Cambridge University Press & Assessment
978-1-009-29152-1 — Advanced Concepts in Particle and Field Theory
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Excerpt
More Information

Part I

Preliminaries

1

The nature of observing Nature

1.1 Fundamental physics as a natural science

The ultimate aim of this course is to present the contemporary attempt to perceive the (fundamental) structure and nature of Nature. First, however, we must examine the (methodo-)logical framework at the foundation of this aim.

1.1.1 Not infrequently, things are not as they seem

Although an erudite historian will certainly and readily cite earlier quotations of the thought expressed in the title of this section, I should like to introduce this *leitmotif* as a Copernican legacy. The readiness to abandon the "obvious," "generally accepted" and "common sense" for unusual insights – those we can actually check – is certainly an essential element. This motif permeates the development of our understanding of Nature, and reappears in its contemporary form as *duality* [☞ Section 11.4].

Of course, not just any unusual insight will do: a *luni*centric or an *iovi*centric system, for example, would offer no advantage over the *geo*centric cosmological system. Most significantly, heliocentricity simplifies both the conceptual structure and the practical application of the planetary system, and makes it more uniform. Although still assuming circular orbits and so in need of corrections,[1] Copernicus' model is *essentially* simpler; maybe this could be regarded as a variant of Ockham's principle.

This idea is not yet Newton's universal law of gravity, but already contains its germ, its unifying motif: all planets follow the same type of regular motion and only appear to wander randomly (as their original Greek name implies). Also, the ultimate test of this model is easily identifiable: the positions and the motions of the planets determined by (the simpler) computations within the heliocentric system agree with astronomical observations.

Examples of this leitmotif begin at such a simple level that they are rarely noticed:

1. The shadow of an object is often distorted and many times larger than the object itself. Nevertheless, only very little children are afraid of the shadow of a wolf or a monster, however aptly conjured by the artists in a puppet theater.

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[1] Only after Kepler's *ad hoc* postulate of elliptical orbits (which Newton explained *a posteriori*) did heliocentricity achieve its really convincing technical simplicity and precision.

2. Viewed from a large plateau (without mountains on the horizon), the Earth does look flat. Yet, Eratosthenes (*c*. 276–195 BC) not only proved that the Earth was round, but even computed its size (to about 10–15% of the modern-day value!). This computation was based on the length of the summer solstice noon shadows in Syene (a.k.a. Aswan) and in Alexandria, the distance between these cities, and using geometry that is two millennia later regarded as elementary. In time, Eratosthenes' results and reasoning became "politically inconvenient," were suppressed and forgotten for some sixteen centuries, and were re-discovered in the West only centuries later, in the Renaissance. Although by now few people doubt that the Earth is round, when (if?) humankind expands into Space, the once obvious flatness of the Earth will become unthinkable; just as once its roundness was.

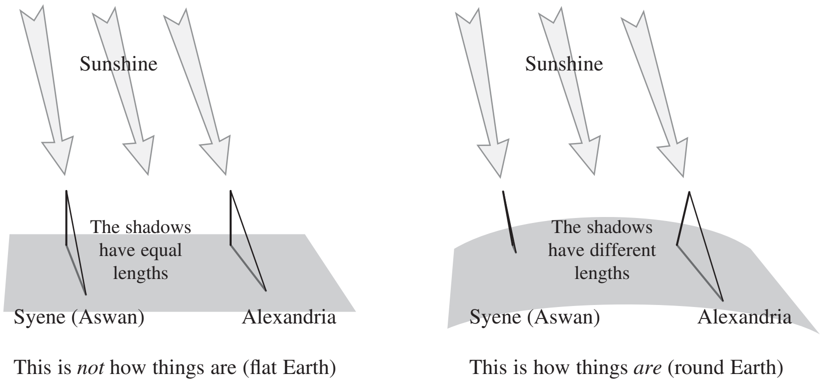

Figure 1.1 Eratosthenes' analysis which, by means of measuring angles and distances, gives (depending on the precise value of ancient units he would have used) the size of our planet Earth to about 16% at worst and 2% at best! (The shadows in the illustration are exaggerated.)

3. Everyday experience convinces us: the Sun and the Moon revolve around the Earth. This was indeed known to the ancient Greek science, as reported in Claudius Ptolemy's (*c*. AD 90–165) *Almagest*. This suppressed the teachings of Aristarchus (*c*. 310–230 BC), who not only advocated the heliocentric system, but also estimated that the Sun is about 20 times further away from Earth than the Moon and about 20 times bigger.[2] It took sixteen centuries for the West to rediscover this.
4. To the naked eye, our blood seems homogeneous and continuous. So it was believed to be until 1683, when the Royal Society published the first detailed pictures of red blood cells, as seen through a microscope and drawn by Antoni van Leeuwenhoek (1632–1723). In 1932, Ernst August Friedrich Ruska (1906–88) designed the first electronic microscope, the modern versions of which permit us to see – in the most direct way possible – individual molecules and even atoms, of which all matter around and within us is composed.

This last insight (quite literally!) is due to technical development, and it fully convinces us of the finite divisibility of *things* around us. Seemingly continuous things: fluids, air, metals… in fact consist of an enormous number of teensy particles! Whence stems the conviction that there exist "elementary particles" – the smallest building blocks of which everything else consists. Although

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[2] The 20-fold error in Aristarchus' result stems from insufficient precision in angular measurements of the time; his reasoning and geometry were essentially correct. Also, the ratio of the diameters of the Sun and the Moon indeed does equal the corresponding ratio of their average distances from the Earth, but is ≈ 400, not 20.

this idea is fantastically successful in explaining Nature and even predicting its behavior, it behooves us to keep in mind that the "particulate nature" of Nature mirrors our gradually improving understanding of Nature, and that this insight is subject to verification and periodic audits.

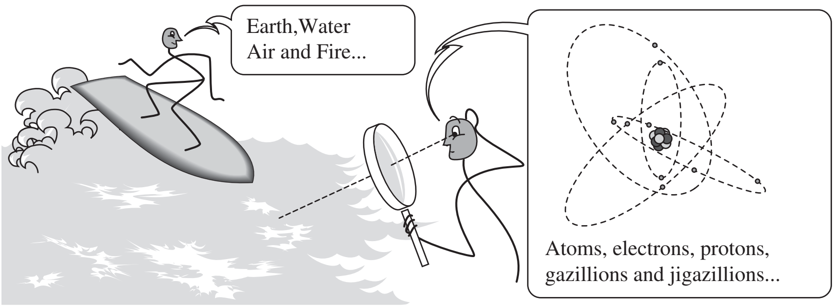

Figure 1.2 What at humanly characteristic scales seems smooth, homogeneous and continuous, may well look completely different under sufficiently closer scrutiny.

The Reader will certainly have no difficulty extending this list with many other and possibly more interesting and amusing examples, evidencing our basic leitmotif. Standard human perception, so well adapted to our daily routine, does not serve us well when concerning scales and proportions that are not as commonplace. From the typical, everyday vantage point and at characteristic human scales, planetary and stellar events appear warped. We must apply our (patiently educated and disciplined) mind to correct this picture. Indeed, once so educated, the Sun in the sky never again seems the same! In our mind's eye, we can actually *see* the Earth upon which we stand, as it rotates around the star we call the Sun. Similarly, once educated about the blood cells, our mind's eye has no difficulty *seeing* the erythrocytes as they stream through the blood plasma in our veins, and the leukocytes as they attack the blood-borne bacterial invaders.

Yesterday's unbelievable and ridiculed "nonsense" (that diseases are caused by germs too tiny to be visible was indeed widely ridiculed) may well turn out to become an evident truth of today – and such realizations turn out well remembered. So-called "evident" truths must not be exempt from verification just because they are considered evident: not infrequently, "evident" is simply that which is familiar and what are we used to. Not yet having doubted something is no guarantee of its truth.

However, we must then inquire which claims should we doubt and how do we establish the truth of any particular claim if everything is to be doubted? Following Descartes' rationale, *everything* that may be doubted without self-contradiction should be doubted. However, physicists are usually more pragmatic than that.[3] With a nod to the principle "if it ain't broke, don't fix it," physics models and theories are doubted and re-examined when they start predicting things that *are not*, or fail to predict the things that *are*. . . And, predictions are derived from a model as much as technically and practically possible.

In fact, it is our *duty* to "churn out" everything one possibly can from every scientific model. This is both for the sake of economy (the predictions of a model are its "products") and in order to establish if the model is in as full an agreement with Nature as it is possible to determine at any given time.

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[3] . . . and even without the persnickety conclusion that Descartes' motto *cogito, ergo sum* leads into solipsism, or recalling Hume's demonstration just how destructive such infinitely regressive doubting may be. . .

1.1.2 The black box: a template of learning

To formalize our approach, let us picture the scrutinized system as a *black box*, representing the lack of knowledge about its contents. What follows may then be regarded as the three pillars of (exact, natural) science.

I. To learn something of the contents of the box, an *input* (controlled or otherwise known) is directed at the box, and we observe the *outcome*. The input may be something as simple as knocking, shaking, or maybe something more technical, such as X-rays or ultrasound. The outcome is whatever *emerges* from the box in response. For example, as the box is shaken, its weight might move in a way suggesting that it is concentrated in several distinct subsystems inside the box. Or, the box may ring hollow to knocking. Or, X-rays may show the image of Thumbelina's skeleton. . .

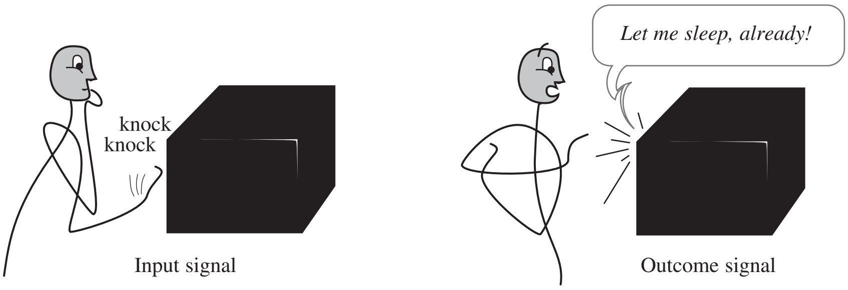

Figure 1.3 The black box experiment template.

II. Using the information about the box in the form of a "response to the input," where both input and outcome are adequately *quantified*, we develop a *mathematical model* that faithfully reproduces all received outcome signals as a response to the corresponding input signals. Needless to say, both input and outcome signals must be measured, and will therefore be known only up to measurement errors. This defines the resolution/precision/tolerance of the model. Of course, a resolution of the mathematical model cannot be guaranteed to be better than this; and this must then be understood as the resolution of the model as a whole.

III. This mathematical model is then used to derive the consequences of the conceptual model: One computes the response of the system (as represented by our model, in the role of the black box) to new, as yet untested input signals. These responses then need to be tested, if and when that becomes possible.

Herein then lies the clue as to "what and when to doubt" and "how to test truthfulness." Physics (and, more generally, scientific) models must be re-verified, wherein one or more of the "ingredients" are doubted and perhaps even replaced, if the model does not reproduce and correctly correspond to Nature to within the resolution of the model [☞ also Comment 10.5 on p. 388]. This shows that:

Conclusion 1.1 *Exact science always errs, but is exact about how much.*

Comment 1.1 *"Physics students learn this very quickly, through a shock, when they proudly obtain the required results of the first lab exercise, and the teaching assistants quiz them about the errors at least as much as about the obtained results." D. Kapor*

This three-step process, “observe–model[4]–predict,” repeats iteratively and infinitely, in counterpoint to the above-cited leitmotif, and guaranteed by Gödel’s incompleteness theorem,[5] since the research subject is sufficiently complex and is not easily exhaustible (unlike, e.g., “tic-tac-toe,” which is exhaustible as a game of strategy) [211]; see the lexicon entry in Appendix B.1, as well as Appendix B.3. When the model is constructed, the predictions of the model are derived and checked experimentally, as well as possible in practice. As human ingenuity incessantly improves the technology, and new techniques and methods (both experimental and theoretical) are being continually developed, new predictions are being continually derived and checked with an increasing precision. Sooner or later, these new checks (both experimental and theoretical) indicate the shortcomings and uncover statements that can be neither proven nor disproven within the given theoretical system.[6] If such a new statement can be experimentally checked as true or false, the model needs to be extended so as to include this new fact about Nature. When the so-extended model successfully reproduces all (known) “new” facts, additional predictions of the now extended model are derived and checked, and these typically indicate further directions of extension and improvement, upon which yet more additional predictions may be derived, and so on.

> **Comment 1.2** *To illustrate, the phenomena we now label as* **electrodynamics** *are describable by equations that are easily written down within the theoretical system of classical mechanics of particles and fields, but can be neither proven nor disproven within this system. The Maxwell equations (5.72) and the electrodynamics laws that they represent, provide* **new axioms** *to the theoretical system of the classical mechanics of particles and fields applied to charged particles and electromagnetic fields. In turn, Section 5.1 shows these equations to follow from the gauge principle, which therefore is the one (overarching)* **new axiom***; see also Appendix B.3.*

1.1.3 Philosophers are not scientists

A second glance at this framework of thought reveals something extraordinary! The scientific models[7] described here, and systems of such models forming theories and theoretical systems, are improved and extended, but not *literally falsified*, i.e., proven to be unconditionally false! (For the most part, it is rather our mental imagery and philosophical “underpinnings” of the scientific model that are taken too seriously, and may have to be abandoned as false.) Radical revisions of course do occur in scientific research – and not so infrequently – but that does not *falsify* established models and theories, only perhaps an unwarranted trust that those models and theories would be exact and absolute truths. Properly understood within their qualifications, models and theories of fundamental physics have not been falsified throughout the past three centuries, but have been and continue to be refined, extended and often united.

Reasons for this are found in comparing scientific models with earlier efforts and doctrines. Scientific models unify the inspiration of (experimental) induction with the rigor, self-consistency and persistence of (rigorous mathematical and logical) deduction.

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[4] In this context, the verb “to model” encompasses the creation of the mathematical model that describes the scrutinized phenomenon, and that can be summarized into an applicable formula. Whence stems the *law* for the system wherein the phenomenon is observed, and “to model” then includes “to introduce as a law of Nature.” However, this is not an absolute and inviolable law by decree, but one that is subject to verifications in comparisons with Nature, and adaptations to this one and ultimate arbiter.

[5] . . . barring the dismal logical possibility of the scientific spirit dying out or becoming exterminated. . .

[6] These are essentially undecideable statements; see the lexicon entry on Gödel’s incompleteness theorem, in Appendix B.1, and Appendix B.3 in particular.

[7] A *scientific model* includes the mathematical model together with its concrete interpretation: formulae, algorithms, programs, together with their physical meaning, i.e., a dictionary between the symbols of the mathematical model and the corresponding quantities in Nature. In this sense, a “model” then also implies a “law” – in the sense of Newton’s, Ampère’s or Gauss’s law, not in the sense of a decree of some legislative body. The notion of “natural law” is thus integrally woven into the scientific modeling of Nature, far from it having been abandoned, as sometimes opined [533].

This complementary combination of *quantitative measurements* and *mathematical modeling* is often attributed to the revolution in the philosophical approach in studying Nature, and is most often linked to Galileo and Newton. However, Eratosthenes' and Aristarchus' above-cited planetological results were clearly based on this same combination of methods. This idea is therefore over two millennia old. Suppressed through most of the past two millennia, this same combination of measurements and mathematics was methodically and consistently revived by Galileo, Newton and their followers. With the development of mathematics – and especially of calculus, invented for that purpose by Newton, Leibniz and contemporaries[8] – physics engaged into *warp* drive (the superluminal propulsion from the sci-fi series *Star Trek*).

Roughly, measurements translate quantities describing observed natural phenomena into corresponding quantities in a mathematical model. This model is then used as a faithful (as best as known) representative and replacement of the natural phenomenon. It is also a persistently rigorous tool for deductive predictions about that natural phenomenon. Those predictions are then checked in turn, the model adapted, corrected and improved, if and when the predictions turn out to differ from what is observed in Nature.

Thus, Einstein's theory of relativity does not *falsify* Newton's mechanics but *extends* it: When all relative speeds in a system are much less than the speed of light in vacuum, relativistic corrections to Newton's mechanics are negligible and Newton's mechanics yields a perfectly usable model of reality. If some of the relative speeds increase, the corresponding corrections become relevant, Newton's mechanics is no longer a good enough approximation (the errors, about which physics always must be precise, become unacceptably large), and we must use the relativistic formulae. In turn, Einstein's relativistic physics cannot be claimed to be absolutely true/exact either, but merely that it is more accurate than Newton's. After all, we already know that quantum physics may well force us to revise the structure (and perhaps even the nature) of spacetime itself when approaching Planck-length scales. Science can only make qualified statements, the "truth" of which will always depend on precision (resolution) – and which continues to improve in ways that no one can foresee.

Insisting on the iteration of this precision-sensitive "observe–model–predict" cycle immediately discards "theories" such as the one about *phlogiston*, the supposed intangible substance of heat. That "theory" neither explained nor predicted quantitative data, and may be called a "theory" only in common, non-technical parlance. A similar fate befell the so-called "plum pudding" model of the atom, which explained and predicted very little (and incorrectly), and which its Author humbly called a "model" worth exploring, and mercilessly abandoning if found faulty; which it was – both faulty and abandoned.

It is absolutely crucial that what we intend to call a *scientific* theory must be subject to verification through comparison with Nature, at least in principle. This implies that a theory must be *quantitative*, i.e., a theory must explain and predict experimental data, which can be checked. Quantitative predictions may be as simple as "yes/no" results; whether one predicts a single bit of information or an entire googolplex[9] of them – predictions must contain *new* information.

A word of warning: "subject to testing" does not mean that we can simply call up the local lab, order some results, and expect a twenty-minute delivery. Nor does it mean that even a planetary budget could fund the required experiment (not that there will be a planetary budget any time soon). Nor does it mean that anyone has even the faintest hint of an idea for a concrete experiment, even with a pan-galactic budget and a post-*Star Trek* technology. However, the theory must be

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[8] It has recently been discovered that Archimedes knew about the concepts of limit and the principle of exhaustion [382], but that this knowledge has been neglected and forgotten for the better part of two millennia.

[9] *Googol* (which must not be confused with *Google*) is the number 10^{100}; *googolplex* is the number $10^{10^{100}}$. For comparison, there are only about $\mathbf{N} := 10^{80} \ll 10^{100}$ particles in the universe, but the number of all their k-fold relations is immensely larger than googol, $\sum_k \binom{\mathbf{N}}{k} = 2^{\mathbf{N}} \ggg 10^{100}$, and the number of all relationships between those relationships (as a second-order estimate of complexity) is much larger than googolplex, $2^{2^{\mathbf{N}}} \ggg 10^{10^{100}}$.

"subject to testing, in principle": thought experiments may be envisioned rigorously, and their execution is obstructed by neither political economy nor practical "minutiae" such as magnetizing a mountain-size apparatus. Of course, the models that may be tested may be either demonstrated as *tentatively* established,[10] or discarded if they can be shown to disagree with Nature.

It cannot be over-emphasized (see, however, also Digression 1.1 below, as well as Sections 8.3.1 and 11.1.4 and Appendix B.3):

Conclusion 1.2 *Models that can (in principle) be refuted are scientific.*

Interestingly, a verb (in Chinese) is, by definition, a word that can be negated [578]. However, the correct application of this criterion, so simply stated, supposes a detailed understanding of the structure of scientific systems, to which we return in Section 8.3.

Digression 1.1 The principle of Conclusion 1.2 reminds us of the principle of falsifiability, popularized by Karl Popper [443, 444]. Intending to describe the historical process of the evolution of science, he concluded that experiments about atoms *falsify* classical physics, which is then *substituted* by quantum physics since that successfully describes atoms. So understood, the *principle of falsifiability* harbors at least two equivocations: (**1**) the naive version equates it with the related "testability" and presupposes direct and unequivocal experimental testing, and (**2**) equivocation in categories. Both equivocations are dangerous to the socio-political status of science. Also, the tacit assumption that all statements of a model are necessarily either confirmable or falsifiable, which simply need not be the case [☞ the lexicon entry on Gödel's incompleteness theorem, in Appendix B.1, and also Appendix B.3].

The first equivocation is based on a restriction of physics as a science to a "directly empirical" science, whereby a theory that we cannot experimentally test is being denied its "scientificity." However, there exist (in the scientific and the sci-fi literature and media) effects that contradict no known science, but for the experimental testing of which [☞ also Refs. [171, 505]]:

1. the resources are too expensive (e.g., a synchrotron around the Earth or around the Sun and Proxima Centauri, not to dream of a tokamak from here to Andromeda),
2. the requisite procedures are prohibited by moral or ethical reasons (e.g., cloning, bionic, and certain educational, behavioral and nutritional experimentation),
3. the resources require an as yet unknown technology (e.g., painting the ceiling of a room with neutronium would cancel gravity in the room – if "only" we knew how to produce neutronium paint and how to paint the ceiling without it caving in),
4. a new concept and/or methodology is needed (e.g., for a direct measurement of an *upper* limit of the proton's lifetime).

It is already intuitively clear that not one of these obstructions for experimental testing should take away from the "scientificity" of a theory. And, even simpler, it is clear that experiments with stars, positions of the constellations and the development of our own universe cannot be performed at will, nor is setting up an experimental control group

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[10] Being forever subject to future and additional testing, "established" can in this context only ever be understood as *tentative*; this is a "small" detail that is rarely stated explicitly, but must always be understood.

possible even in principle! Nevertheless, it is just as clear that astrophysics, astronomy and cosmology are no less “scientific” for this.

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The second possible equivocation is more subtle, and so also more dangerous. Also, it has at least two aspects. On one hand, there is the danger of confusing the category to which a certain *theoretical structure* belongs. For example, “classical physics” is not a particular model with particular predictions that may be experimentally tested, but a scientific system of assumptions (axioms) and procedures of derivation; this then may be applied to concrete phenomena, such as a pendulum, a bob on a spring, or the atom. The incorrectness of any one concrete model – as in the case of the classical model of the atom (see however Footnote *11* on p. 310 as well as example B.2) – may imply an error in the *application* of classical mechanics or in classical mechanics *itself*, or perhaps even elsewhere in the underlying complete chain of reasoning. We must explore precisely which of the assumptions lead to the observed disagreement with Nature. In fact, the application itself may turn out to harbor an error for various reasons, from a minor technicality to a fundamental inappropriateness. That, after all, is the usual advisory about all proofs by contradiction. However, it would evidently be silly to deny the “scientificity” of classical physics as a whole because of its inability to model the atom.

On the other hand, the very idea that a scientific theory *falsifies* another is a dangerous equivocation. Both in common parlance and in legal practice, the verb “to falsify” implies that the statement being falsified is being shown to be a falsehood. This, in turn, implies the tacit expectation of a binary true/false value. However, it is – or should be – very well known that the relation between quantum and classical physics is *continuous* and depends on the context and “resolution.” For any process under scrutiny, we must compute the ratio of $\hbar$ with all characteristic actions and all other commensurate physical quantities.[11] If each of these ratios is *sufficiently smaller* than 1, the numerical errors in the results computed using classical physics are *negligible*. It is evident that “sufficiently small” here implies a finite and an a-priori established tolerance. Therefore, the answer to questions such as “is classical physics applicable even to a single particular event?” essentially depends on at least one *continuous* parameter, and the answer cannot possibly be an unconditional “yes/no.” Classical physics is therefore extended/generalized and not *falsified* by quantum physics. The situations with relativity, field theory, and even superstring theory are analogous.

Generally, physicists understand that quantum physics does not simply *falsify* classical physics, but *extends* it into a domain where classical physics is *not sufficiently precise*. Unfortunately, philosophers of science are not physicists. This pragmatic approach should be compared with a similar vantage point of philosophers of natural sciences such as Thomas S. Kuhn [323], where one needs to know that Kuhn obtained his BS (1943), MS (1946) and PhD (1949) degrees in *physics* at Harvard, where he lectured on history of physics 1948–56. However, Kuhn opines that theories (and paradigms) are chosen by the group of researchers that is more successful than others, and assigns this choice a degree of socio-politically pliable subjectivity. This seems all too alien to most physicists I know,

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[11] In elementary particle physics one uses so-called natural units, based on the natural constants $\hbar$ and c, whereupon these are not written explicitly, and formally one says that “$\hbar = 1 = c$.” This practice may well be used in any complete unit system: once in agreement to use SI units, “length of 10” may only mean “10 m,” “force of 5” may only mean “$5\,\mathrm{N}{=}5\,\mathrm{kg\,m/s^2}$,” etc. However, as the purpose of this book is to *introduce* the Reader into this practice, factors of $\hbar$ and c are herein written explicitly, but in gray ink.

and which I myself (perhaps all too naively) cannot accept for physics itself, nor any other science, but only and at most for the admittedly capricious socio-historical process of development in a particular subfield.

Suffice it here then to just assert without a historically and statistically justified argument – as a manifesto, if need be: Theories and theoretical systems in physics are chosen by Nature itself, through our long and patient communication with it (in the sense of the caricature in Figure 1.3 on p. 6). Albeit extremely challenging and difficult at times, this is always well worth the effort and ardor.

Of course, it is logically impossible for a science to be exact without being quantitative. That is, “exactness” must be accepted as the requirement that it must be possible to develop a system of questions that can be answered by precise yes/no answers. Subsequently, these answers (easily written as a binary number) may quantitatively characterize the events to be modeled – and to be predicted. If these yes/no answers follow a statistical (probabilistic) distribution, this is only a *technical* complication and does not take away from the “exactness” in this sense.[12] This is always true of all branches of physics. While statistical physics and quantum mechanics are probabilistic, this only complicates the techniques and dictates the style of research. In fact, many fine (mathematical) techniques of statistics specify precisely which questions are meaningful to ask, which are meaningless, and which among the former ones have a definitive answer, which “only” a probabilistic one. For example, the temperature (as the average kinetic energy) of a fluid may be predicted precisely, but the kinetic energy of a single molecule is subject to fluctuations; the distribution of these fluctuations is predictable precisely, but not their individual values in practice, owing to the too large number of contributing factors, such as the repeated collisions with 10^{26}’s of molecules. The kinetic energy of a single molecule may in practice then only be known probabilistically – even if it were possible to mark and follow a single molecule without disturbing and changing it.

From this point of view, physics and science in general may be accused of being pragmatic, which they indeed are to a considerable degree. However, it is pragmatic physics and science that brought us Moon rocks, pictures of the surface of Jupiter’s satellites and of distant galaxies and nebulae, and which can find extrasolar planets; that produce artificial heart valves that the human immune system accepts; that can provide early signs of hurricanes, cyclones and tsunamis so as to warn the endangered population. Unfortunately, ethically and morally wrong, and just plain uninformed application of science may also lead to our planet radioactively glowing in the dark of the universe, or “only” to lose all ice and heat up to a point where life as we know it is no longer possible. Through this feedback, science also affects our thinking, our opinions and convictions, and so influences almost everything else, thereby being far more than “just pragma.”

The foregoing also uncovers the price to pay: although physics is about Nature, it describes Nature indirectly, by way of the models that are sufficiently (and ever more) precise.

Example 1.1 The statement “in Rutherford’s planetary model, the atom consists of a nucleus at the center and the electrons orbiting around it” does not mean that atoms literally exist in such simple tinkertoy form. More precisely, one means that the mathematical model developed from this *mental caricature* nevertheless reproduces the so far addressed real observations with sufficient accuracy.

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[12] Let’s recall: exact science always errs, but knows precisely how much [☞ Conclusion 1.1 on p. 6].