

# Applied Longitudinal Data Analysis for Medical Science

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A Practical Guide

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To Marjon, Mike and Nick

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## Preface

The most important feature of this book is the word applied in the title. This implies that the emphasis of this book lies more on the application of statistical methods for longitudinal data analysis and not so much on the mathematical background. In most other books on longitudinal data analysis, the mathematical background is the major issue, which may not be surprising since (nearly) all the books on this topic have been written by statisticians. Although statisticians fully understand the difficult mathematical material underlying longitudinal data analysis, they often have difficulty in explaining this complex material in a way that is understandable for the researchers who have to use the method or interpret the results. Therefore, this book is not written by a statistician, but by an applied medical researcher. In fact, an applied researcher is not primarily interested in the basic (difficult) mathematical background of

the statistical methods, but in finding the answer to a specific research question; the applied researcher wants to know how to apply a statistical method and how to interpret the results. Owing to their different basic interests and different level of thinking, communication problems between statisticians and applied researchers are quite common. This, in addition to the growing interest in longitudinal studies, initiated the writing of this book: a book on longitudinal data analysis, which is especially suitable for non-statistical applied researchers. The aim of this book is to provide a practical guide on how to handle data from a longitudinal study. The purpose of this book is to build a bridge over the communication gap that exists between statisticians and applied researchers regarding the (complicated) topic of longitudinal data analysis.



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I am very grateful to all my colleagues and students who came to me with (mostly) practical questions on longitudinal data analysis. This book is based on all those questions.