

## Index

---

- advection, 162
  - definition, 40
  - distinction with convection, 164
  - equation, 40, 163
  - in a boundary layer, 165
- advective transport term, 39, 82
- body force, 60
  - expression, 61
- boundary condition, 114
  - Dirichlet, 114
    - energy equation, 173
  - Neumann, 115
    - energy equation, 173
  - no slip, 115
  - Robin, 115
    - energy equation, 174
- boundary layer, 165, 169
  - and Reynolds number, 171, 213
  - concept and history, 210
  - equations, 215
  - estimate of thickness, 172, 213
  - thermal, 169
  - thickness for flow past a flat plate, 215
- Burgers' equation, 165
  - inviscid, 40
    - discussion, 46
- Cauchy's first law of motion, 69
- coefficient of thermal expansion, 132
- conduction, 148
  - scaling, 184
- conservation
  - general idea, 3
  - of energy, 125
    - basic equation, 126
  - of mass, 3
    - basic equation, 4
  - of momentum, 102
    - basic equation, 103
  - time rate basis, 3
- continuity equation
  - conservation form, 16, 110
    - in Cartesian coordinates, 17
    - terms labeled, 17, 18
  - general discussion, 16
  - in one dimension, 7
  - incompressible flow, 27
    - in Cartesian coordinates, 28
  - integral form, 14, 111
    - terms labeled, 18
  - Lagrangian form, 51, 108
  - non-conservation form, 51, 109
  - the unknowns, 22
- continuum mechanics, 2
- control volume, 4
- convection, 164
- convection–diffusion equation, 162
- Couette flow, 116
  - scaled equations, 192
  - temperature profile, 174
  - velocity profile, 119
- creeping flow, *see* Stokes flow
- different forms of the equations, 57
- differential equation, 7
  - partial differential equation (PDE), 7
- diffusion, 162
  - general discussion, 155
  - thermal, 148
- diffusive transport, 91

- diffusive transport term, 82
    - thermal (conduction), 148
  - dimensional analysis, 182
  - divergence
    - for Cartesian coordinates, 16
    - general idea, 16
    - in matrix form, 17
    - of a vector, 15
  - divergence theorem, 15
    - general discussion, 18
    - mathematical definition, 22
  - dot product, 11
    - calculation in Cartesian coordinates, 11
  - double dot product, 141
  - dynamic viscosity, 75
  
  - energy
    - definition of, 128
    - specific, 126
  - energy equation
    - conservation form, 138
    - general discussion of, 147
    - in terms of specific heat at constant pressure, 146
    - in terms of specific heat at constant volume, 146
  - incompressible flow
    - in Cartesian coordinates, 152
    - terms labeled, 149
    - with constant thermal conductivity, 149
  - integral form, 138
  - Lagrangian form, 140
  - non-conservation form, 139
  - starting point, 127
  - terms labeled, 147
  - thermal energy equation in Lagrangian form, 143
- energy flow rate, 127
  - equation of state, 25
    - for an ideal gas, 25, 153
  - Euler's equation, 85, 208
  - Eulerian description
    - discussion, 33
    - general idea, 36
  
  - field variables, 22
  - first law of thermodynamics, 124
  - fluid element, 33
  - flux, 13
    - general definition, 13
    - of energy, 127
  
  - Fourier's law, 135
  - Fourier, Jean-Baptiste Joseph, 133
  - Froude number, 202
  - fully-developed flow, 117
  
  - gas constant, 26
  - general solution, 114
    - Couette flow, 118
    - temperature, 176
    - pressure-driven flow
      - temperature, 178
  - governing equations, 152
    - incompressible flows, 153
    - overview, 1
    - table description, 2
  
  - heat, 133
    - sign convention, 125
  - heat equation, 159
  - heat rate
    - expression using a volume integral, 135
    - expression using an area integral, 135
  - Heinrich Blasius, 210
  - hydrostatic pressure, 89
  
  - incompressible flow, 94
    - constant density definition, 27
    - general discussion, 27
    - related to volume change of fluid
      - element, 54
      - relation to Mach number, 208
  - inertia term, 82
  - initial condition, 41, 114
  - integrating out the spatial dependence, 14
  - internal energy
    - specific, 128
  - inviscid flow, 85, 208
  - isothermal compressibility
    - coefficient, 132
  - isothermal expansion coefficient, 148
  - isotropic, 72
  
  - Lagrangian description
    - discussion, 33
    - general idea, 36
  - Laplace's equation, 157
    - averaging property, 159
  - Laplacian operator, 96
    - operating on the velocity vector, 97
  - local time derivative, 39
  - Ludwig Prandtl, 210

- mass flow rate
  - as an integral, 12
  - equation for one dimension, 6
  - obtaining, 5
  - through area, 11
- mass flux, 13
  - in the  $x$ -direction, 6
- mass of arbitrary system, 8
- material derivative
  - advective transport term, 37
  - as an operator, 37
    - in Cartesian coordinates, 38
  - Eulerian description, 37
  - in the continuity equation, 51
  - in the Navier–Stokes equations, 55
  - Lagrangian description, 37
  - local time derivative term, 37
  - of velocity vector, 38
  - of volume of fluid element, 53
  - reason for upper case ‘D’, 39
  - using the nabla symbol, 37
- mechanical energy equation, 142
- mechanical pressure, 79
  
- nabla, 15
  - in Cartesian coordinates, 16
- Navier–Stokes equations
  - conservation form, 107, 110
  - incompressible flow
    - conservation form, 107
    - in Cartesian coordinates, 113
    - integral form, 107, 111
    - Lagrangian form, 98
    - non-conservation form, 98, 109
  - integral form, 106, 111
  - Lagrangian form, 80, 108
  - non-conservation form, 80, 109
  - terms labeled, 80
- Newtonian fluid, 75
- nondimensionalization, *see* scaling
- normal stress, 67
  - compressive stress, 67
  - tensile stress, 67
  
- outward normal, *see* unit normal
  
- Péclet number, 167
- partial derivative, 7
- partial differential equation (PDE), 7, 41
- Poiseuille flow, 120
- power
  - definition, 126
  - expression as a volume integral, 137
  - expression as an area integral, 136
  - sign convention, 126
- pressure
  - discussion of, 72
- pressure driven
  - velocity profile, 122
- pressure gradient, 83
- pressure-driven flow, 120
  - temperature profile, 177
- property, 126
  - extensive, 127
  - intensive, 127
  
- rate of energy due to volume change, 148
- Reynolds number, 170, 183, 202
  
- scaled
  - continuity equation, 200
  - energy equation
    - with temperature difference scaling, 204
  - energy equation for an incompressible flow
    - with insulated walls, 205
  - Navier–Stokes equations, 202, 208
- scaling, 182
  - characteristic scales, 185
  - incompressible Navier–Stokes equations, 199
  - pressure-driven flow, 194
  - the continuity equation, 199
  - the Navier–Stokes equations, 200
  - the steps, 183
- second coefficient of viscosity, 73
  - relationship to dynamic viscosity, 78
- second law of thermodynamics, 134, 155
- shear stress, 67, 74
- source
  - in the energy equation, 137
- specific enthalpy, 131
- specific heat
  - constant pressure, 130
    - expression, 132
  - constant volume, 129
    - expression, 130
- specific internal energy
  - material derivative of in terms of density and temperature, 145
  - material derivative of in terms of pressure and temperature, 146
- state principle, 144
- steady state, 40
  - general discussion, 23

- Stokes flow, 201
- stress tensor
  - divergence of, 70
    - Newtonian fluid in an incompressible flow, 98
  - dotted with normal, 65
  - for a Newtonian fluid, 79
    - for incompressible flow, 94
    - in Cartesian coordinates, 79
    - various terms, 71
  - general description, 62
  - in Cartesian coordinates, 63
  - relationship to traction vector, 63
  - symmetry of, 67
- stress vector, 61
- summation of forces
  - area integral version, 69
  - volume integral version, 69
- surface force, 60
  - in terms of traction vector, 61
- surroundings, 125
- system, 3
- Taylor series, 34, 35
- tensor product, 104
- thermal conductivity
  - reason for being positive, 134
- thermal diffusivity, 160
- thermal energy equation, 142
- traction vector, *see* stress vector
- turbulence, 216, 217
- unit normal, 10
- velocity gradient, 75
  - in Cartesian coordinates, 76
- velocity vector
  - Cartesian components, 9
  - normal component, 11
- viscous dissipation, 141, 148
  - in Cartesian coordinates, 151
- viscous force term, 82, 91, 99
- volume change force, 89
- volume change term, 73
- volumetric heat generation, 138
- vorticity, 217
- work
  - $pdV$  work, 131
  - sign convention, 125