

PHYSICS AND ENGINEERING OF GRADED-INDEX MEDIA

Optical materials with varying refractive indices are called graded-index (GRIN) media and they are widely used within many industries, including telecommunications and medical imaging. Another recent application is space division multiplexing, an enormously improved technique for optical data transmission. The book synthesizes recent research developments in this growing field, presenting both the underlying physical principles behind optical propagation in GRIN media and the most important engineering applications.

The principles of wave optics are employed for solving Maxwell's equations inside a GRIN medium, ensuring that diffractive effects are fully included. The mathematical development builds gradually and a variety of exact and approximate techniques for solving practical problems are included, in addition to coverage of modern topics such as optical vortices, photonic spin-orbit coupling, photonic crystals, and metamaterials. This text will be useful for graduate students and researchers working in optics, photonics, and optical communications.

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Dedicated to my grandchildren

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Preface

Maxwell proposed as early as 1854 the concept of a graded-index (GRIN) device, even before he developed his celebrated equations. Similar ideas were used by Wood in 1906 and by Luneberg in 1954 for the imaging applications. By 1970, GRIN glasses were fabricated whose refractive index varied radially in a cylindrical fashion. Such glasses were used either in a rod form as flat lenses or drawn into a fiber form, depending on the application. By the year 1980, GRIN fibers were used for the first generation of optical telecommunication systems. Plastic GRIN fibers were developed during the 1990s and are used routinely for transferring data between computers. More recently, silica GRIN fibers have been used for mode-division multiplexing in telecommunication systems and for observing novel nonlinear phenomena. The GRIN concept has also been extended to photonic crystals and metamaterials.

This book is intended to bring together a large amount of recent research material in a well-organized form such that a reader can develop physical understanding based on the fundamentals and apply it to emerging novel applications. Two earlier books in this area, published in 1978 and 2003, focused on the imaging applications and made use of mostly geometrical optics for describing light propagation inside a GRIN medium. The book employs the techniques of wave optics for solving Maxwell's equations inside GRIN media and ensures that the diffractive effects are fully included. The mathematical development builds up slowly and presents a variety of exact and approximate techniques for solving practical problems.

The primary role of this book is as a graduate-level text suitable for students and scientists working in the areas of optics, photonics, and imaging science. An attempt is made to include as much recent material as possible so that students are exposed to the recent advances in the areas covered by the book. The book can also serve as a reference text for researchers already engaged in or wishing to enter the fields where GRIN media are employed. The reference list at the end of each chapter is more elaborate than what is common for a typical textbook. The listing of recent

research papers should be useful for researchers using this book as a reference. At the same time, students can benefit from it if they are assigned problems requiring reading of the original research papers. Although written primarily as a research monograph, portions of this book may be useful for graduate-level courses on the subjects of fiber optics, imaging science, and nonlinear optics.

Many persons have contributed to this book either directly or indirectly. It is impossible to mention all of them by name. I thank my Ph.D. and graduate students who took my courses related to electromagnetic theory, optical waveguides, and fiber-optic communication systems for their insightful questions and comments. I am grateful to my colleagues at the Institute of Optics of University of Rochester for numerous discussions and for providing a cordial and productive atmosphere. Last, but not least, I thank my family for understanding why I needed to spend considerable time on preparing the manuscript for this book instead of spending time with them. If the readers find the book useful, I would consider my time was well spent. For the color version of all figures in this book, please visit the resources on our website www.cambridge.org/9781009282079.

Acronyms

Each scientific field has its own jargon. Although an attempt was made to avoid extensive use of acronyms, many still appear throughout the book. Each acronym is defined the first time it appears in a chapter so that the reader does not have to search the entire text to find its meaning. As a further help, all acronyms are listed here in alphabetical order.

| | |
|------|------------------------------|
| CCD | charge-coupled device |
| CVD | chemical vapor deposition |
| CW | continuous wave |
| DGD | differential group delay |
| EDFA | erbium-doped fiber amplifier |
| FFT | fast Fourier transform |
| FWHM | full width at half maximum |
| FWM | four-wave mixing |
| GRIN | graded-index |
| GVD | group-velocity dispersion |
| LCP | left-circularly polarized |
| LED | light-emitting diode |
| LP | linearly polarized |
| MMF | multi-mode fiber |
| NLS | nonlinear Schrödinger |
| OAM | orbital angular momentum |
| OSA | optical spectrum analyzer |
| PBG | photonic bandgap |
| PCF | photonic crystal fiber |
| PMD | polarization-mode dispersion |
| QPM | quasi-phase matching |

| | |
|------|----------------------------------|
| RCP | right-circularly polarized |
| RIFS | Raman-induced frequency shift |
| RMS | root mean square |
| SAM | spin angular momentum |
| SBS | stimulated Brillouin scattering |
| SEM | scanning electron microscope |
| SHG | second harmonic generation |
| SLM | spatial light modulator |
| SMF | single-mode fiber |
| SNR | signal-to-noise ratio |
| SOP | state of polarization |
| SPM | self-phase modulation |
| SRS | stimulated Raman scattering |
| SSFM | split-step Fourier method |
| STML | spatiotemporal mode locking |
| TOD | third-order dispersion |
| WDM | wavelength-division multiplexing |
| WKB | Wentzel–Kramers–Brillouin |
| XPM | cross-phase modulation |
| ZDWL | zero-dispersion wavelength |