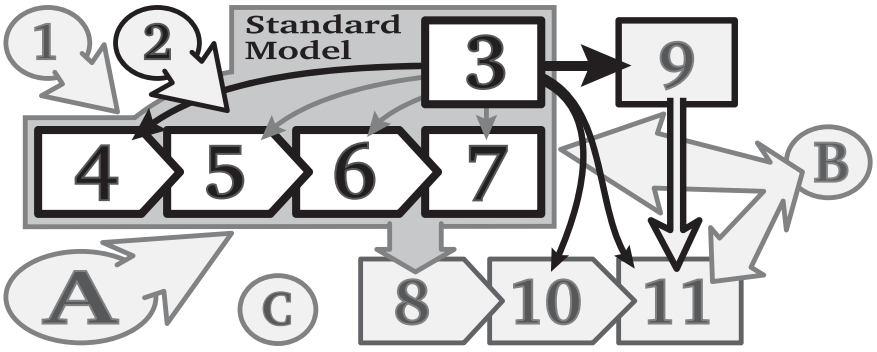


**Figure P.1** The arrows indicate the dependencies between chapters and appendices, implying the recommended sequence of reading. The appearance of the boxes and chapter numbers indicates the relative significance of the chapters. The package-framed boxes indicate the minimal content, (**2**→)**3**→**4**→**5**→**6**→**7** (skipping the technically demanding sections), for a one-semester course "elementary particle physics."

presented herein, then comparing with the cited literature – and certainly before looking up a solution on the internet.

— ❦ —

I am grateful, first of all, to the Department of Physics at the University of Novi Sad, Serbia, where I have been lecturing annually on elementary particle physics since May 2009, and especially to my friend and colleague, Professor Miroslav Vesković (Department of Physics, University of Novi Sad), for making this possible. In turn, I am grateful to the Department of Physics and Astronomy at Howard University, Washington DC, for presenting the opportunity to translate this course a quarter of the Earth's circumference westward, and present this interactively modified version to the Washington DC metropolitan area students. This book would of course not exist were it not for my three decades of research in this field, and I am grateful to all my collaborators and colleagues for their uncountable corrections and comments, which have in so many ways shaped my understanding. Even just listing their names is prohibitive and I cannot but resort to a simple collective "thank you, all." I should like to thank all the Students and Colleagues who have attended my lectures and contributed to the evolution of the course – not the least of which by proofreading. In particular, Shawn Eastmond, Tehani Finch, Philip Kurian, Henry Lovelace, Sidi Maiga and Branislav Nikolić have contributed diligently to the current version. I can only hope for such a continued evolution of both the course and this book. Special thanks go to Prof. Darko Kapor (Department of Physics, University of Novi Sad), whose astute, critical and exacting reading of the first drafts provided the invaluable and inexorable impetus for persevering through the project and eventually completing it. Finally, I should also like to thank the staff at Cambridge University Press for their help in finalizing the project, and especially Ms. Patterson for her constructive proofreading.

All the remaining errors[3] are, however, entirely and solely mine.

Tristan Hübsch

[3] Errare divine est, aliter nos non simus.

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