PRINCIPLES OF MAGNETOSTATICS

The subject of magnetostatics—the mathematical theory that describes the forces and fields resulting from the steady flow of electrical currents—has a long history. By capturing the basic concepts, and building toward the computation of magnetic fields, this book is a self-contained discussion of the major subjects in magnetostatics.

Overviews of Maxwell's equations, the Poisson equation, and boundary value problems pave the way for dealing with fields from transverse, axial, and periodic magnetic arrangements and assemblies of permanent magnets. Examples from accelerator and beam physics give up-to-date context to the theory. Furthermore, both complex contour integration and numerical techniques (including finite difference, finite element, and integral equation methods) for calculating magnetic fields are discussed in detail with plentiful examples.

Both theoretical and practical information on carefully selected topics make this a one-stop reference for magnet designers, as well as for physics and electrical engineering students. This title, first published in 2017, has been reissued as an Open Access publication on Cambridge Core.

RICHARD C. FERNOW received his PhD at Syracuse University for work on particle physics and worked at Brookhaven National Laboratory. He contributed to the optimization of the coil design for collider magnets and made calculations of magnetic fields in solenoid channels. He is a member of the American Physical Society.

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RICHARD C. FERNOW
Formerly Brookhaven National Laboratory

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One Liberty Plaza, 20th Floor, New York, NY 10006, USA

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Preface

My career at Brookhaven National Laboratory began in 1978, when I was hired to work on a particle physics experiment at the alternating gradient synchrotron (AGS). At that time, one of the lab's major projects was a high-energy proton collider known as ISABELLE. Unfortunately, development of the superconducting magnets proposed for the project ran into serious technical difficulties. As a result, a member of the physics department, Dr. Robert Palmer, proposed a radically different design for the collider magnets. He began recruiting a small group from the physics and accelerator departments to work on the alternate magnet design. I was one of the staff members who joined his group.

Because I came from a background in particle physics, the work on the design of high-field magnets was a revelation to me. One of my main responsibilities was to work on the optimization of the dipole conductor cross-section and end designs needed to achieve the demanding field quality requirements for the accelerator. I quickly discovered that the methods needed for practical magnetic field design went far beyond my academic training in electricity and magnetism. Much of the work required frequent feedback with the engineers working on the project. Although I moved on from the magnet division after about four years, my interest in calculating magnetic fields remained with me throughout the rest of my career. A significant part of the contents of this book are based on my notes from that period.

The subject of magnetostatics has a long history. There is a vast literature, so a book of this size has to make difficult choices about which topics to include. My primary objective was to produce a self-contained discussion of the major subjects in magnetostatics with an emphasis on the computation of magnetic fields. For that reason, I have included brief treatments of most standard background material, such as the magnetostatic Maxwell's equations and potential theory. However, the choice of example topics relies heavily on my background and interests. Many of the examples come from the fields of accelerator and beam physics. I also felt it was important to expose a wider audience to a series of very insightful papers by the late Klaus Halbach of Lawrence Berkeley Laboratory. For the discussion of numerical methods, I decided to concentrate on a small number of subjects while including sufficient details to enable readers to begin writing their own computer codes, if they so desired.

The first three chapters are mostly a survey of basic material. Chapter 1 treats the theory of magnetic fields from conductors in free space, and Chapter 2 discusses fields from magnetic materials. Chapter 3 introduces the vector and scalar potentials. It includes general solutions to the Laplace equation and the solution of boundary value problems.

Chapters 4–6 discuss transverse fields in two dimensions. Chapter 4 looks at fields from line currents, current sheets, and current blocks. Field quality is introduced in terms of multipole expansions, and the effects of approximating ideal current distributions are described. Chapter 5 looks at transverse fields using complex variable methods. The powerful techniques for computing the fields of block conductors using contour integration are discussed in detail. Contour integrals are also derived for the fields from image currents and magnetized bodies. Chapter 6 looks at transverse fields that are determined by the shape of the iron. The discussion here mainly concerns the iron surfaces used in dipole and quadrupole magnets.

Chapters 7–9 discuss some other field configurations. Chapter 7 looks at axial field arrangements. This includes the fields from current loops, solenoids, and systems of coils. The solution of the solenoid field using the sheet model is treated in detail. Chapter 8 considers periodic magnetic channels. First the field from a helical conductor winding is discussed. Then inverse problems are introduced, and some of the field configurations used for magnetic wigglers and particle beam-focusing channels are examined. Chapter 9 begins with a standard treatment of the properties of permanent magnets. This is followed by a discussion of Halbach's model for rare-earth cobalt magnets and his analysis of assemblies of permanent magnets.

In Chapter 10, we relax the strict conditions of magnetostatics and allow for the case of slowly varying currents. This leads to brief discussions of some standard subjects such as Faraday's law, but also some more engineering-related topics such as eddy currents and the skin effect. This also seemed an appropriate place to include a brief discussion of magnetic field measurements using rotating coils.

Chapter 11 discusses numerical methods. No attempt is made here to survey the thousands of papers devoted to numerical solutions of magnetic field problems. Instead, three methods for solving the Poisson equation are presented with a significant amount of detail. This chapter also includes a discussion of the POISSON code, which is freely available and extremely useful for investigating

2D problems. The chapter ends by returning to the inverse problem and presenting several examples of using optimization methods.

The appendices collect some important details about mathematical techniques and special functions used in the book.

The level of the treatment of background magnetostatic topics in the book is typical of those encountered by undergraduate physics majors. Some of the material in Chapters 4–11 will likely be unfamiliar to many readers. However, an attempt has been made to include sufficient details and references, so interested physics and engineering majors should be able to follow the discussions.

At a number of places in the text, I have indicated the source of a mathematical relation in footnotes using the following notation.

AS	M. Abramowitz & I. Stegun (eds.), *Handbook of Mathematical Functions*, Dover Publications, 1972.
CRC	S. Selby (ed.), *Standard Mathematical Tables*, 14th ed., The Chemical Rubber Company, 1965.
GR	I. Gradshteyn & I. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, 1980.

I would like to thank several of my former colleagues, especially Gerry Morgan, Steve Kahn, Bob Palmer, Juan Gallardo, and Scott Berg, for many interesting discussions concerning magnetic fields and the methods used for calculating them. I would like to thank Peter Wanderer and Animesh Jain of the Superconducting Magnet Division at Brookhaven National Laboratory for providing the image of field lines in a RHIC dipole, which has been used on the cover of this book. I would also like to thank Simon Capelin and the staff at Cambridge University Press for their collaboration on this project. Finally, I would like to thank my wife Ruth for her constant support and encouragement.