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978-1-009-29038-8 — Quarks, Gluons and Lattices
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1
Quarks and gluons

Our prime candidate for a fundamental theory of strong hadronic forces is a model of quarks interacting through the exchange of non-Abelian gauge fields. The quark model represents a new level of substructure within hadronic particles such as the proton. We have several compelling reasons to believe in this next layer of matter.

First, the large cross sections observed in deeply inelastic lepton–hadron scattering indicate important structure at distance scales of less than 10^{-16} centimeters, whereas the overall proton electromagnetic radius is of order 10^{-13} centimeters. The angular dependences observed in these experiments suggest that the underlying charged constituents carry half-integer spin. These studies have raised the question of whether it is theoretically possible to have pointlike objects in a strongly interacting theory. Asymptotically free non-Abelian gauge interactions offer this hope (Perkins, 1977).

A second impetus for a theory of quarks lies in low energy hadronic spectroscopy. Indeed, it was the successes of the eightfold way (Gell-Mann and Ne'eman, 1964) which originally motivated the quark model. We now believe that the existence of two 'flavors' of low mass quarks lies at the heart of the isospin symmetry in nuclear physics. Adding a somewhat heavier 'strange' quark to the theory gives rise to the celebrated multiplet structure in terms of representations of the group $SU(3)$.

Third, we have further evidence for compositeness in the excitations of the low-lying hadrons. Particles differing in angular momentum fall neatly into place on the famous 'Regge trajectories' (Collins and Squires, 1968). In this way families of states group together as orbital excitations of some underlying system. The sustained rising of these trajectories with increasing angular momentum points toward strong long-range forces. This originally motivated the stringlike models of hadrons.

Finally, the idea of quarks became incontrovertible with the discovery of the 'hydrogen atoms' of elementary particle physics. The intricate spectroscopy of the charmonium and upsilon families is admirably explained in potential models for non-relativistic bound states of heavy quarks (Eichten *et al.*, 1980).

Despite these successes of the quark model, an isolated quark has never been observed. (Some hints of fractionally charged macroscopic pieces of matter may eventually prove to contain unbound quarks, or might be a sign of some new and even more exciting type of matter (LaRue, Phillips and Fairbank, 1981).) These basic constituents of matter do not copiously appear as free particles emerging from present laboratory experiments. This is in marked contrast to the empirical observation in hadronic physics that anything which can be created will be. The difficulty in producing quarks has led to the speculation of an exact confinement. Indeed, it may be simpler to imagine a constituent which can never be produced than an approximate imprisonment relying on some unnaturally effective suppression factor in a theory seemingly devoid of any large dimensionless parameters.

But how can we ascribe any reality to an object which cannot be produced? Are we just dealing with some sort of mathematical trick? We will now argue that gauge theories potentially possess a simple mechanism for giving constituents infinite energy when in isolation. In this picture a quark–antiquark pair will experience an attractive force which remains non-vanishing even for asymptotically large separations. This linearly rising long-distance potential energy forms the basis of essentially all models of quark confinement.

We begin by coupling the quarks to a conserved 'gluo-electric' flux. In usual electromagnetism the electric field lines thus produced spread and give rise to the inverse square law Coulombic field. If in our theory we can now somehow eliminate massless fields, then a Coulombic spreading will no longer be a solution to the equations. If in removing the massless fields we do not destroy the Gauss law constraint that the quarks are the sources of electric flux, the electric lines must form into tubes of conserved flux, schematically illustrated in figure 1.1. These tubes will only end on the quarks and their antiparticles. A flux tube is a real physical object carrying a finite energy per unit length. This is the storage medium for the linearly rising interquark potential (Kogut and Susskind, 1974).

A simple model for this phenomenon is a type II superconductor containing magnetic monopole impurities. Because of the Meissner effect (Meissner and Ochsenfeld, 1933), a superconductor does not admit magnetic fields. However, if we force a hypothetical magnetic monopole into the system, its lines of magnetic flux must go somewhere. Here the role of the 'gluo-electric' flux is played by the magnetic field, which will bore a tube of normal material through the superconductor until it ends on an antimonopole or it leaves the boundary of the system. Such flux tubes have been experimentally observed in applied magnetic fields (Huebner and Clem, 1974).

Another example of this mechanism occurs in the bag model (Chodos *et al.*, 1975). Here the gluonic fields are unrestricted in the baglike interior of a hadron but forbidden by ad hoc boundary conditions from extending outside. In attempting to extract a single quark from a proton, one would draw out a long skinny bag carrying the gluo-electric flux of the quark back to the remaining constituents.

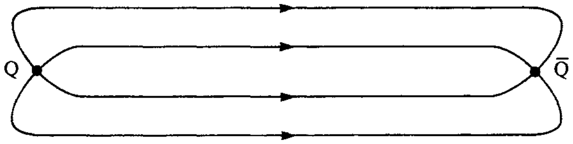

Fig. 1.1. A flux tube from a quark to an antiquark.

The above models may be interesting phenomenologically, but they are too arbitrary to be considered as the basis for fundamental theories. In their search for a more elegant model, theorists have been drawn to non-Abelian gauge fields. This dynamical system of coupled gluons begins like electrodynamics with a set of massless gauge fields interacting with the quarks. Using the freedom of an internal symmetry, the action includes self-couplings of the gluons. The bare massless fields are all charged with respect to each other. The confinement conjecture is that this input theory of massless charged particles is unstable to a condensation of the vacuum to a state in which only massive excitations can propagate. In such a state the gluonic flux around quarks should form into the tubes needed for linear confinement. Much of the recent effort in elementary particle theory has gone into attempts to show that this indeed takes place.

The confinement phenomenon makes the theory of the strong interactions qualitatively different from theories of the electromagnetic and weak forces. The fundamental fields of the Lagrangian do not manifest themselves in free hadronic spectrum. In not observing free quarks and gluons, we are led to the conjecture that all observable strongly interacting particles are gauge singlet bound states of these fundamental constituents.

In the usual quark model baryons are bound states of three quarks. Thus the gauge group should permit singlets to be formed from three objects in the fundamental representation. This motivates the use of $SU(3)$ as the underlying group of the strong interactions. This internal symmetry must not be confused with the broken $SU(3)$ represented in spectroscopic multiplets. Ironically, one of the original motivations for quarks has now

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become an accidental symmetry. The symmetry considered here is hidden behind the confinement mechanism, which only permits us to observe singlet states.

For the presentation in this book we assume, perhaps too naively, that the nuclear interactions can be considered in isolation from the much weaker effects of electromagnetism, weak interactions, and gravitation. This does not preclude the possible application of the techniques presented here to the other interactions. Indeed, grand unification may be crucial for a consistent theory of the world. To describe physics at normal laboratory energies, however, only for the strong interaction must we go beyond well-established perturbative methods. Thus we frame our discussion around quarks and gluons.

2
Lattices

The best evidence we have for confinement in a non-Abelian gauge theory of the strong interactions comes by way of Wilson's (1974) formulation on a space-time lattice. At first this prescription seems a little peculiar because the vacuum is not a crystal. Indeed, experimentalists work daily with relativistic particles showing no deviations from the continuous symmetries of the Lorentz group. Why, then, have theorists in recent years spent so much time describing field theory on the scaffolding of a space-time lattice?

The lattice represents a mathematical trick. It provides a cutoff removing the ultraviolet infinities so rampant in quantum field theory. As with any regulator, it must be removed after renormalization. Physics can only be extracted in the continuum limit, where the lattice spacing is taken to zero.

But infinities and the resulting need for renormalization have been with us since the beginnings of relativistic quantum mechanics. The program for electrodynamics has had immense success without recourse to discrete space. Why reject the time-honored perturbative renormalization procedures in favor of a new cutoff scheme?

We are driven to the lattice by the rather unique feature of confinement in the strong interactions. This phenomenon is inherently non-perturbative. The free theory with vanishing coupling constant has no resemblance to the observed physical world. Renormalization group arguments, to be presented in detail in later chapters, indicate severe essential singularities when hadronic properties are regarded as functions of the gauge coupling. This contrasts sharply with the great successes of quantum electrodynamics, where perturbation theory was central. Most conventional regularization schemes are based on the Feynman expansion; some process is calculated until a divergence is met in a particular diagram, and this divergence is then removed. To go beyond the diagrammatic approach, one needs a non-perturbative cutoff. Herein lies the main virtue of the lattice, which directly eliminates all wavelengths less than twice the lattice spacing. This occurs before any expansions or approximations are begun.

On a lattice, a field theory becomes mathematically well-defined and can be studied in various ways. Lattice perturbation theory, although somewhat awkward, recovers all the conventional results of other regularization schemes. Discrete space-time, however, is particularly well-suited for a strong coupling expansion. Remarkably, confinement is automatic in this limit where the theory reduces to one of quarks on the ends of strings with a finite energy per unit length. Most recent research has concentrated on showing that this phenomenon survives the continuum limit.

A lattice formulation emphasizes the close connections between field theory and statistical mechanics. Indeed, the strong coupling treatment is equivalent to a high temperature expansion. The deep ties between these disciplines are manifest in the Feynman path integral formulation of quantum mechanics (Feynman, 1948; Dirac, 1933, 1945). In Euclidian space, a path integral is equivalent to a partition function for an analogous statistical system. The square of the field theoretical coupling constant corresponds directly to the temperature. Thus, the particle physicist has available the full technology of the condensed matter theorist.

Confinement is natural in the strong coupling limit of the lattice theory; however, this is not the region of direct physical interest, for which a continuum limit is necessary. The coupling constant on the lattice represents a bare coupling at a length scale of the lattice spacing. Non-Abelian gauge theories possess the property of asymptotic freedom, which means that in the short distance limit the effective coupling goes to zero. This remarkable phenomenon allows predictions for the observed scaling behavior in deeply inelastic collisions. Indeed, this was one of the original motivations for a non-Abelian gauge theory of the strong interactions. The consequence for the lattice theory, however, is that the bare coupling must be taken to zero as the lattice spacing decreases towards the continuum limit. Thus we are inevitably led out of the high temperature regime and into a low temperature domain. Along the way in a general statistical system one might expect to encounter phase transitions. Such qualitative shifts in the physical characteristics of a system can only hamper the task of showing confinement in the non-Abelian theory. In later chapters we will present evidence that such troublesome transitions can be avoided in the four-dimensional $SU(3)$ gauge theory of the nuclear force.

Although our ultimate goal with lattice gauge theory is an understanding of hadronic physics, many interesting phenomena arise which are peculiar to the lattice. We will see non-trivial phase structure occurring in a variety of models, some of which do not correspond to any continuum field theory. The lattice formulation is highly non-unique and thereby spurious transitions can be alternately introduced and removed. We will also see that the statistical mechanics of gauge models displays curious analogies with magnetic systems in half the number of space-time dimensions. Even quantum electrodynamics shows interesting structure in certain lattice formulations. This rich spectrum of phenomena has led to the recent popularity of lattice field theories and motivates this book.

3
Path integrals and statistical mechanics

The Feynman path integral formulation of quantum mechanics reveals deep connections with statistical mechanics. This chapter is concerned with this relationship for the simple case of a non-relativistic particle in a potential. Starting with a partition function representing a path integral on an imaginary time lattice, we will show how a transfer matrix formalism reduces the problem to the diagonalization of an operator in the usual quantum mechanical Hilbert space of square integrable functions (Creutz, 1977). In the continuum limit of the time lattice, we obtain the canonical Hamiltonian. Except for our use of imaginary time, this treatment is identical to that in Feynman's early work (Feynman, 1948).

We begin with the Lagrangian for a free particle of mass m moving in potential $V(x)$

$$L(x, \dot{x}) = K(\dot{x}) + V(x), \tag{3.1}$$

$$K(\dot{x}) = \tfrac{1}{2}m\dot{x}^2, \tag{3.2}$$

where $\dot{x}$ is the time derivative of the coordinate x. Velocity-dependent potentials are beyond the scope of this book. Note the unconventional relative positive sign between the two terms in eq. (3.1). This is because we formulate the path integral directly in imaginary time. This improves mathematical convergence, yet leaves us with the usual Hamiltonian for diagonalization.

For any trajectory we have an action

$$S = \int \mathrm{d}t\, L(\dot{x}(t), x(t)), \tag{3.3}$$

which appears in the path integral

$$Z = \int [\mathrm{d}x(t)]\, \mathrm{e}^{-S}. \tag{3.4}$$

Here the integral is over all trajectories $x(t)$. As it stands, eq. (3.4) is rather poorly defined. To characterize the possible trajectories we introduce a cutoff in the form of a time lattice. Putting our system into a time box of total length τ, we divide this interval into

$$N = \tau/a, \tag{3.5}$$

discrete time slices, where a is the timelike lattice spacing. Associated with the i'th such slice is a coordinate x_i. This construction is sketched in figure 3.1. Replacing the time derivative of x with a nearest-neighbor difference, we reduce the action to a sum

$$S = a\sum_i \left[\tfrac{1}{2}m\left(\frac{x_{i+1}-x_i}{a}\right)^2 + V(x_i)\right]. \tag{3.6}$$

The integral in eq. (3.4) is now defined as an integral over all the coordinates

$$Z = \int \left(\prod_i \mathrm{d}x_i\right) \mathrm{e}^{-S}. \tag{3.7}$$

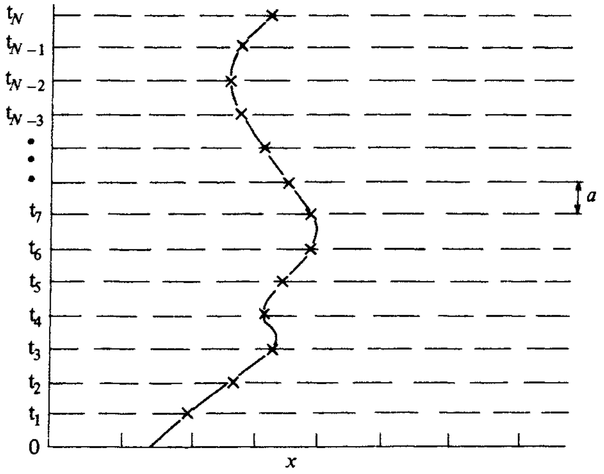

Fig. 3.1. Dividing time into a lattice. (From Creutz and Freedman, 1981.)

Eq. (3.7) is precisely in the form of a partition function for a statistical system. We have a one-dimensional chain of coordinates x_i. The action represents the inverse temperature times the Hamiltonian of the thermal analog. We will now show that evaluation of this partition function is equivalent to diagonalizing a quantum mechanical Hamiltonian obtained from this action with canonical methods. This is done via the transfer matrix.

The key to the transfer-matrix analysis is to note that the local nature of the action in eq. (3.6) permits us to write the partition function in the form of a matrix product

$$Z = \int \prod_i \mathrm{d}x_i \, T_{x_{i+1},\, x_i}, \tag{3.8}$$

where the transfer-matrix elements are

$$T_{x',\,x} = \exp\left[-\frac{m}{2a}(x'-x)^2 - \frac{a}{2}(V(x')+V(x))\right]. \tag{3.9}$$

This operator acts in the Hilbert space of square integrable functions, where the inner product is the standard

$$\langle\psi'|\psi\rangle = \int dx\, \psi'^*(x)\,\psi(x). \tag{3.10}$$

We introduce the non-normalizable basis states $\{|x\rangle\}$ such that

$$|\psi\rangle = \int dx\, \psi(x)\,|x\rangle, \tag{3.11}$$

$$\langle x'|x\rangle = \delta(x'-x), \tag{3.12}$$

$$1 = \int dx\,|x\rangle\langle x|. \tag{3.13}$$

The canonically conjugate operators $\hat{p}$ and $\hat{x}$ satisfy

$$\hat{x}|x\rangle = x|x\rangle, \tag{3.14}$$

$$[\hat{p},\hat{x}] = -\mathrm{i}, \tag{3.15}$$

$$e^{-\mathrm{i}\hat{p}\Delta}|x\rangle = |x+\Delta\rangle. \tag{3.16}$$

In this Hilbert space the operator T is defined via its matrix elements

$$\langle x'|T|x\rangle = T_{x',x}, \tag{3.17}$$

where $T_{x',x}$ is given in eq. (3.8). With periodic boundary conditions for our lattice of N sites, the path integral is compactly expressed

$$Z = \mathrm{Tr}\,(T^N). \tag{3.18}$$

The operator T is easily written in terms of the conjugate variables $\hat{p}$ and $\hat{x}$

$$T = \int d\Delta\, e^{-aV(\hat{x})/2}\, e^{-\Delta^2 m/(2a)-\mathrm{i}\hat{p}\Delta}\, e^{-aV(\hat{x})/2}. \tag{3.19}$$

To prove this equation, simply check that the right hand side has the matrix elements of eq. (3.9). The integral over Δ is Gaussian and gives

$$T = (2\pi a/m)^{\frac{1}{2}}\, e^{-\frac{1}{2}aV(\hat{x})}\, e^{-\frac{1}{2}a\hat{p}^2/m}\, e^{-\frac{1}{2}aV(\hat{x})}. \tag{3.20}$$

Connection with the usual quantum mechanical Hamiltonian appears in the small lattice spacing limit. When a is small, the exponents in eq. (3.20) combine to give

$$T = (2\pi a/m)^{\frac{1}{2}}\, e^{-aH+O(a^3)}, \tag{3.21}$$

where

$$H = \hat{p}^2/(2m) + V(\hat{x}). \tag{3.22}$$

This is just the canonical Hamiltonian corresponding to the Lagrangian in eq. (3.1).

The procedure for going from a path-integral to a Hilbert-space formulation of quantum mechanics consists of three steps. First define the path integral with a time lattice. Then construct the transfer matrix and the Hilbert space on which it operates. Finally, take the logarithm of the transfer matrix and identify the negative of the coefficient of the linear term