Introduction

> We think we know our mine and its stones, but what this engineer is
> constantly investigating here in this way is beyond our comprehension.
>
> Franz Kafka, *A Visit to the Mine*[*]

In 1686, mining official and tax collector Nicolaus Voigtel published
Geometria subterranea, oder Marckscheide-Kunst. Despite its obscure title,
the book was an instant commercial success for a very simple reason: no prac-
titioner before him had ever printed anything about the art of underground
surveying.[1] The frontispiece was designed to catch the eye and the imagi-
nation of early modern readers (Figure I.1). Two surveyors, both wearing
improbable uniforms, surround a mirror's frame ornamented with grimacing
mascarons. Their modest hats symbolize the average miner's protecting head-
gear, indicating that they have practical experience of the matter. Similarly,
their long vests evoke the rough *Arschleder* (butt leathers) then ubiquitous
among miners.[2] Their fine stockings and fancy shoes, however, clearly reveal
a higher status and relate them to the world of learning, knowledge, maybe
even sciences. Both men are depicted in a teaching position, with the senior
surveyor using a measuring stick as a pointer towards the inside of a rocky
tunnel.

The engraver played here with the then-popular genre of the theatres of
machines, in which the world – presented as a construction site – was the cen-
tral object of the frontispiece. Lured by the metaphor of the mirror, sometimes
replaced by a stage, readers were metaphorically invited behind the scenes to
observe hydraulic or hoisting machinery and envision 'a possible, mathemati-
cally-guided future'.[3] In the *Geometria subterranea*, the dark mine at the cen-
tre of the frontispiece could be taken in a literal sense, for the topic of Voigtel's

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[*] Kafka, *A Hunger Artist and Other Stories*, translation by Joyce Crick (2012), p. 26.
[1] Within a generation, it went through at least three editions and nine printings. This work is stud-
ied in more detail in Chapter 4.
[2] For depictions of actual miners and surveyors, see Weigel, *Abbildung und Beschreibung derer
sämtlichen Berg-Wercks* (1721), pp. 6, 14.
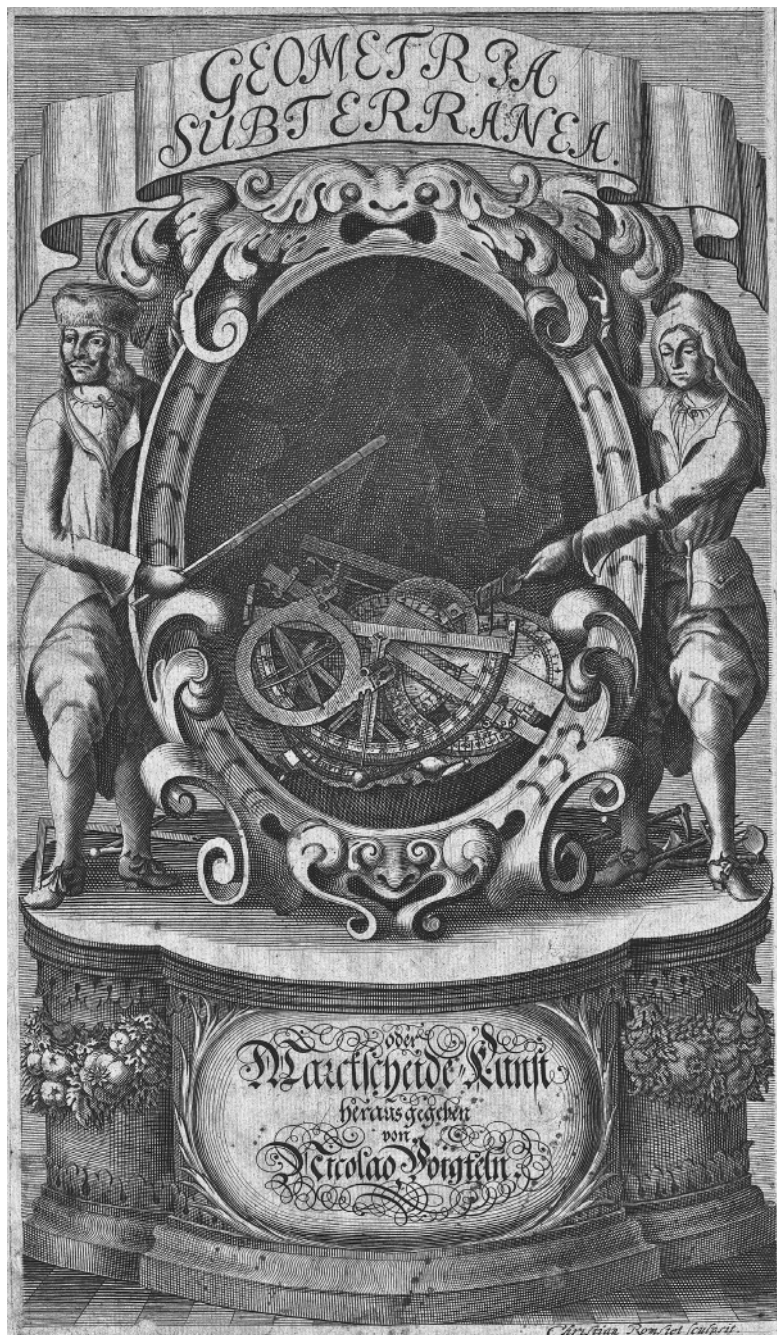[3] Keller, 'Renaissance Theaters of Machines' (1978), p. 495.

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Figure I.1 Frontispiece of Nicolaus Voigtel, *Geometria subterranea, oder Marckscheide-Kunst* (Eisleben, 1686), engraved by Christian Romstet. Courtesy ETH-Bibliothek Zürich, Rar 271.

book was precisely to explain how to survey such obscure and sinuous tunnels. More allegorically, Voigtel presented himself as the first to divulge what had long 'been kept very secret', hidden in esoteric manuscripts and obscured by the dialect of practitioners.[4] On the ground, where an ingenuous reader might expect to find a silver ore vein or precious crystals, the younger miner hints at the instruments of subterranean geometry. One can discern a compass suspended in gimbals and a semi-circle. Both were usually suspended from a surveying chain, here represented in the miner's hand, to find one's direction underground. The implicit message was clear: the riches contained in this book were not made of silver, crystal, or even gold. They could not be extracted using a pick and a hammer, or any of the boring instruments casually represented – below the mirror, separated from the mathematical ones – at the two men's feet. The real treasure presented here was the useful knowledge needed to dig and extract precious metals. Buying this book and following the two miners' instructions, the frontispiece implied, any diligent reader could learn the esoteric art of subterranean geometry.

In the early modern period, much of the European economy relied on metals. Rulers fought for the much-needed silver coins that would fuel costly wars – *pecunia nervus belli*, as the saying went. Metal money powered the nascent capitalist economies and enabled early colonial enterprises.[5] Copper could be used for coins and engravings, or as an alloy in bronze to cast bells and cannons. Zinc, tin, and lead all had numerous uses in manufacturing and construction. Lesser-known metals such as cobalt, bismuth, or quicksilver were vital in countless crafts and processes, from medicine to glass colouring.[6] Yet these metals mostly came from a handful of mining regions in central Europe and Scandinavia, most prominently the Ore Mountains of Saxony, the Harz, and several regions of the Hapsburg dominions. Metal mines were closely monitored by local rulers whose finances crucially depended on the 'bloodstained labour and work' of miners, as Voigtel put it in his introduction.[7] A merchant from Nuremberg, a commercial metropolis known for its goldsmiths and founders, summed up this unique position in the early sixteenth century: 'that much silver cannot be found in no other land than in the Holy Empire, so that

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[4] Voigtel, *Geometria subterranea oder Markscheide-Kunst* (1686), introduction: 'sehr geheim gehalten worden'. (NB. All translations from German, French and Latin to English are my own, unless otherwise indicated.) On this literary tradition, see the recent PhD thesis of Benjamin Ravier, *Voir et concevoir: les théâtres de machines (XVIe–XVIIIe siècle)* (2013).

[5] See Strieder, 'Die deutsche Montan- und Metall-Industrie im Zeitalter der Fugger' (1931), pp. 189–226; Graulau, *The Underground Wealth of Nations* (2019).

[6] For a very short introduction to early modern mining, see Küpker, 'Manufacturing' (2015), pp. 516–519. The seminal work on ore mining – especially in Central Europe – is the four-volume German *Geschichte des deutschen Bergbaus* (2012), here especially vol. 1, pp. 317–452.

[7] Voigtel, *Geometria Subterranea* (1686), *An den Leser*: 'mit blutsauerer Mühe und Arbeit Bergwerck bauen'.

all christian and unchristian countries have to be fed and supplied with silver from the German kingdoms'.[8]

In order to meet this demand, mines became both bigger and more complex. In the late fifteenth century, the discovery of new ore deposits led to a silver rush known as the 'mining clamour' (*Berggeschrey*). Sinuous tunnels expanded, following ore veins to previously unknown depths. Capital flew from Nuremberg and Venice, later from the Low Countries, as mining cities flourished and attracted some of the finest minds of their time. Conversely, miners and assayers from various provinces of the Holy Roman Empire travelled to distant countries, helping to extract tin in Cornwall or copper in the Carpathian Mountains of Eastern Europe.[9] Skilled technicians designed the complex mining pumps, the huge furnaces, and the myriad of technical instruments depicted in early modern books such as Agricola's *De re metallica*.[10] This led Max Weber to present, in his *Economic History*, the mines as a crucible of modern rationality, a place from which 'experimentation was taken over into science'. According to Lewis Mumford, 'the mine is nothing less in fact than the concrete model of the conceptual world which was built up by the physicist of the seventeenth century'.[11] And yet, mining relied first and foremost not on mineralogy or metallurgy but on the obscure *geometria subterranea*. The usefulness of this esoteric branch of mathematics quickly became proverbial, as expressed in 1569 by the Parisian humanist Petrus Ramus: 'It is by means of geometrical hands that the Teutonic Pluto draws to itself the riches of the German soil.'[12]

The fateful interplay between geometrical practices and the nascent earth and mining sciences is an important theme of this book. From the silver rush to the foundation of modern mining academies in the late eighteenth century, humanists and later natural scientists relentlessly sought to uncover what they saw as

[8] Dietrich, *Untersuchungen zum Frühkapitalismus im mitteldeutschen Erzbergbau und Metallhandel* (1991), p. 34: 'alles silber findt man die menig in keinem anderen land denn im Heiligen Reich, sondern alle umbligene christliche und unchristliche land müssen aus teutschen landen mit silber gespeist und versehen werden' (text dates from 1523). The Holy Roman Empire produced half of the European metals by some estimates, and figures are much higher for some precious metals such as silver.

[9] Rapp, *Les origines medievales de l'Allemagne moderne: de Charles IV à Charles Quint (1346–1519)* (1989), pp. 149–152; Graulau, *The Underground Wealth of Nations* (2019), especially p. 4.

[10] Agricola, *De re metallica libri XII* (1556). While Agricola was not the only one, or even the first, to depict the mining world – see Biringuccio, *Pirotechnia* (2005) – the quality of his illustrations stands out until well into the following century. See Déprez-Masson, *Technique, mot et image* (2006).

[11] Weber, *General Economic History* (1927), p. 368; Mumford, *Technics and Civilization* (2010), p. 70: 'Did the mine acclimate us to the views of science? Did science in turn prepare us to accept the products and the environment of the mine? The matter is not susceptible to proof: but the logical relations, if not the historical facts, are plain.'

[12] Ramus, *Scholarum mathematicarum libri unus et triginta* (1569), p. 60: 'Ergo Germanicus ille Pluto geometricis manibus divitias suas Germaniae effodit atque eruit.'

the secrets of subterranean surveyors. At the same time, they often relied on the craftsmen's vernacular expertise, on their instruments and experience of practical geometry to advance their own theoretic inquiries.

What did this mysterious discipline consist of? Its German name (*Markscheidekunst*) literally reads as 'the art of setting limits'. Concretely, it encompassed all measuring operations used in the delimitation of concessions and in the daily running of metal mines. Surveyors ascertained the direction of galleries and knew how to bypass crumbled or dangerous sections. They learnt to draw mining maps and to use them to monitor local districts or to connect existing workings. Over time, subterranean geometry came to include more original tasks such as building dams to create water ponds or planning complex hydraulic systems that powered both mining and smelting machines. This artisanal mathematics gradually became an obvious tool to alleviate the uncertainty of mining, manage the technical issues, and organize the extraction of metals in a rational manner. If a few items of Voigtel's textbook belonged to the academic mathematical curriculum, the better part dealt with matters that did not follow traditional categories and classifications. The use of instruments and the numerous surveying methods were passed on from master to student, with an emphasis put on direct observation and training in the mines. In other words, the discipline fell outside of the academic geometry of its time, and vastly differed from the deductive implementation of theories that early modern scholars labelled 'applied' or 'mixed' mathematics.[13]

Subterranean geometry lays more broadly at the intersection of the history of science, culture, economy, and technology. This 'art of setting limits' hardly fits present-day classifications of knowledge, which is precisely why it has so far fallen out of view. In the last decade, however, our understanding of early modern efforts to understand and master nature has been greatly renewed, building on earlier works by Peter Dear, William Eamon, and Hélène Vérin.[14] It has become clear that mines played a decisive role in the transformation of knowledge about nature, confirming the intuitions of Max Weber and Lewis Mumford. These densely urbanized regions attracted money and skills, drawing scholars including Georgius Agricola (see Chapter 1), Jean-André Deluc (Chapter 7), Paracelsus and Gottfried Wilhelm Leibniz.[15] Pamela O. Long has

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[13] On the distinction between pure, applied, mixed and practical mathematics, see Epple, Kjeldsen, and Siegmund-Schultze, 'From "Mixed" to "Applied" Mathematics: Tracing an Important Dimension of Mathematics and its History' (2013), pp. 657–733. And in particular Jim Bennett's contribution 'How Relevant Is the Category of "Mixed Mathematics" to the Sixteenth Century?' (pp. 677–680).

[14] See Dear, *Discipline and Experience* (1995); Eamon, *Science and the Secrets of Nature* (1994); Vérin, *La gloire des ingénieurs* (1993).

[15] Among recent works, Pamela Long has produced the most influential account on the role of early modern mines. See Long, *Openness, secrecy, authorship* (2001), esp. ch. 6, 'Openness and Authorship I: Mining, Metallurgy, and the Military Arts'; on mining experts and the circulation of knowledge, see Ash, *Power, Knowledge, and Expertise in Elizabethan England* (2004), esp. ch. 1, 'German Miners, English Mistrust, and the Importance of Being "Expert"'.

further argued that historians should devote more attention to artisans in order to understand properly the rise of new sciences. Long describes the mines as important 'trading zones' in which people and knowledge feverishly circulated and adapted to challenging environments, where scholars met and occasionally collaborated with practitioners.[16]

Building on these results, I argue that geometry decisively influenced the new technical landscape of the Holy Roman Empire, with far-reaching epistemological consequences. Despite some brilliant efforts, early modern scholars struggled to understand a culture in which geometry and arithmetic blended with mechanics, mining laws, knowledge of the earth and its minerals. Mine surveyors, for their part, had to cope with capricious economic and political landscapes. From the introduction of gunpowder blasting to the discovery of rich silver mines in New Spain, they incessantly adapted their skills, instruments, and methods. Subterranean geometry was not a distinct science but a craft culture that encompassed a vast tradition of quantification. The following studies thus rely on a broad range of sources: the few textbooks published on the topic – such as Voigtel's *Geometria subterranea* – are contrasted with a rich tradition of manuscripts circulated from masters to pupils. Sketches and survey books, mining maps, and apprentices' examinations, combined with a wide array of administrative documents, reveal the significance of this mathematical culture. Even religion came into play, with Protestant priests writing evocative sermons such as the *Spiritual Mine* or the *Mining Homilies* (see Chapter 2). Sunday sermons indeed played an important role in popularizing the discipline, praising 'the marvellous instrument, the compass, and its use in subterranean geometry'.[17]

More broadly, the present study explores the silent rise of practical mathematics in the early modern period, epitomized here by a discipline which challenges our modern understanding of practice and theory.[18] The history of subterranean geometry illustrates the growing reliance on numbers and geometric figures in civil society at large. Being a cornerstone of the all-important mining *œconomy*, the discipline elicited heated debates about the methods and values of mathematics. Sporadic exchanges with the learned spheres reflect the richness and complexity of the relationship between the science taught in universities and the crafts used underground. The inner logic of the 'art of setting limits', its historical development, and numerous ramifications blur the distinction between crafts and sciences, contributing to current debates and

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[16] See Long, *Openness, Secrecy, Authorship* (2001); Long, *Artisan/Practitioners and the Rise of the New Sciences, 1400–1600* (2011).

[17] Eichholtz, *Geistliches Bergwerck* (1655), p. 131: 'das Wunder *Instrument* den Compaß / und dessen Gebrauch im Marscheiden'.

[18] Morel, 'Mathematics and Technological Change: The Silent Rise of Practical Mathematics' (2023).

redefinitions of what useful knowledge meant and how it was used in the past.[19] It builds and expands on a growing scholarship about technical education and the role of mathematical practitioners. This discipline, born in the mining pits of the Holy Roman Empire and performed over centuries by craftsmen, who slowly turned into engineers, exemplifies a 'vernacular conception of nature' typical of the early modern period.[20]

Underground Mathematics

Using mathematical methods in inquiries about nature is often seen as a trait of modernity *per se*, whereas it should be historically questioned and explained. Why did geometry and arithmetic become credible tools in efforts to understand nature? How did this happen concretely in the early modern period? Who were the actors, how did they train and collaborate, and what methods were actually used? Historians dealing with the mathematization of nature have long worked to clarify the complex relationships between scholars and practitioners, sometimes questioning the relevance of such labels altogether. The once radical ideas of Edgar Zilsel (1891–1944), who argued that 'superior craftsmen' had played a key role in the shaping of the new sciences and the introduction of mathematical laws, have been revived, amended, and widely discussed, most recently by Lesley Cormack and Margaret Schotte.[21] Studies have underlined the crucial role played by mathematics and quantification in the emergence of rationality and the rise of new sciences, a position sometimes presented as a 'mathematization thesis'.[22] While there seems to be a consensus on the importance of mathematics in the early modern period, its precise influence and degree of usefulness is considerably more difficult to ascertain.

By describing the rational culture shaped by miners, its social impact and its gradual improvements, the increasing influence of mathematics can be coherently accounted for. Conversely, this book questions the scholarly depictions of early modern mathematical practices, building on Robert Halleux's seminal remark: 'Those writing books and those practising arts and crafts were not the same persons.'[23] Looking beyond academic knowledge, one can enquire

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[19] Morel, Parolini, and Pastorino, *The Making of Useful Knowledge* (2016); Valleriani, *The Structures of Practical Knowledge* (2017).

[20] Smith, 'Science on the Move: Recent Trends in the History of Early Modern Science' (2009), p. 364.

[21] Cormack, Walton, and Schuster, *Mathematical Practitioners and the Transformation of Natural Knowledge in Early Modern Europe* (2017); Schotte, *Sailing School* (2019).

[22] On the contrary, one can find a critical account of this thesis in Cohen, *The Scientific Revolution: A Historiographical Inquiry* (1994), pp. 309–327, defending the idea that, prior to the seventeenth century, 'mathematics had impinged upon craftsmen's activities only in a few exceptional cases'.

[23] Halleux, *Le savoir de la main* (2009), p. 8.

how laws, religion, and politics converged to present mathematics as the most accurate and efficient method for understanding nature and acting upon it. While many excellent studies focus on the interactions between scholars and their patrons, the present book suggests a methodological workaround. Studying the involvement of craftsmen in human affairs and society at large, one can reconsider the multifaceted influence of quantification on early modern thought.

The historical significance of metals and money, combined with the rich history of earth crafts and sciences, has led to detailed analyses of the early developments of mineralogy, chemistry, or metallurgy. Dowsing rods and transmutative alchemy, mining trolls and popular magic have all been analysed by modern historians.[24] Mathematics, on the other hand, is surprisingly missing from historical accounts of mines, caves, and the underground world. Yet it was ubiquitous in prospecting, extracting, and assaying activities, even if nowadays exact sciences are more commonly associated with university towns or abstract theories. Past uses of arithmetic and geometry in mining are not so much ignored as taken for a static given, or purely empirical rules of thumb. Historians usually recognize that the extraction of ore implied some kind of measurements but see these as mere applications of 'general mathematical laws'.[25] Even as recent studies have recommended using actors' categories and contextualize past knowledge about nature, the growing use of geometric figures and numbers in mining is mostly mentioned only in passing and considered self-explanatory.[26]

In the sixteenth century, this book argues, an intrinsically useful culture of geometry pervaded the mining cities and conditioned their ulterior technical and scientific developments. More importantly, this culture gradually spread beyond the mining states of Saxony, Bohemia or Brunswick and was felt in all of the Holy Roman Empire and beyond. Given that both the mines and the mathematical arts are widely seen as triggering dramatic changes in early modern world-views, one can gain general insights from studying subterranean geometry, which lies precisely at the crossroad of these topics.[27] In order

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[24] On the history of the dowsing rod and the issue of natural magic in early modern mining, see Dym, *Divining Science* (2011). On transmutative *chymistry* and magical knowledge, see Fors, *The Limits of Matter* (2015).

[25] Baumgärtel, 'Von Bergbüchlein zur Bergakademie' (1965), p. 93.

[26] Graulau, *The Underground Wealth of Nations* (2019), pp. 10–12, rightly states that 'the medieval evolution of mathematics worked to the advantage of early capitalist mining business' – the topic of her book – without engaging with the nature of this evolution or its exact influence.

[27] A series of articles on subterranean geometry was produced in the 1930s and 1940s by Walther Nehm, himself a professor of subterranean geometry. In recent times, the subject has been touched upon by Ziegenbalg, 'Aspekte des Markscheidewesens' (1984), pp. 40–49; Ziegenbald, 'An Interdisciplinary Cooperation' (1993), pp. 313–324; Ziegenbalg, 'Von der Markscheidekunst zur Kunst des Markscheiders' (1997). It is briefly mentioned in Ash, *Power, Knowledge, and Expertise in Elizabethan England* (2004), pp. 28–30.

to understand the practitioners' crafts, it is necessary to immerse oneself in the geology, laws, and religion of the time. Recent works have shown how the global contributions of mathematical practitioners, from navigation to fortification, were intrinsically linked to the specific domains they were part of.[28] To that extent, our modern ignorance of subterranean geometry simply illustrates its idiosyncratic, at times even esoteric, character. These practitioners were literally and metaphorically underground engineers, whose discipline flourished in the early modern period only to lose its global echo in the nineteenth century. Once mining academies were founded in the aftermath of the Seven Years War (1756–1763), public interest gradually shifted to the more challenging puzzles of a quickly industrializing world. The efficiency of mathematics in mine engineering, a source of wonder during the early modern period, was now simply taken for granted. A once much-discussed discipline had been all but forgotten.[29]

What makes the craft of subterranean geometry relevant is that it was routinely practised by and for 'common men' and performed publicly, as detailed in Chapter 2.[30] Long before it became a specialized profession, the *Markscheidekunst* was both an art and an administrative position that could be practised by various officials: foremen and mining masters in the sixteenth century, sworn mining officials, technicians, and engineers in later times. Its history will here be told mostly from the point of view of practitioners, many of them anonymous, including not only the surveyors, but the investors, local preachers, and the mining people at large. This choice obviously has its own challenges. Unlike university professors and clerics, most craftsmen did not publish anything – Voigtel's book being in this sense exceptional – although some of their technical accomplishments have survived to this day. The present book is therefore based on a diverse set of archival documents, handwritten textbooks, maps and sketches, travel diaries and trial reports, calculation sheets as well as the prodigiously comprehensive records of mining administrations.

Writing and publishing simply did not belong to the professional duties of mining officials, as they obeyed a logic of administration rather than a logic of

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[28] Johnston*, Making Mathematical Practice* (1994), p. 2. Recent works in the history of mathematical practices include the domains of navigation, architecture and fortification. See Schotte, *Sailing School* (2019); Métin, *La fortification géométrique de Jean Errard et l'école française de fortification (1550–1650)* (2016); Lefèvre, 'Architectural Knowledge' (2017), pp. 247–270.

[29] At the turn of the twentieth century, the then-called *Markscheidekunde* had firmly been established as an engineering discipline with proven methods. See Wilski, 'Über die heutige Markscheidekunde' (1933), pp. 61–66.

[30] On the role of the 'common men', see Lutz, *Wer war der gemeine Mann?* (1979); Whaley, *Germany and the Holy Roman Empire* (2013), pp. 12, 187, 221. The notion of *Gemeiner Mann* refers not only to the lowest rank of society, here the illiterate miners, but to the citizen at large, whose 'participation … in the governmental process was undoubtedly an important feature of early modern German society' (Whaley, p. 12).

patronage. Johann Andreas Scheidhauer, a mining master presented in Chapter 6, carefully recorded for several decades his theoretical considerations and experiments on subterranean geometry, water wheels, and all kind of practical mathematics. These documents were known and circulated within a circle of trusted colleagues during his lifetime. However, they were not intended to be printed and thus remained after his death buried in mining archives, where their significance and originality went unnoticed. Surveyors and mining officials were mostly literate and eagerly read the vernacular booklets about geometry or mercantile arithmetic, but few of them were learned. Their lack of university education and insufficient skills in High German were repeatedly mocked by scholars. Such difficulties were compounded when they tried to describe their technical procedures using the *Bergmannsprache*, the heavy dialect spoken in mining regions. German-speaking contemporaries could not understand even a simple discourse on subterranean geometry, and would have needed to peruse the *Mathematisches Lexicon* of Christian Wolff to decipher it. On the title page of his dictionary, the famous philosopher and mathematician conveniently promised to 'describe the dialect and expressions of the subterranean surveyors, as well as of artists and artisans'.[31]

Scholars are not absent from the following case studies, as they repeatedly try to understand, systematize, and divulge what they consider to be 'books of secrets'. During the mining boom, scholars and rulers routinely complained about practitioners 'so jealous and begrudging about their art, that they do not want anybody to see it', at a time when underground surveying was mostly a hands-on know-how that could hardly be put on paper. Knowledge had to be open, scholars argued, 'so that henceforth one or two hundreds craftsmen do not have to believe a single one, without sufficient evidence' and face the potentially severe consequences.[32] These debates, as we shall see, were ultimately less about the openness of knowledge than about who had authority over it. Even the publication of Voigtel's comprehensive *Geometria subterranea*, analysed in Chapter 4, did not put an end to the criticism of practitioners. Leonhard Christoph Sturm, university professor and member of the Prussian Academy of Science, confessed that he had 'little experience of the mining language'. Having never found 'the time and patience to read the bespoken book', it was enough for him to 'leaf through it and consider the images'. Having visited a mining site once, he had witnessed surveying operations but 'could not understand anything about it'. Faced with the incommunicability of practical

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[31] Wolff, *Vollständiges Mathematisches Lexicon* (1734), title page: 'Die Mund- und Redens-Arten derer Marckscheider auch hieher gehöriger Künstler und Handwercker, beschrieben.' For a modern analysis, see Drissen, *Das Sprachgut des Markscheiders* (1939).

[32] Reinhold, *Gründlicher vnd Warer Bericht. Vom Feldmessen, Sampt allem, was dem anhengig*, (1574), dedication: 'damit forthin nicht ein 100. oder 200. gewercken / einem allein / ohne gnungsame beweiß / mußten glauben geben.'