CONFORMAL METHODS IN GENERAL RELATIVITY

This book is a systematic exposition of conformal methods and how they can be used to study the global properties of solutions to the equations of Einstein's theory of gravity. It shows that combining these ideas with techniques of the theory of partial differential equations can elucidate the stability of the basic solutions of the theory. Introducing the differential geometric, spinorial and PDE background required to gain a deep understanding of conformal methods, this text provides an accessible account of key results in mathematical relativity over the last 30 years, including the stability of de Sitter and Minkowski spacetimes.

For graduate students and researchers, this self-contained account includes useful visual models to help the reader grasp abstract concepts and a list of further reading, making this the perfect reference companion on the topic.

This title, first published in 2017, has been reissued as an Open Access publication on Cambridge Core.

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ἀγεωμέτρητος μηδεὶς εἰςίτω (Let no one untrained in geometry enter)

– *Epigram at the Academy of Plato*

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Preface

This book discusses an approach to the study of global properties of solutions to the equations of general relativity, the Einstein field equations, in which the notion of conformal transformation plays a central role. The use of conformal transformations in differential geometry dates back, at least, to the work of *Hermann Weyl* in the 1920s.[1] Their application to global questions in general relativity, as presented in this book, stems from the seminal work of *Roger Penrose* in the 1960s in which the close connection between the global causal structure of the solutions to the equations of general relativity and conformal geometry was established.[2] Penrose's key insights are that the close relation between the propagation of the gravitational field and the structure of light cones which holds locally in a spacetime is also preserved in the case of large scales and that the asymptotic behaviour of the gravitational field can be conveniently analysed in terms of conformal extensions of the spacetime. In the following decade Penrose's ideas were polished, extended and absorbed into the mainstream research of general relativity by a considerable number of researchers[3] – finally leading to the influential notion of *asymptotic simplicity*. The subject reached its maturity when this *formal* theory was combined with the methods of the theory of partial differential equations (PDEs). This breakthrough is mainly due to the work of *Helmut Friedrich* in the early 1980s, who – through the *conformal Einstein field equations*[4] – showed that ideas of conformal geometry can be used to establish the existence of large classes of solutions to the Einstein field equations satisfying Penrose's notion of asymptotic simplicity. As a result of this work it is now clear that Penrose's original insights hold for large classes of spacetimes and not only for special explicitly known solutions.

This book develops the theory of the conformal Einstein field equations from the ground up and discusses their applications to the study of asymptotically simple spacetimes. Special attention is paid to results concerning the existence and stability of *de Sitter-like spacetimes*, the semiglobal existence and stability of *Minkowski-like spacetimes* using hyperboloidal Cauchy problems and the

[1] See Weyl (1968).
[2] See Penrose (1963, 1964).
[3] See e.g. Hawking and Ellis (1973); Geroch (1976).
[4] See Friedrich (1981a,b, 1983).

construction of *anti-de Sitter-like spacetimes* from initial boundary value problems. These results belong to the canon of modern mathematical relativity. In addition to their mathematical interest, they are of great physical relevance as they express, among other things, the internal consistency of general relativity and provide an approach for the global evaluation of spacetimes by means of numerical methods.

Why a book on the subject? The applications of conformal methods in general relativity constitute a mature subject with a number of *core results* which will withstand the pass of time. Still, it provides a number of challenging open questions whose resolution will strengthen its connections with other research strands in general relativity. This book aims at making the subject accessible to physicists and mathematicians alike who want to make use of conformal methods to analyse the global structure and properties of spacetimes. Hopefully, this book will provide an alternative to the use of original references while learning the subject or doing research.

Anyone who wants to engage with the subject of this book faces a number of challenges. To begin with, one has a vast literature spreading over more than 50 years. As it is to be expected from a living subject, the perspectives change through time, the importance of certain problems rise and wane and it is sometimes hard to differentiate the fundamental from the subsidiary. The combination of results from various references is often hindered by changing notation and conflicting conventions. Moreover, to appreciate and understand the results of the theory one requires a considerable amount of background material: conformal geometry, spinors, PDE theory, causal theory, etc. These methods are an essential part of the toolkit of a modern mathematical relativist. This book endeavours to bring together in a single volume all the required background material in a concise and coherent manner.

As a cautionary note, it should be mentioned what this book is not intended to be. This book is not an introductory book to general relativity. A certain familiarity with the subject is assumed from the outset – ideally at the level of Part I of R. Wald's book *General Relativity*.[5] This is also not a book on the applications of the theory of PDEs in general relativity. For this, there are other books available.[6] Also, although the Cauchy problem in general relativity is a leading theme, this book should not be viewed as a monograph on the topic – for this, I refer the interested reader to H. Ringström's monograph.[7]

I have endeavoured to write a book which not only serves as an *introduction to the subject* but also is a *tool for research*. With this idea in mind, I have striven to provide as much detail as possible of the arguments and calculations. However, at some stages supplying further details is neither possible nor desirable. Indeed, quoting the preface of J. L. Synge's classical book on general relativity: "There

[5] See Wald (1984).
[6] See e.g. Choquet-Bruhat (2008); Rendall (2008).
[7] See Ringström (2009).

are heavy calculations in the book, but there are places where the reader will find me sitting on the fence, whistling, instead of rushing into the fray"; see Synge (1960).[8] In an attempt to keep the readability and the length of the text under control, I have not endeavoured to provide completely general or optimal theorems – the attentive reader will realise this and is referred to the literature for further details, if required. As a picture is better than a thousand words, I have complemented the text with a considerable number of figures and diagrams which, I hope, will help to explain the content of the main text and provide useful visual models.

In writing this book, I have assumed the reader to have a certain mathematical maturity. Some basic knowledge of topology is needed – Appendix A in Wald's book contains the required background – as well as familiarity with basic tensorial calculus. I have, however, not assumed any prior knowledge of 2-spinors. The necessary toolkit is developed in the course of two chapters. Readers looking for a supplementary source on the topic are referred to J. Stewart's book.[9] The applications of conformal methods discussed in this book require certain knowledge of the theory of PDEs. I provide all the required material in a chapter of its own – nevertheless, some previous exposure to the basic ideas of the theory of PDEs is an advantage. Some arguments in the book make use of very concrete results of analysis. In these cases, I have included the necessary ideas in appendices to the various chapters.

[8] I am thankful to R. Beig for bringing my attention to this quote.
[9] See Stewart (1991).

Acknowledgements

First and foremost, I would like to acknowledge the influence that Helmut Friedrich had in the development of my understanding of the theory of the conformal Einstein field equations. He has been an inexhaustible source of knowledge and insights. I also would like to acknowledge the role that Robert Beig, Sergio Dain and Malcolm MacCallum have played in shaping complementary perspectives on the subject. I am also grateful for the extensive discussions with my collaborators Christian Lübbe and Jose Luis Jaramillo which have allowed to form, define and refine some of the points of view and concerns presented in this book.

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Although all the calculations presented in this book have been carried out by hand, the system `xAct` for `Mathematica` has been a valuable resource in case of trouble. I am thankful to Thomas Bäckdahl and Alfonso García-Parrado for their help at various stages with this system. I also thank Peter Hübner and Anil Zenginoglu for allowing me to use figures and diagrams from their publications.

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Juan A. Valiente Kroon

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Symbols

$d^*_{\boldsymbol{ij}}$ alternative description of the components of the magnetic part of the rescaled Weyl tensor, page 264

$(\boldsymbol{e}_{\boldsymbol{a}}, \Gamma_{\boldsymbol{a}}{}^{\boldsymbol{b}}{}_{\boldsymbol{c}}, \Xi, s, L_{\boldsymbol{ab}}, d^{\boldsymbol{a}}{}_{\boldsymbol{bcd}}, T_{\boldsymbol{ab}})$ unknowns in the frame version of the standard conformal field equations, page 196

$(\boldsymbol{e}_{\boldsymbol{a}}, \hat{\Gamma}_{\boldsymbol{a}}{}^{\boldsymbol{b}}{}_{\boldsymbol{c}}, \hat{L}_{\boldsymbol{ab}}, d^{\boldsymbol{a}}{}_{\boldsymbol{bcd}}, T_{\boldsymbol{ab}}, \Xi, d_{\boldsymbol{a}})$ unknowns in the frame version of the extended conformal field equations, page 205

$(\boldsymbol{e}_{\boldsymbol{AA'}}, \Gamma_{\boldsymbol{AA'BC}}, \Xi, s, L_{\boldsymbol{AA'BB'}}, \phi_{\boldsymbol{ABCD}}, T_{\boldsymbol{AA'BB'}})$ unknowns in the spinorial version of the standard conformal field equations, page 199

$(\boldsymbol{e}_{\boldsymbol{AA'}}, \hat{\Gamma}_{\boldsymbol{AA'BC}}, \hat{L}_{\boldsymbol{AA'BB'}}, \phi_{\boldsymbol{ABCD}}, T_{\boldsymbol{AA'BB'}}, \Xi, d_{\boldsymbol{AA'}})$ unknowns in the spinorial version of the extended conformal field equations, page 208

$(\boldsymbol{o}, \boldsymbol{\iota})$ spin basis in index-free notation, page 66

$(\hat{\Sigma}_{\boldsymbol{AA'BB'}}, \hat{\Xi}^{\boldsymbol{C}}{}_{\boldsymbol{DAA'BB'}}, \hat{\Delta}_{\boldsymbol{CC'DD'BB'}}, \hat{\Lambda}_{\boldsymbol{BB'CD}}, \delta_{\boldsymbol{AA'}}, \varsigma_{\boldsymbol{AA'BB'}}, \gamma_{\boldsymbol{AA'BB'}})$ zero quantities in the spinorial extended conformal field equations, page 208

$(\hat{\Sigma}_{\boldsymbol{ab}}, \hat{\Xi}^{\boldsymbol{c}}{}_{\boldsymbol{dab}}, \hat{\Delta}_{\boldsymbol{cdb}}, \Lambda_{\boldsymbol{bcd}}, \delta_{\boldsymbol{a}}, \gamma_{\boldsymbol{ab}}, \varsigma_{\boldsymbol{ab}})$ zero quantities in the frame extended conformal field equations, page 205

$(\mathcal{M}, \boldsymbol{g})$ generic spacetime, page 45

$(\mathcal{U}, \varphi)$ coordinate chart, page 28

$(\Sigma_{\boldsymbol{AA'BB'}}, \Xi^{\boldsymbol{C}}{}_{\boldsymbol{DAA'BB'}}, Z_{\boldsymbol{AA'BB'}}, Z_{\boldsymbol{AA'}}, \Delta_{\boldsymbol{CDBB'}}, \Lambda_{\boldsymbol{BB'CD}}, Z, M_{\boldsymbol{AA'}})$ zero quantities in the spinorial version of the standard conformal equations, page 199

$(\Sigma_{\boldsymbol{ab}}, \Xi^{\boldsymbol{c}}{}_{\boldsymbol{dab}}, Z_{\boldsymbol{ab}}, Z_{\boldsymbol{a}}, \Delta_{\boldsymbol{cdb}}, \Lambda_{\boldsymbol{bcd}}, Z, M_{\boldsymbol{a}})$ zero quantities in the frame version of the standard conformal field equations, page 196

$(g_{ab}, \Xi, s, L_{ab}, d^a{}_{bcd}, T_{ab})$ unknowns in the metric standard conformal field equations, page 191

$(h_{ij},\ s_{ij},\ \zeta,\ \varsigma)$ unknowns in the conformal static equations, page 511

(o^A, ι^A) spin basis in abstract index notation, page 71

$(u, r, \theta^{\mathcal{A}})$ Bondi coordinates, page 236

$(x(\tau), \boldsymbol{\beta}(\tau))$ conformal geodesic with parameter τ, page 127

(x^μ) local coordinates in a four-dimensional manifold, page 28

$[\nabla_a, \nabla_b]$ commutator of covariant derivatives, page 39

$[[\boldsymbol{\xi}, \boldsymbol{\eta}]]$ antisymmetric product of $\boldsymbol{\xi}$, $\boldsymbol{\eta} \in \mathfrak{S}$, page 65

$[\boldsymbol{g}]$ conformal class of the metric $\boldsymbol{g}$, page 113

$[\boldsymbol{u}, \boldsymbol{v}]$ commutator of the vector fields $\boldsymbol{u}$ and $\boldsymbol{v}$, page 34

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$\alpha_{\boldsymbol{a}}$, $\beta_{\boldsymbol{a}}$, $\omega_{\boldsymbol{a}}, \ldots$ components of the covectors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\omega}, \ldots$ with respect to the frame $\{\boldsymbol{e_a}\}$, page 51
α_a, β_a, $\omega_a, \ldots$ generic covectors in abstract index notation
$\approx$ diffeomorphism between sets, page 27
$\bar{\xi}^{A'}$, $\bar{\eta}_{A'}, \ldots$ complex conjugates of the spinors ξ^A, $\eta_B, \ldots$, page 72
β^2 norm of the covector $\tilde{\boldsymbol{\beta}}$, page 134
β_a covector associated to a conformal geodesic in abstract index notation, page 127
$\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\omega}, \ldots$ generic covectors in index-free notation
$\boldsymbol{\beta}$ covector associated to a conformal geodesic in index-free notation, page 127
$\boldsymbol{\chi}$ Weingarten map, page 56
$\boldsymbol{D}$ generic three-dimensional connection in index-free notation
$\boldsymbol{d}$ rescaled conformal geodesic covector, page 201
$\boldsymbol{\delta}$ Euclidean metric on $\mathbb{R}^3$, page 143
$\boldsymbol{e}$, $\boldsymbol{e_{AB}}$ space spinor irreducible components of the frame vector $\boldsymbol{e_{AA'}}$, page 104
$\boldsymbol{\ell}$ three-dimensional Lorentzian metric on the conformal boundary of an anti-de Sitter-like spacetime, page 456
$\boldsymbol{f}$ covector defining a Weyl connection in index-free notation, page 119
$\boldsymbol{f}$ unphysical conformal geodesic covector, page 201
$\boldsymbol{g}$ generic four-dimensional Lorentzian metric tensor in index-free notation
$\boldsymbol{g}^\sharp$ generic contravariant four-dimensional Lorentzian metric tensor in index-free notation
$\boldsymbol{g}_{\mathscr{E}}$ standard metric on the Einstein cylinder, page 144
$\boldsymbol{\gamma}$ metric in the quotient manifold, page 141
$\boldsymbol{h}$ generic (negative definite) Riemannian three-dimensional metric
$\boldsymbol{\hbar}$ standard metric on the unit 3-sphere, page 142
$\boldsymbol{K}$ extrinsic curvature tensor of a hypersurface in index-free notation, page 61
$\boldsymbol{k}$ intrinsic metric of compact two-dimensional surfaces
$\boldsymbol{M}$, $\boldsymbol{N}, \ldots$ generic higher rank tensors in index-free notation
$\boldsymbol{N}$ tangent to the generators of null infinity
$\boldsymbol{\nabla}$, $\bar{\boldsymbol{\nabla}}$ generic linear connections in index-free notation, page 38
$\boldsymbol{\nu}$ unit normal to a hypersurface $\mathcal{S}$, page 54
$\boldsymbol{\omega}$, $\boldsymbol{\omega^{AB}}$ space spinor irreducible components of the frame covector $\boldsymbol{\omega^{AA'}}$, page 104
$\boldsymbol{\partial}_\mu$ coordinate basis vector
$\boldsymbol{Q}$ transition tensor between connections in index-free notation, page 42
$\boldsymbol{q}$ intrinsic metric of null infinity
$\boldsymbol{\Sigma}$ torsion tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 39
$\boldsymbol{\sigma}$ standard metric on the unit 2-sphere
$\boldsymbol{\sigma}_{\mathbf{L}}(\boldsymbol{\xi})$ symbol of a differential operator $\mathbf{L}$, page 252
$\boldsymbol{t}$ vector field generating a timelike congruence
$\boldsymbol{\tau}$ vector counterpart of the spinor $\tau_{AA'}$, page 102
$\boldsymbol{v}$, $\boldsymbol{u}$, $\boldsymbol{w}, \ldots$ generic vectors in index-free notation

ς shear tensor, page 226
$\boldsymbol{z}$, $\boldsymbol{\zeta}$ deviation vector and covector, respectively, page 135
$\boldsymbol{\mathcal{L}_h}$ conformal Killing operator of the metric $\boldsymbol{h}$, page 257
$\breve{\mathbf{u}}$ perturbation quantity in an evolution system
$\chi_{(AB)CD}$ spinorial counterpart of the Weingarten tensor
χ_{AB} spinorial counterpart of the acceleration vector
$\circ$ composition of functions, page 36
$\coprod$ disjoint union of sets
$\delta(i)$ Dirac's delta, page 279
$\Delta_{\boldsymbol{h}}$ Laplacian operator of the Riemannian metric $\boldsymbol{h}$
$\delta_\mu{}^\nu$, $\delta_a{}^b$, $\delta_i{}^j$, $\delta_\alpha{}^\beta$, $\delta_A{}^B$, $\delta_{\boldsymbol{A}}{}^{\boldsymbol{B}}$, $\delta_{\boldsymbol{a}}{}^{\boldsymbol{b}}$, $\delta_{\boldsymbol{i}}{}^{\boldsymbol{j}}$ Kronecker's delta
$\delta_{\alpha\beta}$ components of the three-dimensional Euclidean metric in Cartesian coordinates, page 47
$\delta_{\boldsymbol{AB}}$ Sen connection on a timelike conformal boundary, page 471
$\delta_{\boldsymbol{ij}}$ components of a three-dimensional Riemannian metric with respect to an orthonormal basis, page 45
$\dot{\boldsymbol{\gamma}}(\mathrm{s})$ tangent vector to a curve, page 30
$\dot{\boldsymbol{x}}(\mathrm{s})$ alternative notation for the tangent vector to a curve, page 30
$\epsilon = \pm 1$ encodes the causal character of a hypersurface, page 54
$\epsilon_{\boldsymbol{abcd}}$ components of the volume form with respect to an orthonormal basis
$\epsilon_{\boldsymbol{AB}}$, $\epsilon^{\boldsymbol{AB}}$ components of the spinors ϵ_{AB}, ϵ^{AB} with respect to a spin basis, page 71
$\epsilon_{A'B'}$, $\epsilon^{A'B'}$ complex conjugates of the spinors ϵ_{AB}, ϵ^{AB}
$\epsilon_{AA'BB'CC'DD'}$ spinorial counterpart of the volume form, page 78
ϵ_{ABCDEF} spinorial counterpart of the three-dimensional volume form, page 105
ϵ_{abcd} volume form of a metric g_{ab}, page 49
ϵ_{AB}, ϵ^{AB} antisymmetric spinors, page 67
$\equiv$ definition
$\eta_{\boldsymbol{ABCD}}$ components of the electric part of the Weyl spinor, page 373
$\eta_{\boldsymbol{ab}}$ components of a four-dimensional Lorentzian metric with respect to an orthonormal basis, page 45
$\eta_{\mu\nu}$ components of the Minkowski metric tensor in Cartesian coordinates, page 47
ð, $\bar{ð}$ eth and eth-bar operators, page 241
exp exponential map, page 275
Γ geodesic distance, page 276
$\gamma(\mathrm{s})$ curve in a manifold with parameter s, page 30
$\Gamma_{\boldsymbol{a}}{}^{\boldsymbol{c}}{}_{\boldsymbol{b}}$ connection coefficients of $\boldsymbol{\nabla}$ with respect to $\{\boldsymbol{e_a}\}$
$\gamma_{\boldsymbol{i}}{}^{\boldsymbol{j}}{}_{\boldsymbol{k}}$ connection coefficients of the three-dimensional connection $\boldsymbol{D}$ with respect to the frame $\{\boldsymbol{e_i}\}$, page 59
$\Gamma_\mu{}^\nu{}_\lambda$ Christoffel symbols of the metric $\boldsymbol{g}$ in the coordinates (x^μ)
$\Gamma_{\boldsymbol{A'A'}}{}^{\boldsymbol{BB'}}{}_{\boldsymbol{CC'}}$ spinorial counterpart of the connection coefficients $\Gamma_{\boldsymbol{a}}{}^{\boldsymbol{b}}{}_{\boldsymbol{c}}$, page 82
$\Gamma_{\boldsymbol{AA'}}{}^{\boldsymbol{B}}{}_{\boldsymbol{C}}$ reduced spin connection coefficients, page 82

$\Gamma_{\boldsymbol{ABCD}}$ space spinor counterpart of the reduced spin connection coefficients $\Gamma_{\boldsymbol{AA'CD}}$, page 107
$\gamma_{\boldsymbol{AB}}{}^{\boldsymbol{C}}{}_{\boldsymbol{D}}$ reduced spatial spin connection coefficients, page 109
$\gamma_{\boldsymbol{AB}}{}^{\boldsymbol{CD}}{}_{\boldsymbol{EF}}$ spinorial counterpart of the three-dimensional connection coefficients $\gamma_{\boldsymbol{i}}{}^{\boldsymbol{j}}{}_{\boldsymbol{k}}$, page 109
$\grave{\Omega}$, $\check{\Omega}$ massless and, respectively, massive part of the conformal factor associated to Euclidean initial data sets, page 529
$\hat{\boldsymbol{\nabla}}$ generic Weyl connection in index-free notation, page 119
$\hat{\Gamma}_{\boldsymbol{a}}{}^{\boldsymbol{b}}{}_{\boldsymbol{c}}$ connection coefficients of a Weyl connection $\hat{\boldsymbol{\nabla}}$, page 119
$\hat{\Gamma}_{\boldsymbol{AA'}}{}^{\boldsymbol{B}}{}_{\boldsymbol{C}}$ reduced Weyl connection spin coefficients, page 206
$\hat{\nabla}_a$ generic Weyl connection in abstract index notation, page 119
$\hat{\rho}^{\boldsymbol{c}}{}_{\boldsymbol{dab}}$ Weyl connection algebraic curvature, page 203
$\hat{\rho}_{\boldsymbol{ABCC'DD'}}$ Weyl connection reduced spinorial algebraic curvature, page 207
$\hat{P}^{\boldsymbol{c}}{}_{\boldsymbol{dab}}$ Weyl connection geometric curvature, page 203
$\hat{P}_{\boldsymbol{ABCC'DD'}}$ Weyl connection reduced spinorial geometric curvature, page 207
κ conformal factor associated to the construction of the cylinder at spatial infinity, page 541
Λ Newman-Penrose Ricci scalar, page 87
λ cosmological constant, page 2
$\Lambda_{(\boldsymbol{ABCD})}$, $\Lambda_{\boldsymbol{AB}}$ irreducible components of the spinorial Bianchi equation, page 351
$\langle \boldsymbol{\omega}, \boldsymbol{v} \rangle$ action of the covector $\boldsymbol{\omega}$ on the vector $\boldsymbol{v}$
$\langle \boldsymbol{t} \rangle^{\perp}|_p$ subspace orthogonal to $\boldsymbol{t}$, page 55
$\langle \boldsymbol{t} \rangle$ one-dimensional subspace spanned by $\boldsymbol{t}$, page 55
$\langle\langle \boldsymbol{\xi}, \boldsymbol{\eta} \rangle\rangle$ Hermitian product of $\boldsymbol{\xi}, \boldsymbol{\eta} \in \mathfrak{S}$, page 94
$[\![\nabla_a, \nabla_b]\!]$ modified commutator of covariant derivatives, page 40
$\mathbb{H}^n$ n-dimensional half Euclidean space, page 29
$\mathbb{R}^+$ non-negative real numbers
$\mathbb{R}^2$ Euclidean plane
$\mathbb{R}^n$ n-dimensional Euclidean space
$\mathbb{S}^2$ 2-sphere
$\mathbb{S}^3$ three-dimensional unit sphere, page 142
$\mathbf{A}^*$ transpose of the complex conjugate of the matrix $\mathbf{A}$
$\mathbf{A}^3$ normal matrix in an initial boundary value problem, page 314
$\mathbf{A}^\mu$ symmetric matrices in a symmetric hyperbolic system, page 294
$\mathbf{d}$ exterior derivative (differential), page 31
$\mathbf{d}x^\mu$ coordinate basis covector
$\mathbf{L}$ generic differential operator
$\mathbf{L}^*$ formal adjoint of the differential operator $\mathbf{L}$
$\mathbf{L}_{\boldsymbol{h}}$ Yamabe operator, page 256
$\mathbf{T}$ map associated to the prescription of boundary conditions in an initial boundary value problem, page 314
$\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}, \ldots$ $\mathbb{C}^N$-valued functions

$\mathcal{B}_a(p)$ ball of radius $a > 0$ centred at the point p
$\mathcal{C}_p$ null cone at a point $p \in \mathcal{M}$, page 45
$\mathcal{C}_p^+$, $\mathcal{C}_p^-$ future and, respectively, past null cone at a point $p \in \mathcal{M}$, page 45
$\mathcal{D}$ a generic derivation, page 30
$\mathcal{D}_{AB}$ Sen connection of $\nabla_{AA'}$ induced by $\tau_{AA'}$, page 105
$\mathcal{E}$ corner in an initial boundary value problem, page 314
$\mathcal{G}$ generic lens-shaped domain, page 301
$\mathcal{H}_k$ standard hyperboloids, page 154
$\mathcal{I}$ cylinder at spatial infinity, page 542
$\mathcal{I}^0$ intersection of the cylinder at spatial infinity with a Cauchy initial hypersurface, page 542
$\mathcal{I}^\pm$ critical sets where null infinity touches spatial infinity, page 542
$\mathcal{M}$, $\mathcal{N}$ generic (unphysical) spacetime manifolds
$\mathcal{N}$, $\mathcal{N}'$ initial null hypersurfaces in a characteristic problem, page 320
$\mathcal{N}_i$ complex null cone at i, page 522
$\mathcal{N}_{\mathbb{C}}(i)$ complexification of the null cone through i, page 532
$\mathcal{P}$ covariant derivative in the direction of $\tau_{AA'}$, page 105
$\mathcal{Q}$ generic quotient manifold, page 141
$\mathcal{R}$ generic subset of a hypersurface $\mathcal{S}$
$\mathcal{S}$ generic hypersurface on a manifold $\mathcal{M}$
$\mathcal{T}$ timelike boundary, page 314
$\mathcal{U}$, $\mathcal{V}$ generic open subsets of a manifold or $\mathbb{R}^n$
$\mathcal{U}_{\mathbb{C}}$ complexification of a neighbourhood $\mathcal{U}$ of the point at infinity, page 532
$\mathcal{Z}$ intersection of initial null hypersurfaces in a characteristic problem, page 320
$\mathfrak{S}$ complex vector space, page 65
$\mathfrak{S}(\mathcal{M})$ spin structure (spin bundle) over $\mathcal{M}$, page 81
$\mathfrak{S}(\mathcal{S})$ space spinor structure over a three-dimensional manifold $\mathcal{S}$, page 101
$\mathfrak{S}^*$ dual of the complex vector space $\mathfrak{S}$, page 65
$\mathfrak{S}^\bullet(\mathcal{M})$, $\mathfrak{S}_A(\mathcal{M})$, $\mathfrak{S}^A(\mathcal{M})$, $\mathfrak{S}_{AA'}{}^B(\mathcal{M}), \ldots$ various spin bundles over $\mathcal{M}$
$\mathfrak{S}^\bullet$ spin algebra, page 66
$\mathfrak{S}^A$, $\mathfrak{S}_A$, ... alternative notation for the vector spaces $\mathfrak{S}$, $\mathfrak{S}^*, \ldots$, page 66
$\mathfrak{S}^{A'}$, $\mathfrak{S}_{A'B'}$, ... complex conjugates of the spaces $\mathfrak{S}^A$, $\mathfrak{S}_{AB}, \ldots$, page 72
$\mathfrak{T}^\bullet(\mathcal{M})$ tensor bundle over $\mathcal{M}$, page 34
$\mathfrak{T}^a(\mathcal{M})$ alternative notation for the tangent bundle over $\mathcal{M}$, page 36
$\mathfrak{T}^{a_1\cdots a_k}{}_{b_1\cdots b_l}(\mathcal{M})$ alternative notation for the tensor bundle over $\mathcal{M}$, page 36
$\mathfrak{T}_a(\mathcal{M})$ alternative notation for the cotangent bundle over $\mathcal{M}$, page 36
$\mathfrak{X}(\mathcal{M})$ set of of scalar fields over $\mathcal{M}$, page 30
$\mathring{\mathbf{u}}$ background quantity in an evolution system
$\mathscr{C}$ generic cut of null infinity
$\mathscr{C}_\star$ fiduciary cut of null infinity
$\mathscr{E}$ extension operator of functions between Sobolev spaces, page 308
$\mathscr{I}$ part of the conformal boundary that is a hypersurface, page 178
$\mathscr{I}^\pm$ future and, respectively, past null infinity

$\mathscr{N}_i^+$, $\mathscr{N}_i^-$ null cones generated by the null geodesics through i, page 531
$\mathscr{N}_u$ outgoing null hypersurface associated to the retarded time u
$\mathscr{R}_{\boldsymbol{h}}$ linearised Ricci operator, page 289
$\mathscr{Z}$ generic intersection of null infinity with a null hypersurface
int $\mathcal{A}$ topological interior of the set $\mathcal{A}$, page 397
i square root of -1
$\mu_{\boldsymbol{ABCD}}$ components of the magnetic part of the Weyl spinor, page 373
$\nabla_{\boldsymbol{a}}$ covariant directional derivative in the direction of $\boldsymbol{e_a}$, page 51
$\nabla_{\boldsymbol{u}}\boldsymbol{v}$ covariant derivative of $\boldsymbol{v}$ with respect to $\boldsymbol{u}$, page 38
∇_a, $\bar{\nabla}_a$ generic linear connections in abstract index notation, page 38
$\nabla_{\boldsymbol{AA'}}$ directional spinorial covariant derivative, page 82
$\nabla_{AA'}$, $\tilde{\nabla}_{AA'}, \ldots$ spinor covariant derivatives, page 81
∇_{AB} space spinor counterpart of $\nabla_{AA'}$, page 105
Ω generic three-dimensional conformal factor
$\oplus$ direct sum
$\otimes$ tensor product between tensors or tensor spaces
$\overline{\mathcal{A}}$ topological closure of the set $\mathcal{A}$, page 394
$\parallel \mathbf{u} \parallel_{\mathcal{S},m}$ Sobolev norm of order m of a function over $\mathcal{S}$, page 306
$\partial\mathbb{H}^n$ boundary of the n-dimensional half Euclidean space, page 29
$\partial\mathcal{M}$ boundary of $\mathcal{M}$
ϕ unphysical conformally coupled scalar field, page 216
ϕ_0 radiation field in the asymptotic characteristic problem on a cone, page 500
$\Phi_{ABA'B'}$ spinorial counterpart of the trace-free Ricci tensor, page 89
Φ_{ab} trace-free Ricci tensor of a connection ∇_a in abstract index notation, page 48
ϕ_{AB} unphysical Maxwell spinor, page 215
Π generic distribution, page 55
$\Pi\,|_p$ hyperplane induced by a distribution at a point $p \in \mathcal{M}$, page 55
$\pounds_{\boldsymbol{v}}$ Lie derivative in the direction of $\boldsymbol{v}$, page 37
Ψ_{ABCD} Weyl spinor, page 87
ρ boundary-defining function, page 285
ρ polar radial coordinate, page 514
ρ^α three-dimensional unit position vector, page 514
$\rho^{\boldsymbol{C}}{}_{\boldsymbol{DAA'BB'}}$ reduced spinorial algebraic curvature, page 198
$\rho^{\boldsymbol{c}}{}_{\boldsymbol{dab}}$ components of the algebraic curvature, page 195
$\rho^{AA'}$ spatial spinor used to introduce a $1+1+2$ spinor formalism, page 464
$\boldsymbol{Ric}$, $\boldsymbol{Ric}[\boldsymbol{g}]$ Ricci tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 48
$\boldsymbol{Riem}$ Riemann curvature tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 40
$\boldsymbol{Schouten}$, $\boldsymbol{Schouten}[\boldsymbol{g}]$ Schouten tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 48
σ Newman-Penrose spin connection coefficient corresponding to $\Gamma_{\boldsymbol{01'00}}$
$\sigma^{\boldsymbol{a}}{}_{\boldsymbol{AA'}}$, $\sigma_{\boldsymbol{a}}{}^{\boldsymbol{AA'}}$ spacetime Infeld-van der Waerden symbols, page 74

$\Sigma_{\boldsymbol{a}}{}^{\boldsymbol{c}}{}_{\boldsymbol{b}}$ components of the torsion tensor with respect to an orthonormal frame, page 53
$\sigma_{\boldsymbol{i}}{}^{\boldsymbol{k}}{}_{\boldsymbol{j}}$, $\Pi^{\boldsymbol{k}}{}_{\boldsymbol{lij}}$, $\pi_{\boldsymbol{klij}}$ components of the three-dimensional torsion, geometric and algebraic curvatures, page 264
$\sigma_{\boldsymbol{i}}{}^{\boldsymbol{AB}}$, $\sigma^{\boldsymbol{i}}{}_{\boldsymbol{AB}}$ spatial Infeld-van der Waerden symbols, page 99
$\Sigma_a{}^c{}_b$ torsion tensor of a connection ∇_a in abstract index notation, page 39
$\simeq$ equality at the conformal boundary
$\square$ D'Alembertian operator, page 89
$\square_{AB}$ box commutator, page 89
$\overset{\star}{\simeq}$ equality at a fiduciary cut of null infinity
$\tau_{AA'}$ privileged timelike spinor inducing a space spinor formalism, page 102
Θ conformal factor associated to a conformal geodesic, page 132
$\theta = (\theta^{\mathcal{A}})$ local coordinates on $\mathbb{S}^2$
$\Theta_{\boldsymbol{ABCD}}$ space spinor counterpart of the components of the Schouten tensor of a Weyl connection, page 373
$\tilde{\boldsymbol{\eta}}$ Minkowski metric
$\tilde{\boldsymbol{g}}_{\mathscr{E}}$ metric of the anti-de Sitter spacetime, page 159
$\tilde{\boldsymbol{g}}_{\mathscr{S}}$ metric of the Schwarzschild spacetime, page 163
$\tilde{\boldsymbol{g}}_{dS}$ metric of the de Sitter spacetime, page 155
$\tilde{\mathcal{E}}_k$ asymptotic ends of asymptotically Euclidean manifold $\tilde{\mathcal{S}}$, page 272
$\tilde{\mathcal{F}}_{ab}$ self-dual Faraday tensor, page 213
$\tilde{\mathcal{M}}$ generic (physical) spacetime manifold
$\tilde{\phi}$ physical conformally coupled scalar field, page 216
$\tilde{\phi}_{AB}$ physical Maxwell spinor, page 215
$\tilde{\varrho}$ density of a perfect fluid, page 219
$\tilde{\varrho}$ energy density, page 254
$\tilde{F}_{ab}$ physical Faraday tensor, page 213
$\tilde{j}_k$ energy flux vector, page 254
$\tilde{p}$ pressure of a perfect fluid, page 219
$\tilde{T}_{ab}$ physical energy-momentum tensor
$\tilde{u}^a$ physical 4-velocity of a perfect fluid, page 219
$\underline{x}$ spatial coordinates (x^1, x^2, x^3)
Υ_a logarithmic gradient of a conformal factor, page 116
$\Upsilon_{AA'}$ spinorial counterpart of the logarithmic gradient of a conformal factor, page 123
φ^* pull-back, page 36
φ_* push-forward, page 36
$\varpi_{\boldsymbol{AA'}}$ components of $\varpi_{AA'}$ with respect to a spin basis, page 95
$\varpi_{AA'}$ Hermitian spinor assocated to a Hermitian inner product, page 95
ϱ conformally rescaled density of a perfect fluid, page 220
ϱ unphysical energy density, page 255
$\boldsymbol{Weyl}$, $\boldsymbol{Weyl}[\boldsymbol{g}]$ Weyl tensor of a connection $\boldsymbol{\nabla}$ in index-free notation, page 48
$\xi^{\boldsymbol{A}}$, $\eta_{\boldsymbol{A}}, \ldots$ components of the spinors ξ^A, $\eta_A, \ldots$ with respect to a spin basis

ξ^A, $\eta_A, \ldots$ generic spinors in abstract-index notation
$\xi_{\boldsymbol{ABCC}}$, $\chi_{\boldsymbol{ABCD}}$ real and imaginary parts of $\Gamma_{\boldsymbol{ABCD}}$, page 107
Ξ_{ij}, $S_{\boldsymbol{i}}$, S_{ij}, H_{kij} zero quantities associated to the conformal static field equations, page 511
$\zeta_0, \ldots \zeta_4$ components of the spin-2 zero-rest mass field ζ_{ABCD}, page 551
ζ_{ABCD} spin-2 zero-rest mass field, page 551
$\{\boldsymbol{c_i}\}$ global orthonormal frame on $\mathbb{S}^3$, page 142
$\{\boldsymbol{e_a}\}$ vector basis in index-free notation, page 31
$\{\boldsymbol{\omega^a}\}$ covector basis in index-free notation, page 31
$\{\mathcal{S}_t\}_{t\in\mathbb{R}}$ foliation of $\mathcal{M}$, page 54
$\{\boldsymbol{e_i}\}$ three-dimensional vector basis in index-free notation, page 59
$\{\boldsymbol{e_{AA'}}\}$ alternative index-free notation for the Newman-Penrose null tetrad, page 79
$\{\boldsymbol{e_{AB}}\}$, $\{\boldsymbol{\omega^{AB}}\}$ three-dimensional basis and cobasis with spin frame indices, page 109
$\{\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}}\}$ Newman-Penrose null tetrad in index-free notation, page 77
$\{\boldsymbol{\omega^i}\}$ three-dimensional covector basis in index-free notation, page 59
$\{\boldsymbol{\omega^{AA'}}\}$ soldering form, page 79
$\{\epsilon_{\boldsymbol{A}}{}^A\}$, $\{\epsilon^{\boldsymbol{A}}{}_A\}$ alternative abstract index notation for a spin basis and its dual, page 71
$\{\omega^{\boldsymbol{a}}{}_a\}$ covector basis in abstract index notation, page 36
$\{\omega^{\boldsymbol{i}}{}_i\}$ three-dimensional covector basis in index-free notation, page 59
$\{e_{\boldsymbol{a}}{}^a\}$ vector basis in abstract index notation, page 36
$\{e_{\boldsymbol{i}}{}^i\}$ three-dimensional vector basis in abstract index notation, page 59
$\{l^a, n^a, m^a, \bar{m}^a\}$ Newman-Penrose null tetrad in abstract index notation, page 77
$\{m,\, m_\alpha,\, m_{\alpha_1\alpha_2}, \ldots\}$ sequence of multipole moments of a static spacetime, page 519
b_{ABCD} Cotton spinor, page 512
C^*_p characteristic set of a symmetric hyperbolic system at the point p, page 297
C^∞ class of infinitely differentiable (smooth) functions
$C^\infty(\mathbb{R}^3, \mathbb{C}^N)$ space of smooth functions from $\mathbb{R}^3$ to $\mathbb{C}^N$, page 306
$C^c{}_{dab}$ Weyl tensor of a connection ∇_a in abstract index notation, page 48
C^k class of k-times differentiable functions
$C^k(\mathbb{R}^3, \mathbb{C}^N)$ set of C^k functions from $\mathbb{R}^3$ to $\mathbb{C}^N$, page 307
$C^k\big([0,T]; H^m(\mathbb{R}^3, \mathbb{C}^N)\big)$ set of C^k functions from $[0,T]$ to $H^m(\mathbb{R}^3, \mathbb{C}^N)$, page 307
D bounded open subset of $H^m(\mathbb{R}^3, \mathbb{C}^N)$ such that for $\mathbf{w} \in D$ the matrix $\mathbf{A}^0(0, \underline{x}, \mathbf{w})$ is positive definite bounded away from zero by $\delta > 0$ for all $p \in \mathbb{R}^3$, page 309
$D(\mathcal{R})$ domain of dependence of $\mathcal{R}$, page 304
D, Δ, δ, $\bar{\delta}$ Newman-Penrose directional covariant derivatives, page 92
$D^\pm(\mathcal{A})$, $D(\mathcal{A})$ future/past and total domain of dependence of a set $\mathcal{A}$, page 392

$d^a{}_{bcd}$ rescaled Weyl tensor, page 188
$d_{\boldsymbol{a}}$ components of the rescaled physical conformal geodesics covector, page 203
$D_{\boldsymbol{i}}$ three-dimensional directional covariant derivative in the direction of $\boldsymbol{e_i}$, page 59
D_i generic three-dimensional connection in abstract index notation
$D_{\boldsymbol{AB}}$ three-dimensional covariant directional derivative, page 109
$d_{\boldsymbol{ij}}$, $d_{\boldsymbol{ijk}}$ components of the electric and magnetic parts of the rescaled Weyl tensor, page 261
D_{AB} spinorial counterpart of a three-dimensional Levi-Civita connection $\boldsymbol{D}$, page 106
$F^{\boldsymbol{a}}(x)$, $F^\mu(x)$ coordinate gauge source functions, page 339
$f_{\boldsymbol{a}}$ components of the unphysical conformal geodesics covector, page 203
f_a covector defining a Weyl connection in abstract index notation, page 119
$F_{\boldsymbol{AB}}(x)$ frame gauge source functions, page 345
F_{ab} unphysical Faraday tensor, page 214
g^{ab} generic contravariant four-dimensional Lorentzian metric tensor in abstract index notation
G_{ab} Einstein tensor of a metric g_{ab}
g_{ab} generic four-dimensional Lorentzian metric tensor in abstract index notation
$H^\pm(\mathcal{A})$, $H(\mathcal{A})$ future/past and total Cauchy horizons of the set $\mathcal{A}$, page 394
$H^m(\mathbb{R}^3, \mathbb{C}^N)$ Sobolev space of order m of functions from $\mathbb{R}^3$ to $\mathbb{C}^N$, page 307
$h_a{}^b$ projector associated to a distribution Π, page 55
$h_{\boldsymbol{ABCD}}$ components of h_{ABCD} with respect to a spin frame $\{\epsilon_{\boldsymbol{A}}{}^A\}$, page 99
$h_{AA'}{}^{BB'}$ spinorial counterpart of the projector $h_{\boldsymbol{a}}{}^{\boldsymbol{b}}$, page 98
h_{ABCD} space spinor counterpart of $h_{AA'}{}^{BB'}$ and of a three-dimensional Riemannian metric, page 98
I generic interval in $\mathbb{R}$
i^0 spatial infinity
$i^\pm$ future and, respectively, past timelike infinity
$I^\pm(\mathcal{U})$ chronological future and, respectively, past of a set $\mathcal{U}$, page 391
$J^+(o, \mathcal{M}')$ set consisting of o and all points of $\mathcal{M}'$ which can be joined to o by a causal curve in $\mathcal{M}'$, page 497
$J^\pm(\mathcal{U})$ causal future and, respectively, past of a set $\mathcal{U}$, page 391
j_k unphysical flux vector, page 255
$J_{\boldsymbol{jk}}$, $J_{\boldsymbol{j}}$ normal components of the rescaled Cotton tensor, page 262
K_{ij} extrinsic curvature tensor of a hypersurface in abstract index notation, page 61
L_{ab} Schouten tensor of a connection ∇_a in abstract index notation, page 48
l_{ij} three-dimensional Schouten tensor, page 60
$p \prec\prec q$ timelike related points, page 391
p conformally rescaled pressure of a perfect fluid, page 220
$p \prec q$ strictly causally related points, page 391

$p \preceq q$ causally related points, page 391
$P^{\boldsymbol{C}}{}_{\boldsymbol{DAA'BB'}}$ reduced spinorial geometric curvature, page 198
$P^{\boldsymbol{c}}{}_{\boldsymbol{dab}}$ components of the geometric curvature, page 194
$P^{\boldsymbol{CC'}}{}_{\boldsymbol{DD'AA'BB'}}$ spinorial geometric curvature, page 197
$P_n^{(\alpha,\beta)}(\tau)$ Jacobi polynomial of degree n with parameters (α,β), page 553
$Q_a{}^b{}_c$ transition tensor between connections in abstract index notation, page 42
r three-dimensional Ricci scalar, page 60
$R(x)$ conformal gauge source function, page 348
R, $R[\boldsymbol{g}]$ Ricci scalar of a connection ∇_a, page 48
$R^{\boldsymbol{c}}{}_{\boldsymbol{dab}}$ components of the Riemann tensor with respect to an orthonormal frame, page 53
$R^d{}_{cab}$ Riemann curvature tensor of a connection ∇_a in abstract index notation, page 40
$r^k{}_{lij}$ three-dimensional Riemann curvature tensor in abstract index notation, page 60
$r_{ABCDEFGH}$ spinorial counterpart of the three-dimensional Riemann curvature tensor, page 110
R_{ab} Ricci tensor of a connection ∇_a in abstract index notation, page 48
r_{ACEFGH}, r_{ABCE} reduced three-dimensional curvature spinors, page 110
$R_{CC'DD'AA'BB'}$ spinorial counterpart of the Riemann curvature tensor, page 86
$R_{CDAA'BB'}$ reduced Riemann curvature spinor, page 86
r_{ij} three-dimensional Ricci tensor in abstract index notation, page 60
s the Friedrich scalar, page 186
s_{ABCD} spinorial counterpart of the three-dimensional trace-free Ricci tensor, page 110
s_{ij} three-dimensional trace-free Ricci tensor, page 60
$SO(3)$ three-dimensional special orthogonal group
$T(\mathcal{M})$ tangent bundle over $\mathcal{M}$, page 34
$T|_p(\mathcal{M})$ tangent space at a point $p \in \mathcal{M}$, page 31
$T^*(\mathcal{M})$ cotangent bundle over $\mathcal{M}$, page 34
$T^*|_p(\mathcal{M})$ cotangent space at a point $p \in \mathcal{M}$, page 31
$T^\bullet|_p(\mathcal{M})$ tensor algebra at $p \in \mathcal{M}$, page 33
$T_l^k|_p(\mathcal{M})$ space of (k,l)-tensors at the point $p \in \mathcal{M}$, page 33
$T^{a_1\cdots a_k}{}_{b_1\cdots b_l}$ arbitrary (k,l)-tensor in abstract index notation
T_{ab} unphysical energy-momentum tensor
T_{cdb} rescaled Cotton tensor, page 189
u retarded time
U, $X^{\mathcal{A}}$, ω, $\xi^{\mathcal{A}}$ components of an adapted frame in the asymptotic characteristic problem, page 482
u, v retarded and, respectively, advanced time coordinates
$u^{\boldsymbol{a}}$, $v^{\boldsymbol{a}}$, $w^{\boldsymbol{a}}$, ... components of the vectors $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ with respect to the coframe $\{\boldsymbol{\omega}^{\boldsymbol{a}}\}$, page 51

u^a unphysical 4-velocity of a perfect fluid, page 220
u^a, v^a, w^a, ... generic vectors in abstract index notation
v norm of a static Killing vector, page 504
$x(\mathrm{s})$ alternative notation for a curve with parameter s, page 30
X_{CDAB}, $Y_{CDA'B'}$ curvature spinors, page 86
$Y[\boldsymbol{h}]$ Yamabe invariant, page 280
Y_{abc} four-dimensional Cotton tensor, page 116
y_{ijk} three-dimensional Cotton tensor, page 118
y_{ij} three-dimensional Cotton-York tensor, page 118
$z_{\boldsymbol{AA'}}$, z, $z_{(AB)}$ spacetime and space spinor components of the spinorial counterpart of the deviation vector of a congruence of conformal geodesics, page 383
${}^*F_{ab}$ Hodge dual of an antisymmetric tensor F_{ab}, page 50
${}^*R_{abcd}$, R^*_{abcd} left and, respectively, right duals of the tensor R_{abcd}, page 50
$^+$ Hermitian conjugation, page 96
†, ‡ generalised dualisation operations, page 50
$^\sharp$, $^\flat$ musical operators, page 44
$_\alpha$, $_\beta$, $_\gamma$, ... spatial coordinate indices
$_{\boldsymbol{A}}$, $_{\boldsymbol{B}}$, $_{\boldsymbol{C}}$, ... spinor frame indices, page 74
$_{\boldsymbol{a}}$, $_{\boldsymbol{b}}$, ... spacetime frame indices ranging $\boldsymbol{0}, \ldots, \boldsymbol{3}$
$_{\boldsymbol{i}}$, $_{\boldsymbol{j}}$, $_{\boldsymbol{k}}$, ... frame indices ranging either $\boldsymbol{0}$, $\boldsymbol{1}$, $\boldsymbol{2}$ or $\boldsymbol{1}$, $\boldsymbol{2}$, $\boldsymbol{3}$
$_\perp$ perpendicular component
$_\mu$, $_\nu$, $_\lambda$, ... spacetime coordinate indices
$_A$, $_B$, $_C$, ... abstract spinor indices, page 66
$_a$, $_b$, $_c$ ... abstract spacetime indices
$_i$, $_j$, $_k$, ... abstract spatial indices
${}_sY_{lm}$ spin-weighted spherical harmonics
$_{(a_1\cdots a_l)}$ symmetrisation over the indices $_{a_1\cdots a_l}$, page 36
$_{[a_1\cdots a_l]}$ antisymmetrisation over the indices $_{a_1\cdots a_l}$, page 36
$_\mathcal{A}$, $_\mathcal{B}$, ... arbitrary string of indices
$_{\{a_1\cdots a_l\}}$ symmetric trace-free part over the indices $_{a_1\cdots a_l}$, page 47