

NONEQUILIBRIUM QUANTUM FIELD THEORY

Research into the nonequilibrium processes of quantum many body systems, and the statistical mechanical properties of interacting quantum fields, constitutes a fast developing and widely applicable area of theoretical physics.

Bringing together the key ideas from nonequilibrium statistical mechanics and powerful methodology from quantum field theory, this book captures the essence of nonequilibrium quantum field theory.

Beginning with the foundational aspects of the theory, the book presents important concepts and useful techniques, discusses issues of basic interest such as decoherence and entropy generation, and shows how thermal field, linear response, kinetic theories and hydrodynamics emerge. It also illustrates how these concepts and methodology are applied to current research topics such as nonequilibrium phase transitions, thermalization in relativistic heavy ion collisions, the nonequilibrium dynamics of Bose-Einstein condensation, and the generation of structures from quantum fluctuations in the early Universe.

The book is divided into five parts, with each part addressing a particular stage in the conceptual and technical development of the subject. Full derivations or detailed plausibility arguments are presented throughout. This self-contained book is a valuable reference for graduate students and researchers in particle physics, gravitation, cosmology, atomic-optical and condensed matter physics.

This title, first published in 2009, has been reissued as an Open Access publication on Cambridge Core.

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Dedicated to—
María Isabel and Francisco from E.C.
Tung-Hui, Tung-Fei and Chun-Chu from B-L.

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Preface

In the last decade or two we see increasing research activities in areas where quantum field processes of nonequilibrium many-body systems prevail. This includes nuclear particle physics in the relativistic heavy ion collision (RHIC) experiments, early universe cosmology in the wake of high-precision observations (such as WMAP), cold atom (such as Bose–Einstein) condensation (BEC) physics in highly controllable environments, quantum mesoscopic processes and collective phenomena in condensed matter systems. There is a demand for a new set of tools and concepts from quantum field theory to treat the nonequilibrium dynamics of relativistic many-particle systems and for the understanding of basic issues like dissipation, entropy, fluctuations, noise and decoherence in these systems.

The subject matter of this book is at the intersection of nonequilibrium statistical mechanics (NEqSM) and quantum field theory (QFT). It deals with the nonequilibrium quantum processes of relativistic many-body systems with techniques from quantum field theory. To a lesser extent it also touches on the nonequilibrium statistical mechanical aspects of interacting quantum field theory itself. This subject matter is a natural extension of thermal field theory from equilibrium (finite temperature) to nonequilibrium systems. One major technical challenge is that the usual Euclidean or imaginary time quantum field theoretical methods applicable to stationary quantum systems are no longer valid (except for linear response in near-equilibrium conditions) and real-time formulations are required.

The book has five parts: The first part comprising Chapters 1–3 deals with the basics. After an introductory chapter on basic notions and issues in NEqSM, two chapters are devoted to the basic ideas and techniques of nonequilibrium systems. The second part comprising Chapters 4–6 begins with Chapter 4 on quantum field processes in dynamical backgrounds. Chapters 5–6 form the backbone of the book, in establishing the real-time quantum field theory framework based on the so-called closed time path (CTP or Schwinger–Keldysh) effective action and the influence functional (IF, or Feynman–Vernon) formalisms. This is followed by three chapters in Part III to illustrate the use of these formalisms for addressing issues like gauge invariance, dissipation, entropy, noise and decoherence. From these formalisms we proceed to Part IV, including Chapters 10–12 in the development of thermal, kinetic and hydrodynamics theories for interacting quantum fields, with linear response and thermal field theory as the near-equilibrium limits. Part V of this book shows how to apply this body of knowledge with examples

drawn from three areas: Bose–Einstein condensates (BEC), relativistic heavy ion collisions (RHIC) and early universe cosmology discussed in Chapters 13–15 respectively. The range of application is much broader than that represented by these chosen examples.

We assume the reader has a good knowledge of quantum field theory as taught in a typical first or second year graduate course, with standard textbooks, but we do not require the reader to have much more knowledge of nonequilibrium statistical mechanics beyond those discussed (regrettably light) in a standard first/second year statistical mechanics course. Prior exposure to nonrelativistic many-body theory is helpful but not required, so is thermal field theory, as they will be developed as subcases of the fully relativistic nonequilibrium quantum field theory, the main theme of this book.

Below is a quick guide to the use of this book for readers with different backgrounds. Readers with some good understanding of NEqSM may go directly to Part II while readers familiar with the CTP-IF formalism may start with Part I and go to Part III. Readers more interested in the structure of the kinetic and hydrodynamic theories can delve into Part IV after Part II, while readers more interested in statistical mechanical issues manifested in quantum field theory may want to focus on Chapters 1, 8, 9. Recognizing that readers may come from different disciplines with solid knowledge of their own field who want to learn nonequilibrium quantum field theory for applications to their own problems, we suggest the following streams:

- (1) Atomic-optical and condensed matter physics: Chapters 2, 3, 5, 6, 8, 10, 11, 13
- (2) Nuclear-particle physics: Chapters 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
- (3) Gravitation and cosmology: Chapters 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15

A book on this relatively new but fast developing subject can be of some substance or usefulness only if the authors make a serious attempt to capture or represent the collective research effort by their colleagues working in this field. With this belief we sent out each chapter before it saw its final form to experts on that particular topic, and were blessed with many careful comments and insightful suggestions. For this we are indebted to Alejandra Kandus, Gabor Kunstatter, Da-Shin Lee, Daniel Litim, Fernando Lombardo, Sabino Matarrese, Diego Mazzitelli, Stanislav Mrowczynski, Marcello Musso, Kin-Wang Ng, Juan Pablo Paz, Robert Pisarski, Ana Maria Rey, Antonio Riotto, Ray Rivers, Albert Roura, Dam Son, Rafael Sorkin, Enric Verdaguer, Alexander Vilenkin, Serge Winitzki and Laurence Yaffe. We received able help from A. Eftekharzadeh and Taihung Wu in the preparation of the bibliography and from Chad Galley in checking the consistency of some notations and conventions.

Many of our colleagues and co-workers have contributed to our understanding and development of nonequilibrium quantum field theory over the years. We would like to thank Mario Castagnino, the late Bryce DeWitt, J. Robert

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The writing of this book as an intellectual challenge was a joy for the authors, but it also meant significant sacrifice at the personal level for a long duration, taking our time and attention away from our families and friends; without their understanding and forbearance it would not have been possible.

EC dedicates this book to María Isabel and Francisco. To be able to dedicate it to them was one of the main reasons to write it in the first place.

BLH would like to take this special opportunity to express his sincere appreciation to his wife of 35 years, Chun-Chu Yee, for her companionship in meeting the challenges of life, inward and outward; and her understanding and forbearance at both the ethereal and mundane levels of coexistence and communion. He wants to tell his beloved son Tung-Hui and daughter Tung-Fei that they are the best that have ever happened to him and the most precious in his life: Just being with you or simply hearing from you gives me the greatest pleasure. I value you each for being a fiercely independent individual with such keen or even painful senses of being, and yet able to reach out and connect with the closest and farthest reaches of humanity. He wants to express his love and appreciation to his brothers Bambi Hu and Shiu-Lok Hu, and to his cousin Kwen-Wai Lau for their constant encouragement and unfailing support. Finally he expresses his deep gratitude and indebtedness to his late parents Mr. and Mrs. I-Ping and Pie Wang Hu who raised and cared for their sons with love and dedication, inspiring within them the virtues of kindness and generosity; and to his aunt Sheng-Shuen Hu for her selfless devotion to our family. He is grateful to his many loving friends and the many lovely persons who share with him the laughter and tears, the daring and the follies of life.