1

Introduction

This chapter provides an introduction to the subject known as gradient-index optics. Section 1.1 provides a historical perspective on this subject before introducing the essential concepts needed in later chapters. Section 1.2 is devoted to various types of refractive-index profiles that are employed for making gradient-index devices, with particular emphasis to the parabolic index profile because of its practical importance. In Section 1.3, we discuss the relevant properties of such devices such as optical losses, chromatic dispersion, and intensity dependence of the refractive index occurring at high power levels. The focus of Section 1.4 is on the materials and the techniques used for fabricating gradient-index devices in the form of a rod or a thin fiber. Section 1.5 provides an overview of how the book is organized for presenting a wide body of research carried out during the last 50 years in the area of gradient-index optics.

1.1 Historical Perspective

Propagation of electromagnetic radiation in any medium is affected by its refractive index, denoted as $n(\mathbf{r}, \omega)$ because of its dependence on the frequency ω of the radiation and on the location $\mathbf{r}$ within the medium. In the case of a homogeneous material with uniform density, the dependence of $n(\mathbf{r}, \omega)$ on $\mathbf{r}$ can be ignored. However, the $\mathbf{r}$ dependence of the refractive index must be considered when density variations occur, either naturally (such as in air) or are introduced artificially by grading the refractive index of a material in some fashion. As an example, the phenomenon of mirage results from an index gradient formed in air on a hot day. Such index gradients change with time because of changes in air's temperature and pressure. When density variations in a medium are static (time independent), the medium is referred to as a graded-index (GRIN) medium. We only consider static density variations in this book.

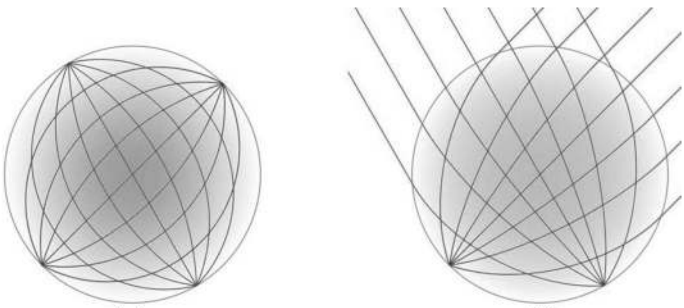

Figure 1.1 Schematic illustration of the GRIN lenses proposed by (a) Maxwell and (b) Luneberg. In both cases, refractive index is the largest at the center and decreases radially toward's the sphere's surface.

Historically, Maxwell proposed more than 160 years ago the concept of a GRIN device, known as the fisheye lens, even before he developed his celebrated equations [1]. The refractive index for such a lens exhibits spherical symmetry and depends on the magnitude of the vector **r**, but not on its direction. Similar ideas were used by Wood [2] in 1906, and by Luneberg in 1954, for imaging applications [3]. Figure 1.1 shows schematically how optical rays bend because of changes in the refractive index inside the GRIN lenses proposed by Maxwell and Luneberg. In both cases, optical rays follow curved paths to come to focus at a point on the sphere's surface.

With advances in glass technology, GRIN glasses could be fabricated by 1970 in which the refractive index varied in a cylindrically symmetric fashion in the plane normal to the direction of propagation. Such GRIN glasses were used either in a rod form [4] or drawn into a fiber form [5], depending on the application. At the same time, planar waveguides were developed in which the refractive index $n(x)$ varied only in one direction normal to the direction of propagation [6–8]. Two books published around 1977 provided a comprehensive account of such GRIN devices [9, 10].

The GRIN fibers were developed during the 1970s and their properties studied extensively in view of their potential applications in the emerging area of optical communications [11]. Indeed, by the year 1980, GRIN fibers were used for the first generation of such systems [12]. Even though telecommunication systems began using single-mode, step-index fibers by 1985, the development of new GRIN materials and devices remained an active area of research. For example, plastic-based GRIN fibers are used routinely for data-transfer applications [13]. One can get a good idea of the intense activity during the 1980s and 1990s by consulting several special issues of the *Applied Optics* journal [14]. Two books also describe the progress realized during this period [15, 16].

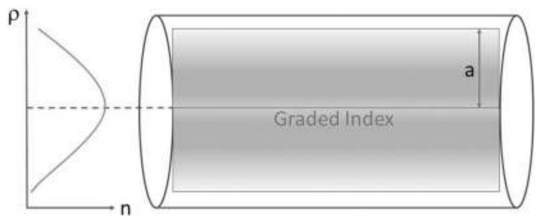

Figure 1.2 Schematic illustration of the refractive-index gradient along a GRIN device.

Starting around 2010, the advent of space-division multiplexing for modern telecommunication systems led to a renewed interest in the use of glass-based GRIN fibers [12, 17, 18]. Since then, the investigation of nonlinear optical phenomena in GRIN fibers has led to major advances. Among these are the topics such as spatiotemporal modulation instability, GRIN solitons, and spatial beam cleanup [19–21]. This book is intended to cover recent research advances and to provide, at the same time, comprehensive coverage of electromagnetic wave propagation inside a GRIN medium.

1.2 Refractive-Index Profiles

The focus of this book is on a GRIN medium whose refractive index varies in a plane normal to the direction of propagation (commonly taken to be the z axis) in a cylindrically symmetric fashion. Figure 1.2 shows schematically how the refractive index varies in such a GRIN rod around its central axis, chosen to be the z axis of the coordinate system. For practical reasons, the refractive index is the largest at the central axis and decreases gradually in all radial directions moving away from the center. In its most general form, the refractive index varies with the radial distance $\rho = \sqrt{x^2 + y^2}$ as [22–24]

$$n^2(\rho) = \begin{cases} n_0^2[1 - 2\Delta f(\rho/a)] & (0 \le \rho \le a) \\ n_0^2(1 - 2\Delta) = n_c^2 & (\rho \ge a), \end{cases} \tag{1.2.1}$$

where n_0 is the maximum value of the refractive index at the center and n_c is its minimum value at $\rho = a$, which is the radius of the cylindrical core enclosing the GRIN region. The function $f(x)$ governs shape of the index profile such that its value is 1 for $x = 1$.

The parameter Δ can be deduced from Eq. (1.2.1) and has the form

$$\Delta = \frac{n_0^2 - n_c^2}{2n_0^2} \approx \frac{n_0 - n_c}{n_0}. \tag{1.2.2}$$

where the approximate form holds when n_c differs from n_0 by at most a few percent so that $\Delta \ll 1$. This is often the case in practice for most GRIN devices. Using $\Delta \ll 1$, the refractive index in Eq. (1.2.1) can be approximated as

$$n(\rho) \approx \begin{cases} n_0[1 - \Delta f(\rho/a)] & (0 \le \rho \le a) \\ n_0(1 - \Delta) = n_c & (\rho > a). \end{cases} \tag{1.2.3}$$

This equation shows that $n(\rho)$ decreases as one moves away from the central axis up to a distance $\rho = a$ in a fashion dictated by the function $f(\rho)$ and takes minimum value n_c in the cladding region $\rho > a$. The GRIN region of radius a constitutes the core of such a GRIN device. The parameter Δ, given in Eq. (1.2.2), represents the fractional decrease in the refractive index across the core and its value is a design parameter for GRIN devices.

The function $f(x)$ governs the shape of the index profile for a GRIN device. This shape depends on the application for which the device is fabricated for and can vary over a wide range. In the case of planar waveguides, even an error function has been used for the shape [8]. In the case of GRIN rods and fibers, it is common to employ a power-law index profile with $f(x) = x^p$, where the exponent p governs the shape of the GRIN region. In this case, the refractive index in the core region varies as [22–24]

$$n^2(\rho) = n_0^2\left[1 - 2\Delta\left(\frac{\rho}{a}\right)^p\right] \qquad (\rho \le a). \tag{1.2.4}$$

Figure 1.3 shows how the refractive-index profile changes when p is varied in the range 1–10 using $n_0 = 1.5$ and $\Delta = 0.06$. The case $p = 2$ corresponds to a parabolic shape of the index profile. Note that the shape becomes closer to a step function for a large value of p such that n remains close to n_0 until one approaches the region near $\rho = a$, where it decreases rapidly and takes the value n_c. A step-index profile, occurring in the limit $p \to \infty$, is used routinely for making step-index fibers. Its use confines light within the core of a step-index fiber through the phenomenon of total internal reflection.

A parabolic index profile, realized for the choice $p = 2$ in Eq. (1.2.4), plays an important role in the literature on GRIN media, and many GRIN devices are designed with such a profile. In this case, we can write Eq. (1.2.4) in the simple form

$$n^2(\rho) = n_0^2(1 - b^2\rho^2), \qquad b = \sqrt{2\Delta}/a. \tag{1.2.5}$$

The parameter b is a measure of the index gradient such that its larger values indicate a faster reduction in the refractive index as ρ is increased. This parameter will play a prominent role in later chapters. As seen from the definition of b in Eq. (1.2.5), its value depends both on the core's radius a and the relative index difference Δ.

Depending on the application, numerical values of the three parameters, a, n_0, and Δ, associated with a GRIN device can vary over a wide range. We classify

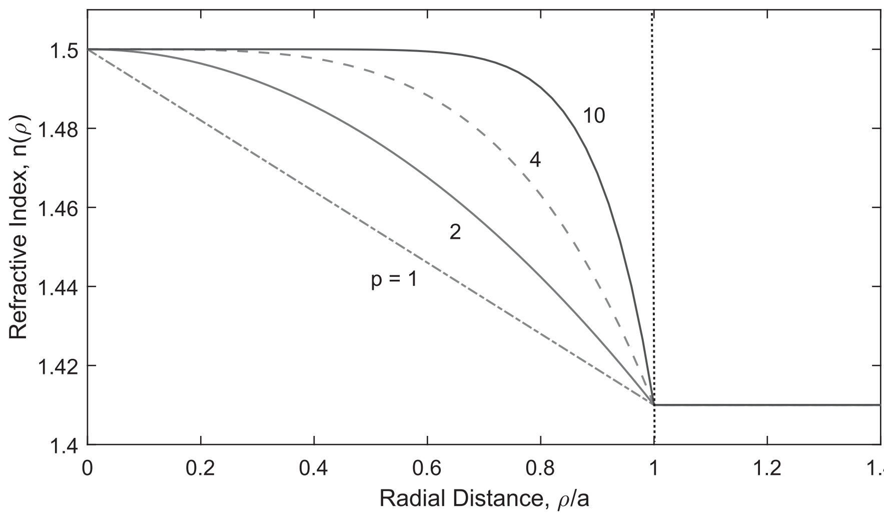

Figure 1.3 Refractive index $n(\rho)$ plotted as a function of the ratio ρ/a for several values of the parameter p. The vertical dotted line at $\rho = a$ separates the core and cladding regions of such GRIN devices.

GRIN devices into two broad groups based on the core's radius a. The value of a exceeds 1 mm for GRIN rods used to make lenses and similar optical elements. In contrast, a is restricted to much smaller values in the range of 10–30 μm for GRIN fibers used for telecommunication applications, among other things. The parameter Δ also varies for these two groups of GRIN devices. Its typical value is around 0.01 for GRIN fibers but can exceed 0.05 for GRIN rods. The value of n_0 depends on the material used for making a GRIN device. In the case of silica glass, n_0 is about 1.45. For plastics, n_0 is closer to 1.5.

We can estimate the value of the parameter b for GRIN rods and fibers from Eq. (1.2.5) by using the values of a and Δ. For GRIN rods, typical values of b are near 0.3 mm^{-1}. In contrast, b is around 5 mm^{-1} for GRIN fibers. Another relevant parameter of a GRIN device is its numerical aperture (NA). As indicated in Section 3.1, it depends on the values of n_0 and Δ as NA = $n_0\sqrt{2\Delta}$. The NA of a GRIN rod is close to 0.5 when $n_0 = 1.5$ and $\Delta = 0.05$. It is lower for GRIN fibers and has values of 0.2 or less.

1.3 Relevant Optical Processes

All materials affect the electromagnetic radiation propagating through them. The most relevant effects are (i) loss of power with distance owing to absorption and scattering, (ii) chromatic dispersion or a frequency-dependent refractive index, and

(iii) intensity-dependent changes in the refractive index of the material. All three are discussed in this section.

1.3.1 Power-Loss Mechanisms

Under quite general conditions, changes in the average power P of an optical beam propagating through a GRIN medium are governed by the Beer–Lambert law [25]:

$$\frac{dP}{dz} = -\alpha P, \tag{1.3.1}$$

where α is called the attenuation coefficient. It includes not only absorption of power by the material but also other sources of power attenuation such as Rayleigh scattering. If P_{in} is the power launched inside a GRIN medium of length L, the output power P_{out} is found by integrating Eq. (1.3.1) to be

$$P_{\text{out}} = P_{\text{in}} \exp(-\alpha L). \tag{1.3.2}$$

It is customary to express α in the decibel units using the relation [12]

$$\alpha\,(\text{dB/m}) = -\frac{10}{L}\log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \approx 4.343\alpha. \tag{1.3.3}$$

Numerical values of the attenuation coefficient α depend both on the material used to make a GRIN device and the wavelength of light launched into it. Figure 1.4 compares the wavelength dependence of measured loss in silica-glass fibers to losses in two types of plastic fibers [26]. As seen there, plastic fibers exhibit much larger losses (> 10 dB/km) compared to those of silica fibers, whose losses can be reduced to below 0.2 dB/km in the wavelength region near 1550 nm. Absorption by the plastic material is the source of high losses in plastic GRIN fibers.

In the case of optical glasses, absorption by the material of the glass is relatively small in the visible and near-infrared regions. However, even small amounts of impurities can increase this loss considerably. In the case of silica fibers, losses can be reduced to below 1 dB/km by eliminating all impurities. For such fibers, the dominant contribution to α arises from Rayleigh scattering, which is a fundamental loss mechanism arising from local microscopic fluctuations in the density of glass used to make the fiber. Glass molecules move randomly in the molten state and freeze in place during cooling. Resulting density fluctuations produce random fluctuations in the refractive index on a scale smaller than the optical wavelength λ. These fluctuations are the source of Rayleigh scattering, whose cross section varies as λ^{-4} [25]. As seen in Figure 1.4, silica's loss resulting from Rayleigh scattering exceeds 1 dB/km in the visible region but is reduced to below 0.2 dB/km in the infrared region near 1550 nm used for modern optical communication systems.

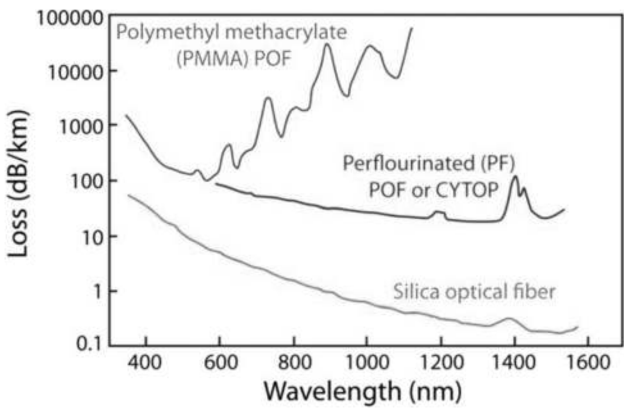

Figure 1.4 Wavelength dependence of the loss in silica fibers and losses in two types of plastic fibers. Note the logarithmic scale in units of dB/km. (After Ref. [26]; ©2014 IOP.)

1.3.2 Chromatic Dispersion

In the case of a GRIN medium, the refractive index $n(\rho, \omega)$ depends both on the spatial location ρ and the frequency ω. Chromatic dispersion has its origin in the frequency dependence of the refractive index. As we shall see in Section 2.1, it is the frequency dependence of the propagation constant, defined as $\beta(\omega) = n(\omega)(\omega/c)$, where c is the speed of light in vacuum, that governs the dispersive properties of any material. When the spectrum of incident light is narrower compared to its central frequency ω_0, we can expand $\beta(\omega)$ in a Taylor series as

$$\beta(\omega) = \beta_0 + \beta_1(\Delta\omega) + \tfrac{1}{2}\beta_2(\Delta\omega)^2 + \dots, \tag{1.3.4}$$

where $\Delta\omega = \omega - \omega_0$ and $\beta_m = (d^m\beta/d\omega^m)_{\omega=\omega_0}$.

In Eq. (1.3.4), β_1 is related inversely to the group velocity v_g and is responsible for the group delay, $\tau_g = \beta_1 L$, over a length L. The parameter β_2, representing the second derivative of β, is called the group-velocity dispersion (GVD) parameter. This parameter will play an important role in chapters dealing with the propagation of optical pulses inside a GRIN medium. For pulses shorter than 1 ps, it is sometimes necessary to consider the cubic term containing β_3 in the Taylor series in Eq. (1.3.4). This parameter is referred to as the third-order dispersion parameter.

The dispersion parameter β_1 can be calculated for any GRIN medium by taking the frequency derivative of β as

$$\beta_1 = \frac{d\beta}{d\omega} = \frac{n_g}{c}, \qquad n_g = n + \omega\frac{dn}{d\omega}, \tag{1.3.5}$$

where n_g is called the group index. It can be employed to calculate the GVD parameter in the form

$$\beta_2 = \frac{d\beta_1}{d\omega} = \frac{1}{c}\frac{dn_g}{d\omega}. \tag{1.3.6}$$

The sign of β_2 depends on the sign of the derivative $dn_g/d\omega$ and can be positive or negative in different spectral regions for glasses used to make a GRIN device. A related dispersion parameter D is also used for GVD; it is defined as

$$D = \frac{d\beta_1}{d\lambda} = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}. \tag{1.3.7}$$

It is easy to show that D is related to β_2 by the relation $D = -(2\pi c/\lambda^2)\beta_2$ and its sign is opposite to that of β_2.

On a fundamental level, the origin of dispersion is related to the atomic resonance frequencies at which a material absorbs electromagnetic radiation. Far from such resonances, the refractive index is well approximated by the Sellmeier equation [27],

$$n^2(\omega) = 1 + \sum_{j=1}^{M} \frac{B_j\omega_j^2}{\omega_j^2 - \omega^2}, \tag{1.3.8}$$

where ω_j is the resonance frequency and B_j is the oscillator strength. The parameters B_j and ω_j are obtained empirically by fitting the measured dispersion curve to Eq. (1.3.8) with $M = 3$. For pure silica glass, these parameters are found to be [27] $B_1 = 0.6961663$, $B_2 = 0.4079426$, $B_3 = 0.8974794$, $\lambda_1 = 0.0684043$ μm, $\lambda_2 = 0.1162414$ μm, and $\lambda_3 = 9.896161$ μm, where $\lambda_j = 2\pi c/\omega_j$ for $j = 1$ to 3.

We can use Eq. (1.3.8) to calculate the frequency dependence of n and n_g for the silica glass without an index gradient. Figure 1.5 shows this dependence in the wavelength range 0.6–1.6 μm. The group-delay parameter is obtained using $\beta_1 = n_g/c$. Even though n decreases monotonically with λ in the entire wavelength range, β_1 exhibits a shallow minimum for silica glass at the specific wavelength, $\lambda = 1.276$ μm, marked by the dotted vertical line in Figure 1.5. This wavelength is called the zero-dispersion wavelength (denoted by λ_{ZD}) because the GVD parameter β_2 vanishes at this wavelength.

Figure 1.6 shows how the dispersion parameters β_2 and D vary with wavelength λ for silica glass (no index gradient) using Eqs. (1.3.6) and (1.3.7). As expected, both β_2 and D vanish at λ_{ZD} near 1.27 μm and change sign for longer wavelengths. It is common to refer to negative values of β_2 as the GVD being anomalous. The curve marked d_{12} shows the differential group delay, $d_{12} = \beta_1(\lambda_1) - \beta_1(\lambda_2)$, using a reference wavelength $\lambda_2 = 0.8$ μm. It shows the relative delay of a pulse as its central wavelength λ_1 is varied.

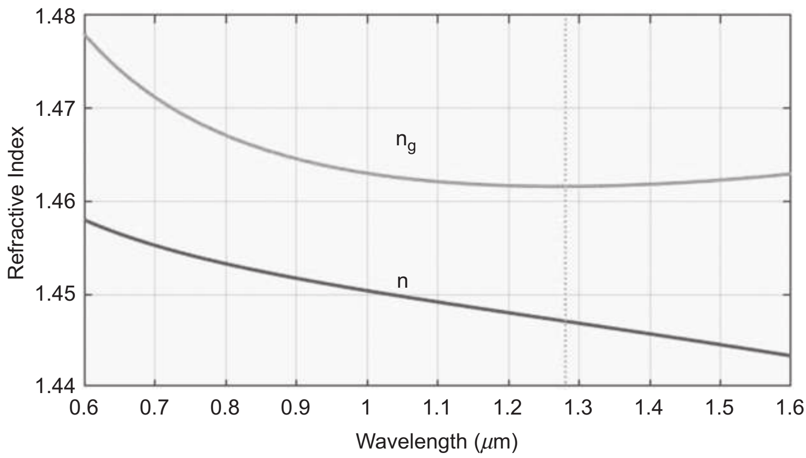

Figure 1.5 Variation of refractive index n and group index n_g with wavelength for fused silica. The dotted line indicates the zero-dispersion wavelength.

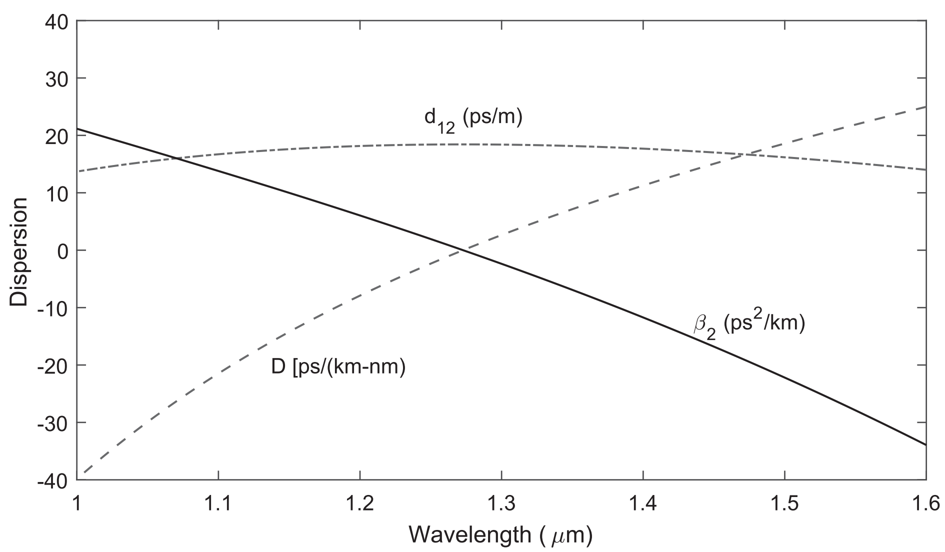

Figure 1.6 Wavelength dependence of β_2, D, and d_{12} for silica glass.

The situation changes considerably when silica glass is used to make a GRIN device. It will be seen in Chapter 2, that the propagation constant β, and hence all dispersion parameters, become mode-dependent for any GRIN medium. In particular, one must consider the intermodal group delay resulting from different values of β_1 for different modes. This topic is covered in Sections 2.4 and 4.1 in the context of optical pulses.

1.3.3 Intensity Dependence of Refractive Index

The response of any dielectric to electromagnetic radiation becomes nonlinear for intense electric fields, and materials used for making GRIN devices are no exception. Several common nonlinear effects have their origin in the Kerr effect. According to it, the refractive index of any material increases at high intensities such that [28]

$$n(I) = n_0 + n_2 I, \tag{1.3.9}$$

where I is the local intensity and n_0 is the low-intensity value of the refractive index. The parameter n_2 is called the *Kerr coefficient*. Its numerical value depends on the material used to make a GRIN device and is about 3×10^{-20} m^2/W for silica glass.

Adding the nonlinear contribution, the refractive index of a GRIN medium has the form

$$n(\mathbf{r}, \omega, I) = n_0(\omega)[1 - \Delta f(\mathbf{r})] + n_2 I(\mathbf{r}), \tag{1.3.10}$$

where the dependence on all three variables is shown explicitly. The maximum value of the nonlinear contribution, $\delta n = n_2 I(\mathbf{r})$, occurs at the location where the intensity peaks. Denoting this peak value with $I_0 = P_0/A_e$, where P_0 is the peak power and A_e is the effective beam area, $\delta n = n_2 P_0/A_e$. As an example, if we use $A_e = 1$ cm^2 and $n_2 = 3 \times 10^{-20}$ m^2/W, $\delta n = 3 \times 10^{-13}$ even at a relatively high peak power of $P_0 = 1$ kW. This value is too small to have any impact when a CW beam is launched inside a GRIN rod.

There are two ways to enhance the nonlinear effects inside a GRIN medium. First, the beam's effective area A_e is reduced considerably when GRIN fibers are used with a core radius close to 10 μm. Second, if a beam containing a train of short optical pulses is used, the peak power P_0 of the pulse can exceed 1 MW. Using $A_e = 10^{-10}$ m^2 for a GRIN fiber, the nonlinear contribution to the refractive index (about 3×10^{-4}) is much smaller than Δ, indicating that it is not likely to affect the GRIN-induced self-imaging phenomenon discussed in Chapter 3. However, if the GRIN fiber is long enough, the nonlinear contribution can affect both the temporal and spectral features of a pulsed beam. As noted in Chapter 5, it also produces novel spatiotemporal features that have been studied extensively in recent years [19–21].

The intensity dependence of the refractive index leads to several nonlinear effects; the two most common ones are known as *self-focusing* and *self-phase modulation* (SPM). The phenomenon of self-focusing is relevant for GRIN media because it can compress an optical beam and compete with the GRIN-induced focusing. Moreover, it leads to a beam's collapse above a certain critical power level [28].

SPM is relevant only for pulsed optical beams. It produces a self-induced phase shift that is different for different parts of the same pulse because of its intensity dependence. Its magnitude can be obtained from Eq. (1.3.10). After a distance L,