
Contents

	<i>Preface</i>	<i>page xi</i>
	Part I The Real Projective Plane	
1	Fundamental Aspects of the Real Projective Plane	3
	1.1 Parallelism	3
	1.2 Perpendicularity	3
	1.3 Duality	6
	1.4 Two Models of the Real Projective Plane	6
	1.5 Recap: The Real Projective Plane as Involving Points and Lines	10
	Exercises	11
2	Collineations	12
	2.1 The Projective General Linear Group	12
	2.2 The Fundamental Quadrangle	15
	2.3 The Fundamental Theorem of Projective Geometry	17
	2.4 Interlude: Homogenisation of Polynomials	26
	Exercises	28
3	Polarities and Conics	31
	3.1 Dualities and Polarities	32
	3.2 Conics	34
	Exercises	40
4	Cross-Ratio	43
	4.1 Introduction to Cross-Ratio	43
	4.2 Perspectivities	47
	4.3 Cross-Ratio of Lines	56
	4.4 Interlude: From Affine Ratio to Cross-Ratio	58

4.5	Harmonic Quadruples	64
4.6	Interlude: Some Invariant Theory	73
4.7	Some Applications of Cross-Ratio	76
	Exercises	80
5	The Group of the Conic	83
5.1	The Projective Line and the Conic	83
5.2	Frégier Involutions	86
5.3	Pencils of Conics	90
5.4	Cross-Ratio on a Conic	93
	Exercises	98
6	Involution	100
6.1	Basics on Involutions	101
6.2	Imaginary Points	102
6.3	The Involution Theorems of Pappus and Desargues	107
6.4	Pascal's Theorem	119
6.5	The Chasles–Steiner Theorem	137
6.6	Quadrangular Sets	156
	Exercises	159
7	Real Affine Plane Geometry from a Projective Perspective	163
7.1	Affinities	165
7.2	Parallel Projection	168
7.3	The Theorems of Ceva and Menelaus	170
7.4	Affine Conics: Ellipse, Hyperbola, Parabola	174
	Exercises	182
8	Euclidean Plane Geometry from a Projective Perspective	184
8.1	Perpendicularity of Lines and Triangle Theorems	184
8.2	Circular Points and Euclidean Conics	197
8.3	Axes, Diameters, Foci	220
	Exercises	236
9	Transformation Geometry: Klein's Point of View	241
9.1	A Tower of Groups	242
9.2	Cayley–Klein Geometries	243
9.3	Descending the Klein Tower	245
10	The Power of Projective Thinking	246
10.1	Euclidean and Affine Connections	246
10.2	Advanced Theory Using Involutions	256
	Exercises	286

11	From Perspective to Projective	288
11.1	Prologue	288
11.2	Naturalism in Art	289
11.3	Brunelleschi	293
11.4	Optics	295
11.5	Mirrors	298
11.6	Camera Obscura	299
11.7	Leon Battista Alberti	301
11.8	The Mathematics of Perspective	302
11.9	Guidobaldo Del Monte	304
11.10	The Infinitude of Space	305
11.11	Does the Infinity of Space Follow from the Use of Perspective Methods?	306
11.12	Points at Infinity	308
11.13	Line at Infinity	311
11.14	Geometry as the Study of Space	313
11.15	Epilogue	314
12	Remarks on the History of Projective Geometry	317
	Part II Real Projective 3-Space	
13	Fundamental Aspects of Real Projective Space	323
13.1	The Complex Projective Plane	332
13.2	The Absolute Conic	334
	Exercises	337
14	Triangles and Tetrahedra	338
14.1	Triangles	338
14.2	Tetrahedra	339
	Exercises	347
15	Reguli and Quadrics	348
15.1	Reguli	348
15.2	Quadrics and Polarities	352
15.3	Eight Associated Points	364
	Exercises	366
16	Line Geometry	368
16.1	Linear Independence of Lines and Plücker Coordinates	368
16.2	Linear Congruences	375
16.3	Linear Complexes	376

16.4	Twisted Cubics	378
	Exercises	386
17	Projections	388
17.1	Central Projections	388
17.2	Singular Projections and Singular Polarities	391
17.3	Computer Vision	393
	Exercises	396
18	A Glance at Inversive Geometry	397
18.1	The Real Inversive Plane	397
18.2	Lester's Theorem	402
	Exercises	403
	Part III Higher Dimensions	
19	Generalising to Higher Dimensions	407
19.1	The Basic Properties of Higher-Dimensional Space	407
19.2	Quadrics and Polarities	410
19.3	Associated Secunda and Lines	412
	Exercises	420
20	The Klein Quadric and the Veronese Surface	421
20.1	The Klein Correspondence	421
20.2	Rational Normal Curves	428
20.3	The Veronese Surface	429
	Exercises	433
	<i>Appendix: Group Actions</i>	434
	<i>References</i>	436
	<i>Index</i>	457