

Physics of Wave Turbulence

A century ago, Lewis Fry Richardson introduced the concept of energy cascades in turbulence. Since this conceptual breakthrough, turbulence has been studied in diverse systems and our knowledge has increased considerably through theoretical, numerical, experimental, and observational advances. Eddy turbulence and wave turbulence are the two regimes we can find in nature. So far, most attention has been devoted to the former regime, eddy turbulence, which is often observed in water. However, physicists are often interested in systems for which wave turbulence is relevant. This textbook deals with wave turbulence and systems composed of a sea of weak waves interacting nonlinearly. After a general introduction which includes a brief history of the field, the theory of wave turbulence is introduced rigorously for surface waves. The theory is then applied to examples in hydrodynamics, plasma physics, astrophysics, and cosmology, giving the reader a modern and interdisciplinary view of the subject.

Sébastien Galtier is a professor at the University of Paris-Saclay. His research focuses on fundamental aspects of turbulence with applications to space plasmas and cosmology. He has published over 100 refereed papers and a graduate text, *Introduction to Modern Magnetohydrodynamics* (Cambridge University Press, 2016). He is a senior fellow of the prestigious Institut Universitaire de France.

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Sébastien Galtier

University of Paris-Saclay



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Preface

Ink, this darkness from which a light comes out
Victor Hugo, Dernière gerbe

Anyone who has ever flown in an aircraft knows how to define a turbulence zone: it is characterized by unpredictable, sometimes violent, often unpleasant jolts, which can even cause some anxiety in the passenger. For the physicist, on the other hand, turbulence is a pleasant, fascinating, and mysterious subject. This book proposes a journey into the world of turbulence in which we will gradually unveil the main fundamental laws governing the physics of turbulence where waves are omnipresent. We will see that since Reynolds' first historical experiment on liquids in 1883, turbulence has been studied in a wide variety of systems: from surface waves on the sea to gravitational waves, turbulence is now ubiquitous in physics.

Eddy turbulence and *wave turbulence* are the two regimes that we may encounter in nature. The attention of fluid mechanics being mainly focused on incompressible hydrodynamics, it is usually the first regime that is treated in books on turbulence. However, physicists are interested in much more diverse systems where waves are often present and for which the second regime (the subject of this book) is relevant. Wave turbulence offers the possibility of developing an analytical theory. Beyond its mathematical beauty, this spectral theory allows a deep understanding of weakly nonlinear systems and to develop a physical intuition on strong wave turbulence. Weak and strong wave turbulence are not independent of each other. On the contrary, one can emerge from the other during the cascade process; the two regimes can also coexist and be in permanent interaction. Without being exhaustive, this book offers a relatively broad overview on wave turbulence which should enable beginning researchers to acquire fundamental knowledge on subjects which are sometimes under development.

The theoretical framework chosen in this book will be that of statistically homogeneous turbulence for which a universal behavior is expected. In Chapter 1, a

general introduction to turbulence is given where we find a brief history of the evolution of ideas, and the emergence of the main concepts and results. This history is of particular importance today, a century after Richardson (1922) introduced the concept of energy cascade. The fundamentals of turbulence are outlined in the physical (Chapter 2) and spectral (Chapter 3) spaces, which constitutes Part I. This first part focuses on incompressible hydrodynamics and thus on eddy turbulence. With Part II, we enter into the core of the book. Wave turbulence is introduced in Chapter 4, with a brief history and a presentation of the multiple scale method for weakly nonlinear systems. In Chapter 5, the theory of weak wave turbulence is presented in great detail for capillary waves, which is one of the simplest systems (three-wave interactions, two-dimensional, Navier–Stokes equations). Various examples dealing with three-wave interactions are discussed in Chapters 6, 7, and 8. In Chapter 9, we conclude with a new topic – gravitational wave turbulence – which is far more complex and involves four-wave interactions.

This book is based on a course on turbulence that I have been giving for several years at École polytechnique to students of the Master’s degree in plasma physics (from the University of Paris-Saclay, Institut Polytechnique de Paris, and Sorbonne University). It is thus, in part, the result of fruitful interactions with my students, whom I would like to thank. I would also like to thank all my colleagues with whom I share my passion on this subject and who have contributed, in their own way, to the writing of this book; I would like to thank in particular Nahuel Andrés, Supratik Banerjee, Amitava Battacharjee, Éric Buchlin, Pierre-Philippe Cortet, Vincent David, Éric Falcon, Stephan Fauve, Özgür Gürçan, Lina Hadid, Romain Meyrand, Frédéric Moisy, Sergey Nazarenko, Alan Newell, Hélène Politano, Fouad Sahaoui, and, of course, Annick Pouquet, who introduced me to turbulence.