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### Final Remark

Indexing is the author's last task. Apart from reflecting on the probable appearance in the near future of a 'Journal of Fuzzy Indexing Theory' all that I can now do is to thank readers in advance for all corrections and suggestions for improvements they may care to send me.