Einstein's General Theory of Relativity

Einstein's general theory of relativity can be a notoriously difficult subject for anyone approaching it for the first time, with arcane mathematical concepts such as connection coefficients and tensors adorned with a forest of indices. This book is an elementary introduction to Einstein's theory and the physics of curved space-times that avoids these complications as much as possible. Its first half describes the physics of black holes, gravitational waves, and the expanding Universe, without using tensors. Only in the second half are Einstein's field equations derived and used to explain the dynamical evolution of the early Universe and the creation of the first elements. Each chapter concludes with problem sets, and technical mathematical details are given in the appendices. This short text assumes a familiarity with special relativity and advanced mechanics.

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A Concise Introduction

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and
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To Mary Mulvihill,
the best science communicator I have ever known.

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Preface

This book is based on lecture notes for an undergraduate course on general relativity taught at Maynooth University in Ireland over a number of years. General relativity is a notoriously difficult subject, famous for a forest of indices that obscure the underlying physics, making it difficult to extract what are often counter-intuitive phenomena such as the bending of light and the physics of black holes. In this book tensors are avoided for as long as possible; the only prerequisites are a good understanding of Lagrangian mechanics and an introductory course on special relativity, though some elementary electrodynamics, thermodynamics, and elementary quantum mechanics would be useful.

Many fascinating results from general relativity, such as the precession of the perihelion of Mercury and the bending of light by the Sun, can easily be understood without the full mathematical machinery of differential geometry. One merely needs to grasp the concept of a geodesic, the shortest path between two points in a given geometry, and learn how to determine the geodesics associated with that geometry, without needing to ask where the geometry comes from. Anyone who understands Lagrange's variational approach to classical mechanics can do this using Lagrangians quadratic in generalised coordinate velocities without the need to introduce tensors and Christoffel symbols, and this is the approach adopted here. Exceptions to this are the analytic form of the cosmic scale factor in Robertson–Walker metrics and the production of gravitational waves: the former is postponed here till after the introduction of Einstein's equations, and the latter is not treated at all, being too advanced for the level adopted here, though the propagation of gravitational waves through empty space is discussed.

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After a brief introduction to the equivalence principle and the notion that the gravitational force is due to the curved geometry of 4-dimensional space-time, the first half of the book examines geodesics in various space-times before tensors are introduced. The line elements for various geometries are just written down, without justification, and the geodesics studied. The philosophy is similar to the way most students learn electromagnetism; the Coulomb field of static point charges and the magnetic field of solenoids and long, straight wires carrying electric currents are usually studied before encountering Maxwell's equations in all their glory. Here students are introduced to the Schwarzschild geometry and general properties of Robertson–Walker line elements before being faced with the full complication of Einstein's equations.

There is a distinct change of gear in the second half of the book where the mathematical description of curved geometries is developed, parallel transport is discussed, connections and Christoffel symbols are introduced, and the Riemann tensor is defined as a prelude to deriving Einstein's equations. Only then can cosmology and the Big Bang be described properly, as the dynamics of the early Universe depend crucially on the equations of state for matter and radiation through Einstein's equations.

In Chapter 4 the mathematics necessary for describing higher-dimensional curved spaces is developed. This chapter is unavoidably more abstract than the other chapters in the book, and the main results necessary for understanding Chapters 5, 6, and 7 are summarised on page 116. Chapter 4 can be omitted on a first reading except for the equations on page 116.

With a view to making the text as readable as possible, properties of the Riemann tensor are stated without proof in the body of the text, with technical details of the proofs being relegated to an appendix, as is the derivation of the Riemann tensor for Schwarzschild and Robertson–Walker space-times. The energy-momentum tensor for a relativistic fluid is also explained in a separate appendix.

It is a pleasure to thank Charles Nash and Sally Lindsay for a careful reading of the manuscript and useful suggestions, though any errors or omissions are entirely the author's own responsibility.