

Contents

<i>Preface</i>	ix
<i>Notation</i>	xi
1. Preliminaries	1
1.1 Review of Linear Algebra	1
1.2 Review of Measure Theory	7
2. Normed Linear Spaces	17
2.1 Definitions and Examples	17
2.2 Bounded Linear Operators	28
2.3 Banach Spaces	37
2.4 Finite Dimensional Spaces	46
2.5 Quotient Spaces	52
2.6 Exercises	57
3. Hilbert Spaces	71
3.1 Orthogonality	71
3.2 The Riesz Representation Theorem	78
3.3 Orthonormal Bases	82
3.4 Isomorphisms of Hilbert Spaces	88
3.5 Fourier Series of L^2 Functions	92
3.6 Exercises	95
4. Dual Spaces	111
4.1 The Duals of L^p Spaces	111
4.2 The Hahn–Banach Extension Theorem	122
4.3 Duals of Subspaces and Quotient Spaces	131
4.4 Separability and Reflexivity	135
4.5 The Transpose of an Operator	143
4.6 Exercises	149

5. Operators on Banach Spaces	161
5.1 Baire Category Theorem	161
5.2 The Principle of Uniform Boundedness	163
5.3 The Open Mapping and Closed Graph Theorems	169
5.4 Fourier Series of L^1 Functions	179
5.5 Schauder Bases for Banach Spaces	183
5.6 Compact Operators	190
5.7 Exercises	199
6. Weak Topologies	211
6.1 Weak Convergence	211
6.2 The Hahn–Banach Separation Theorem	218
6.3 The Weak Topology	224
6.4 Weak Sequential Compactness	232
6.5 The Weak-* Topology	236
6.6 Weak-* Compactness	242
6.7 Locally Convex Spaces	251
6.8 The Krein–Milman Theorem	258
6.9 Uniformly Convex Spaces	264
6.10 Exercises	272
7. Spectral Theory	287
7.1 Banach Algebras	287
7.2 Invertible Elements	295
7.3 Spectrum of an Element of a Banach Algebra	302
7.4 Spectrum of an Operator	311
7.5 Spectrum of a Compact Operator	319
7.6 Exercises	326
8. C*-Algebras	337
8.1 Operators on Hilbert Spaces	337
8.2 C*-Algebras	346
8.3 Commutative C*-Algebras	357
8.4 Spectral Permanence Theorem	368
8.5 Continuous Functional Calculus	373
8.6 Ideals and Quotients of C*-Algebras	380
8.7 Exercises	386
9. Measure and Integration	397
9.1 Positive Linear Functionals on C*-Algebras	397
9.2 The Riesz–Markov–Kakutani Theorem	406
9.3 Complex Measures	416
9.4 The Dual of $C(X)$	424
9.5 Exercises	433

Contents

vii

10. Normal Operators on Hilbert Spaces	445
10.1 Compact Normal Operators	445
10.2 Multiplication Operators	449
10.3 The Spectral Theorem	452
10.4 Borel Functional Calculus	458
10.5 Von Neumann Algebras	470
10.6 Spectral Measures	480
10.7 Applications of the Spectral Theorem	489
10.8 The Gelfand–Naimark Representation	502
10.9 Exercises	511
Appendices	523
A.1 The Stone–Weierstrass Theorem	523
A.2 The Radon–Nikodym Theorem	527
<i>Bibliography</i>	533
<i>Index</i>	539