A Student's Guide to the Navier–Stokes Equations

The Navier–Stokes equations describe the motion of fluids and are an invaluable addition to the toolbox of every physicist, applied mathematician, and engineer. The equations arise from applying Newton's laws of motion to a moving fluid and are considered, when used in combination with mass and energy conservation rules, to be the fundamental governing equations of fluid motion. They are relevant across many disciplines, from astrophysics and oceanic sciences to aerospace engineering and materials science. This *Student's Guide* provides a clear and focused presentation of the derivation, significance, and applications of the Navier–Stokes equations, along with the associated continuity and energy equations. Designed as a useful supplementary resource for undergraduate and graduate students, each chapter concludes with a selection of exercises intended to reinforce and extend important concepts. Video podcasts demonstrating the solutions in full are provided online, along with written solutions and other additional resources.

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Preface

The topic of this book deals with some of the most useful, yet complicated, equations that govern a considerable amount of our natural world. These equations are the equations of fluid motion (both liquids and gases), with the famed Navier–Stokes equations being front and center. The equations show up in a variety of areas of physics and engineering. These areas include astrophysics, condensed matter physics, materials science, geophysics, meteorology, oceanic sciences, biological sciences, as well as mechanical, aerospace, chemical, and civil engineering. In addition, the Navier–Stokes equations are also used to simulate fluids in visual effects models for movies as well as in video games.

The main goal of this book is to provide you, the student, with a friendly, but still comprehensive, path to understanding the Navier–Stokes equations as well as the mass continuity equation and energy equation (which are often seen alongside the Navier–Stokes equations). These equations as a whole make up what is commonly called the governing equations of fluid motion. This book differs from standard texts on fluid mechanics in that it focuses almost entirely on the governing equations with minimal distractions from other, albeit important, aspects of fluid mechanics. Using a somewhat conversational writing style, the book spends a lot of time discussing the meaning of the various terms of the equations. In addition, care has been taken to ensure that steps are not skipped when deriving the equations or when discussing the concepts associated with its various terms. The last chapter of the book introduces the concept of nondimensionalization (i.e., scaling) and how scaling the governing equations can reduce the number of parameters involved in a problem as well as provide insight into the physics of a problem. The examples in the book will focus mainly on what are called incompressible flows, however, there will still be a fair amount of information regarding the version of the equations that pertain to compressible flows. Along with the book, you can take advantage of video podcasts showing the solutions to the end of the chapter problems as well as some additional written material that did not make it into the final version of the book, all of which are available on the book's website.

The book is generally geared toward fourth-year students in physics and engineering. As such, in order to get the most out of the book, having some background in differential equations as well as the calculus of vectors would be helpful, if not necessary. The book would also likely be of great use if you are a beginning graduate student in science or engineering whose chosen field of study is heavily reliant on fluid mechanics. In particular, the last chapter on nondimensionalization might come in handy for those engaged in fluid mechanics research activities. If you are a mathematics student studying the mathematical side of the Navier–Stokes equations, you may find the description of the equations on a more physical basis to be insightful. In addition, if you are a lifelong learner who wants to know more about the equations that describe the motion of everyday fluids, then I hope you will find this book to be a useful addition to your self-study.

Whatever your motivation is for learning the Navier–Stokes equations, I wish you the best of luck in your quest in understanding some of the most amazing equations of nature.

Acknowledgments

When I first embarked on the journey to write a book for Cambridge's Student's Guides series, I was motivated by a little book I read on the Maxwell's equations by Professor Dan Fleisch. The book was called *A Student's Guide to Maxwell's Equations.* Even though the material was nothing new to me, I was blown away by how seemingly effortless the book flowed. It motivated me to try to write a similar book for the Navier–Stokes equations (key word here is try, as the Maxwell's equations book is a very high bar). I contacted Professor Fleisch, who turned out to be the series editor, to see if he could direct me on the steps needed to go about such a project. He put me in contact with Dr. Nicholas Gibbons, senior commissioning editor in physics, and the rest is history. I owe a great deal of thanks to these two gentlemen for their patience and their thoughtful guidance.

Additional thanks goes to Sarah Armstrong and the staff at Cambridge University Press. They are fantastic. A thank you also goes to Dr. John Mousel for reading over parts of the manuscript and providing feedback. I would also like to thank my students, in particular Colin Fisher, who have provided me with encouragement in pursuing this goal.

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