THE LARGE SCALE STRUCTURE OF SPACE-TIME

First published in 1973, this influential work discusses Einstein's General Theory of Relativity to show how two of its predictions arise: first, that the ultimate fate of many massive stars is to undergo gravitational collapse to form 'black holes'; and second, that there was a singularity in the past at the beginning of the universe. Starting with a precise formulation of the theory, including the necessary differential geometry, the authors discuss the significance of spacetime curvature and examine the properties of a number of exact solutions of Einstein's field equations. They develop the theory of the causal structure of a general spacetime, and use it to prove a number of theorems establishing the inevitability of singularities under certain conditions. A foreword contributed by Abhay Ashtekar and a new preface from George Ellis help put the volume into context of the developments in the field over the past 50 years.

STEPHEN W. HAWKING (1942–2018) was an English theoretical physicist, cosmologist, and author who was director of research at the Centre for Theoretical Cosmology at the University of Cambridge. He was the Lucasian Professor of Mathematics at Cambridge from 1979 to 2009 and is the author of numerous books, including the international best-seller *A Brief History of Time*.

GEORGE F. R. ELLIS is the emeritus distinguished professor of complex systems in the Department of Mathematics and Applied Mathematics at the University of Cape Town, South Africa. He is considered one of the world's leading theorists in cosmology and, in recent years, he has been prolific in areas relating to the philosophy of science. He is author or co-author of more than a dozen books, including *Relativistic Cosmology* (with Roy Maartens and Malcolm MacCallum).

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THE LARGE SCALE STRUCTURE OF SPACE-TIME

50TH ANNIVERSARY EDITION

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To
D. W. SCIAMA

Contents

Foreword to the 50th Anniversary Edition

In 1921, Cambridge University Press published Arthur Eddington's monograph, *The Mathematical Theory of Relativity,* arguably the first systematic and comprehensive textbook on the theory. It embodies Eddington's view that 'The investigation of the external world is a quest for structure rather than substance'. It had a deep influence on how researchers thought of general relativity in subsequent decades.

Five decades later, the Press published another monograph, *The Large Scale Structure of Space-Time* by Stephen Hawking and George Ellis in 1973. Hailed immediately as 'a masterpiece, written by sure hands' it too focuses on 'structure' – but now on *global aspects* of spacetime structure, which had been almost entirely ignored in earlier books. The monograph solidified the new approach to understand gravitational phenomena, introduced by Roger Penrose through his use of global methods and causal structures, which transformed the way the community thought of strong gravity. It has had even greater impact on the development of relativistic gravity than Eddington's monograph because it helped shape the 'golden age' of general relativity during the 1970s.

Before the appearance of this monograph, contributions to general relativity were by and large dominated by tensor calculus and partial differential equations in local coordinates. The monograph served as a powerful catalyst that changed our way of understanding the physics of general relativity. Thanks in large part to its influence, a sizable fraction of researchers started thinking invariantly, in geometrical terms, using spacetime diagrams and light cones. The emphasis shifted to global issues. In subsequent years, this shift led to numerous novel directions that created new frontiers of research: black hole uniqueness theorems; detailed investigations of the cosmic censorship hypothesis; introduction of quasi-local horizons that now play a key role in numerical relativity; and unforeseen connections between relativistic gravitation, quantum physics, and statistical mechanics, through black holes.

The transformative impact of the monograph is not confined to physics and astrophysics. Even in the mathematical community that provides us with rigorous proofs, the emphasis has shifted from local results based on partial differential equations to ‘geometric analysis’ that focuses on global existence and uniqueness results for solutions to Einstein’s equations, obtained using geometric structures that emphasize causality.

Hallmarks of this monograph are its conceptual clarity, mathematical rigor, and concise and precise statements that capture the essential underlying structures. The authors reverse the Machian view that the local laws are determined by large scale structure, and instead ‘take the local physical laws to be experimentally determined’ and explore ‘what these laws imply about the large scale structure of the universe’. This insightful switch guides their discussion throughout the monograph.

The organization of the monograph was also novel at the time. It used invariantly defined structures in differential geometry to present general relativity through a systematic set of postulates. Five decades have passed and yet this approach continues to be contemporary! Similarly, almost nothing new can be added to the presentation of the physical effects of curvature on test particles, the detailed mathematical discussion of energy conditions and the masterful treatment of the global structure of space-times – such as de Sitter, anti-de Sitter, Schwarzschild, and Kerr – that continue to feature prominently in the contemporary literature. The discussion of singularity theorems and strong field dynamics associated with gravitational collapse and binary black hole mergers are the crowning achievements of the monograph. A series of influential works from the then Soviet school led by Khalatnikov and Lifshitz suggested that the formation of singularities in gravitational collapse is an artifact of the high degree of symmetry assumed in the analysis, and generic solutions would be singularity-free. The comprehensive treatment of singularity theorems in the monograph was instrumental in causing a decisive shift in the community, away from this paradigm. Similarly, at the time, many astronomers and physicists did not believe that black holes were physical entities. Inclusion of a detailed discussion of black hole dynamics in a monograph shows incredible foresight and confidence. It has been handsomely rewarded through discoveries of binary black hole mergers by the LIGO–Virgo collaboration. Discussions of these events routinely include not only the technical statements from the monograph, but even some of the diagrams!

In his preface to this golden jubilee edition, George Ellis has included a list of topics that are not covered by the book. Almost all of them refer to discoveries that were made since publication. However, the omission of gravitational waves is somewhat puzzling, given that Bondi, Sachs, Penrose, Newman, and others had developed the subject in detail during the preceding decade, and the subject matter is intimately related to the large scale structure of spacetime. Its inclusion would have made the work even more prescient! Perhaps it was left out because the volume is already close to 400 pages. Indeed, even as it stands, the monograph is peerless in the way it served to guide the subsequent developments in the field.

When it first appeared, I was a graduate student. I distinctly remember the excitement we all felt as we slowly absorbed the grandeur of the new vistas that the monograph opened before us. When I moved to Oxford as a postdoctoral researcher, I eagerly went to Blackwell's to buy my own paperback copy, which had just appeared. At £3.95, it is the best book purchase I have ever made! I still refer to it.

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Frontmatter
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Preface to the 50th Anniversary Edition

This book, written by Stephen Hawking (see Carr *et al.* 2019) and myself between 1971 and 1973, presents a systematic overview of Einstein's General Theory of Relativity as a theory of gravity. We wrote it in the middle of what has come to be called the 'golden age' of general relativity: a time when a largely ignored theory, regarded by many as being at a dead end, transitioned to being truly dynamic, with the foundations being laid for developments in many directions in later years.

The book is dedicated to our research supervisor Dennis Sciama, FRS (Ellis and Penrose 2010), who was an outstanding physicist and supervisor. I arrived in Cambridge from South Africa in 1961, and started as his first research student in the University Department of Applied Mathematics and Theoretical Physics (DAMTP) in January 1962. Stephen arrived from Oxford in 1962, and the third student in the group who would focus on related issues in general relativity and cosmology was Brandon Carter, who came from Australia in 1962. The convivial way the research group was run is recalled in Ellis and Penrose (2010) and Ellis (2014).

The key issue we were involved in at the time was whether the universe had a beginning or not. Dennis was debating with Fred Hoyle, Hermann Bondi, and Tommy Gold whether their Steady State theory of the universe, which had no start, was a better model than the Standard Model, which did have a beginning. However, the Steady State model did not obey the field equations of general relativity: would models obeying those equations necessarily have a beginning? This would represent the earliest time the universe existed. Using the data of the time, this seemed to be the case: the universe would start at a *singularity*, an edge to spacetime where physical quantities such as the density would diverge. The universe – and physics – did not exist before that time.

However, the standard cosmological models had a highly simplified geometry: they were spherically symmetric about every point as well as being spatially homogeneous, hence there was no rotation or acceleration that could avoid a singularity. We wanted to know if more general geometries could allow a non-singular start. Our method was to look at specific anisotropic but spatially homogeneous models; but we could not prove it either way.

A related issue, driven primarily by John Wheeler at Princeton, was whether a spacetime singularity would occur at the centre of gravitational collapse when a star had used up all its nuclear energy. The same issue arose: simple models said this would happen when they were over a certain mass, and collapse to a singularity was unavoidable if they were massive enough. But they were spherically symmetric models. Could rotation of a collapsing star avoid a singularity?

The whole topic was blown wide open in 1965 by a truly innovative paper by Roger Penrose, who was then at Birkbeck College, London, showing singularities would indeed occur at the endpoint of gravitational collapse (Penrose 1965); he would much later receive the Nobel Prize in Physics for this work (Nobel Prize 2020). The paper involved innovative examination of global properties and causal structures of spacetimes, energy inequalities rather than exact equations, the crucial concept of a closed trapped surface, and a characterization of existence of singularities via geodesic incompleteness.

The Cambridge group (mainly Hawking, Carter, and myself) went into overdrive to learn the details of these new methods, jointly with colleagues Felix Pirani and others from King's College, London, and DAMTP visitors John Wheeler and Charles Misner from Princeton and Maryland, respectively. Stephen and I wrote a paper showing that these methods would indeed work in the restricted case of spatially homogeneous models (Hawking and Ellis 1965), and he then rapidly produced a series of existence proofs for generic cases, based on the idea of a time-reversed closed trapped surface together with suitable causal conditions. An initial such theorem was given in both Hawking (1965) and the last chapter of his PhD thesis (Hawking 1966a); a further series of singularity theorems with different details were presented in Hawking (1966b, 1966c, 1966d, 1967).

The Adams Prize is awarded jointly by the Cambridge University Faculty of Mathematics and St John's College for an essay in a stipulated topic in mathematics. In 1966 the topic was 'Geometric Problems of Relativity, with special reference to the foundations of general relativity and cosmology'. The adjudicators were H. Bondi, W. V. D. Hodge, and A. G. Walker. The prize was awarded to Roger Penrose for his essay entitled 'An analysis of the structure of spacetime', presenting the methods he had used in his 1965 paper, while Stephen was awarded an auxiliary prize for his essay 'Singularities and the geometry of spacetime' (Hawking 1966e), reprinted with commentary in Ellis (2014). This essay summarized global properties of general relativity theory, and on this basis developed a series of cosmological singularity theorems he had proved. Neither Adams Prize essay was published as a book, although preprint versions of both were circulated in the relativity community.

Further important work developing causal relations and global properties was carried out *inter alia* by Penrose, Robert Geroch (who was at Birkbeck with Penrose), Carter, Hawking, Werner Israel, Misner, and others; see for example Hawking (1968, 1970, 1971). A major summary theorem was developed by Hawking and Penrose (1970).

On the observational side, crucial new data became available about the nature of the expanding universe via the discovery in 1965 of the Cosmic Microwave Background (CMB) radiation, giving evidence of the nature of the evolution of the early universe and the existence of a Hot Big Bang epoch. Its implications were rapidly explored by Sciama, his students John Stewart and Martin Rees, and many others. Stephen and I wrote a paper (Hawking and Ellis 1968) showing how the very existence of that radiation showed a time-reversed closed trapped universe must exist in the early universe, and so provide evidence of the existence of an initial singularity.

To follow these developments in detail required pulling together a variety of mathematical topics that were not well known to the relativity community at that time, so a summary book was discussed between Stephen and myself in 1966, encouraged by Dennis. A contract for such a book with Cambridge University Press was accepted by them on 24 April 1967 and signed on 18 May 1967 under the title *Singularities, Causality and Cosmology,* to be published in the *Cambridge Series on General Relativity*, edited by W. H. McCrea and D. W. Sciama. By the time of publication in 1973 this had become the *Cambridge Monographs on Mathematical Physics*, with J. C. Polkinghorne added as third editor of the series.

The real writing of the book only started in 1970, with the focus being in 1971–1972, because we were both doing other things. This was the pre-LaTeX era. Stephen was having trouble coordinating his muscles so I typed the text myself, inserting handwritten equations in the text sent to the Press, who typeset it and sent proofs back. Then several rounds of corrections to proofs followed. The diagrams were drawn by a draftsman in the geography department under my guidance. The writing was completed in January 1973. The title had changed to *The Large Scale Structure of Space-Time.*

PREFACE TO THE 50TH ANNIVERSARY EDITION

The book is a book of its time, and does not include major developments that have come later. In particular, it was written at a key time in the development of black hole theory; while it made a contribution to that development, later papers and books developed that theory much further. The same is true for cosmology and gravitational waves. Nobel prizes have been awarded in each of these areas since those times.

We were asked later on if we wished to do an updated version to take some of these developments into account, but declined. Given Stephen's physical condition, this would not have been practical. In any case, the book had a terse style because giving all details in depth would have made it much longer, and no more readable. We did not want to change this.

What this book does not cover:

- **Alternative theories of gravity**: Scalar–tensor theories, higher-order gravity theories
- **Experimental tests of general relativity theory**: Solar systems tests, tests via cosmology and astrophysics
- **Inflationary cosmology and structure formation**: Structure formation in cosmology, CMB anisotropies, dark energy/dark matter existence and nature
- **Black hole thermodynamics**: The four laws of black hole thermodynamics**,** Hawking radiation, astrophysical black holes: formation, accretion, and associated radiation
- **Gravitational radiation**: Carrying off energy and momentum, emission and detection of gravitational radiation
- **Quantum gravity**: Supergravity, string theory, loop quantum gravity, etc.; and the wave function of the universe

Major advances have been made in all these areas since the book was written.

What has changed?
A key point to notice is the following: the status of the energy conditions has completely altered due to the advent of inflationary cosmology theory through the pioneering work of Alan Guth (1981), followed by many others (Guth 2007). This is now widely accepted as a correct model of the universe (Mukhanov 2005, Peter and Uzan 2009), with a slow rolling scalar field dominating early universe dynamics so that the energy conditions required for the singularity theorems no longer hold (Ellis 2014).

This possible breakdown of the energy conditions is essentially recognized in our book on page 96, but it is suggested there that this will be on such a small scale as to not alter the conclusions as regards the singularity theorems. But now that dominance of scalar field dynamics in the early universe is generally accepted, that conclusion is called into question. Singularity-free universes are in principle possible (Ellis and Maartens 2003).

However, in the end, the issue of whether the universe has a start or not depends on a resolution of the issue of the nature of quantum gravity: and we simply do not know what that answer is. The key question that led to the book is unsolved. But we acknowledge this in the conclusion on page 364.

George Ellis
Cape Town
June 2022

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Preface

The subject of this book is the structure of space–time on length-scales from 10^{-13} cm, the radius of an elementary particle, up to 10^{28} cm, the radius of the universe. For reasons explained in chapters 1 and 3, we base our treatment on Einstein's General Theory of Relativity. This theory leads to two remarkable predictions about the universe: first, that the final fate of massive stars is to collapse behind an event horizon to form a 'black hole' which will contain a singularity; and secondly, that there is a singularity in our past which constitutes, in some sense, a beginning to the universe. Our discussion is principally aimed at developing these two results. They depend primarily on two areas of study: first, the theory of the behaviour of families of timelike and null curves in space–time, and secondly, the study of the nature of the various causal relations in any space–time. We consider these subjects in detail. In addition we develop the theory of the time-development of solutions of Einstein's equations from given initial data. The discussion is supplemented by an examination of global properties of a variety of exact solutions of Einstein's field equations, many of which show some rather unexpected behaviour.

This book is based in part on an Adams Prize Essay by one of us (S. W. H.). Many of the ideas presented here are due to R. Penrose and R. P. Geroch, and we thank them for their help. We would refer our readers to their review articles in the *Battelle Rencontres* (Penrose (1968)), Midwest Relativity Conference Report (Geroch (1970*c*)), Varenna Summer School Proceedings (Geroch (1971)), and Pittsburgh Conference Report (Penrose (1972*b*)). We have benefited from discussions and suggestions from many of our colleagues, particularly B. Carter and D. W. Sciama. Our thanks are due to them also.

Cambridge S. W. Hawking
January 1973 G. F. R. Ellis