

A VISION FOR DYNAMICS IN THE 21ST CENTURY

A large international conference celebrated the 50-year career of Anatole Katok and the body of research across smooth dynamics and ergodic theory that he touched. In this book many leading experts provide an account of the latest developments at the research frontier and together set an agenda for future work, including an explicit problem list. This includes elliptic, parabolic, and hyperbolic smooth dynamics, ergodic theory, smooth ergodic theory, and actions of higherrank groups. The chapters are written in a readable style and give a broad view of each topic; they blend the most current results with the developments leading up to them and give a perspective on future work. This book is ideal for graduate students, instructors, and researchers across all research areas in dynamical systems and related subjects.

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The Legacy of Anatole Katok

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Editor's Note

The book cover shows the Palace of Bedlewo, a conference center of the Polish Academy of Sciences where a memorial conference for the great dynamicist Anatole Katok took place in August 2019.* This conference inspired this book, and many of the authors participated. During this conference we celebrated 50 years of Katok's leadership by doing what he had always fostered: creating a connected vision of the field of dynamical systems and ergodic theory and shaping an agenda for future research. These are the aims of this book, and its chapters give direction to their respective subjects, as well as a broad view and a contemporary account. A separate problem list provides specific suggestions for investigation.

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