MATHEMATICS OF QUANTIZATION AND QUANTUM FIELDS

Unifying a range of topics that are currently scattered throughout the literature, this book offers a unique and definitive review of some of the basic mathematical aspects of quantization and quantum field theory. The authors present both elementary and more advanced subjects of quantum field theory in a mathematically consistent way, focusing on canonical commutation and anti-commutation relations. They begin with a discussion of the mathematical structures underlying free bosonic or fermionic fields, such as tensors, algebras, Fock spaces, and CCR and CAR representations (including their symplectic and orthogonal invariance). Applications of these topics to physical problems are discussed in later chapters. Although most of the book is devoted to free quantum fields, it also contains an exposition of two important aspects of interacting fields: the diagrammatic method and the Euclidean approach to constructive quantum field theory. With its in-depth coverage, this text is essential reading for graduate students and researchers in departments of mathematics and physics. This title, first published in 2013, has been reissued as an Open Access publication on Cambridge Core.

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Since my high school years, I have kept in my memory the following verses:

Profesor Otto Gottlieb Schmock
Pracuje już dziesiąty rok
Nad dziełem co zadziwić ma świat:
Der Kaiser, Gott und Proletariat.

As I checked recently, it is a somewhat distorted fragment of a poem by Julian Tuwim from 1919. I think that it describes quite well the process of writing our book.

Jan Dereziński

Je dédie ce livre à mon pays.

Que diront tant de Ducs et tant d'hommes guerriers
Qui sont morts d'une plaie au combat les premiers,
Et pour la France ont souffert tant de labeurs extrêmes,
La voyant aujourd'hui détruire par soi-même?
Ils se repentiront d'avoir tant travaillé,
Assailli, défendu, guerroyé, bataillé,
Pour un peuple mutin divisé de courage
Qui perd en se jouant un si bel héritage.
(Pierre de Ronsard, 1524–1585)

Christian Gérard

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