FOURIER ANALYSIS

Fourier analysis is a subject that was born in physics but grew up in mathematics. Now it is part of the standard repertoire for mathematicians, physicists and engineers. This diversity of interest is often overlooked, but in this much-loved book, Tom Körner provides a shop window for some of the ideas, techniques and elegant results of Fourier analysis, and for their applications. These range from number theory, numerical analysis, control theory and statistics, to earth science, astronomy and electrical engineering. The prerequisites are few (a reader with knowledge of second- or third-year undergraduate mathematics should have no difficulty following the text), and the style is lively and entertaining.

This edition of Körner's 1989 text includes a foreword written by Professor Terence Tao introducing it to a new generation of fans.

T. W. Körner is Emeritus Professor of Fourier Analysis at the University of Cambridge. His other books include *The Pleasures of Counting* (Cambridge, 1996) and *Where Do Numbers Come From?* (Cambridge, 2019).

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With a Foreword by

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. . . mathematical ideas originate in empirics, although the genealogy is sometimes long and obscure. But, once they are so conceived, the subject begins to live a peculiar life of its own and is better compared to a creative one, governed by almost entirely aesthetical motivations, than to anything else and, in particular, to an empirical science. There is, however, a further point which, I believe, needs stressing. As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired by ideas coming from 'reality', it is beset with very grave dangers. It becomes more and more purely aestheticising, more and more purely 1'art pour 1'art. This need not be bad, if the field is surrounded by correlated subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste. But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganised mass of details and complexities. In other words, at a great distance from its empirical source, or after much 'abstract' inbreeding, a mathematical subject is in danger of degeneration.

von Neumann (from the first paper in his collected works)

Some calculus tricks are quite easy. Some are enormously difficult. The fools who write the text books of advanced mathematics – and they are mostly clever fools – seldom take the trouble to show you how easy the easy calculations are. On the contrary, they seem to desire to impress you with their tremendous cleverness by going about it in the most difficult way.

Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard. Master these thoroughly, and the rest will follow. What one fool can do, another can.

(from *Calculus Made Easy* by Sylvanus P. Thompson)

'Now,' Herbie says, 'wait a minute. A story goes with it.'

(from *A Story Goes With It* by Damon Runyon)

Contents

Cambridge University Press & Assessment
978-1-009-23005-6 — Fourier Analysis
T. W. Körner , Foreword by Terence Tao
Frontmatter
More Information

Foreword

TERENCE TAO

Mathematical monographs tend to fall, broadly speaking, into two categories. On one hand, one has the undergraduate texts, in which the basics of some subject are covered extensively and systematically, with many examples and exercises for the student to practice core techniques and concepts. On the other hand, one has the graduate texts, where the author assumes that the student is mathematically mature enough to recognise and fill in routine arguments and standard calculations, and is now ready to proceed to the cutting edge of the field.

This book is a rare example of one that stakes a middle ground: it does not comprehensively introduce the fundamental definitions and results in Fourier analysis, but instead provides a sampler of bite-sized (and largely stand-alone) examples of how the themes of the subject (in particular, that of decomposing a general function into simple oscillating modes, and then reconstituting these modes back together to reconstruct the original function) arise naturally in both mathematical and practical contexts.

One feature of this book that is particularly valuable for beginning students is that standard arguments in analysis that would be very quickly breezed through in more advanced texts (using phrases such as ‘by a routine scaling argument’, ‘by linearity we may assume without loss of generality that’, and so forth) are instead carefully worked through by the author. I would encourage such students to study and internalise these arguments when they are presented in this text, as they will certainly encounter them many more times in their studies and their research.

The text does not need to be read in a linear fashion; I myself spent many pleasant hours as a graduate student browsing through whatever topics in the book took my fancy, whether it was reading about the fascinating history of the transatlantic cable, the application of Fourier-analytic ideas to locate primes in arithmetic progressions, or using Fourier-analytic solutions to the heat equation to estimate the age of the Earth. I hope you find this book as enjoyable to dip into as I have.

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Preface

This book is meant neither as a drill book for the successful nor as a lifebelt for the unsuccessful student. Rather, it is intended as a shop window for some of the ideas, techniques and elegant results of Fourier analysis.

I have tried to write a series of interlinked essays accessible to a student with a good general background in mathematics such as an undergraduate at a British university is supposed to have after two years of study. If the reader has not covered the relevant topic, say contour integration or probability, then she can usually omit, or better, skim through any chapters which involve this topic without impairing her ability to cope with subsequent chapters.

It is a consequence of the plan of this book that nothing is done in great depth or generality. If the reader wants to acquire facility with the Laplace transform or to study the L^2 convergence of the Fourier series of an L^2 function she must look elsewhere. It is very much easier to acquire a skill or to generalise a theorem when one is under the pressure of immediate necessity than when one is told that such a skill or generalisation might just possibly come in useful some day.

Another consequence is that, although anything specifically presented as a proof or statement of a result is intended to meet the pure mathematician's criteria for accuracy, the rigour of the accompanying discussion will vary according to the subject discussed. (Compare Chapter 3, Chapter 8 and Chapter 14.) For this I make no apology. 'It is the mark of the educated mind to use for each subject the degree of exactness which it admits' (Aristotle).

I must however apologise for a major, though perhaps unavoidable, fault in this book. The historical remarks which I make in connection with certain problems are brief and, if only for that reason, paint only a small part of a very complicated picture. Moreover, a glance at the average history of mathematics shows that mathematicians are remarkably incompetent historians. I make no claim to superiority and can only advise that the reader consults the original sources before accepting the truth of any historical sketch drawn in this book.

Any textbook owes more to the books and lectures of others than to the nominal author. At one stage I had a list of over 25 names of unwitting contributors to this one. However, a long list prompts more interest in its omissions than its inclusions so I shall simply record my immense debt to the inspiring lectures of G. Friedlander, J.P. Kahane, H.P. Swinnerton-Dyer and H. Shapiro.

This book would never have seen the light of day without the labours of several generations of Cambridge typists. I should like to thank in particular Robyn Bringan, Debbie McCleland and Betty Sharples. It would have contained many more great and small mathematical errors without the careful scrutiny of Jonathan Partington, Richard Hildich, Chris Budd and an anonymous referee.

I close the preface by dedicating this book to my parents with love and respect.

T.W. Körner