Contents

List of Figures xi
List of Tables xvii

1. Matrices and Vector Spaces 1
   1.1 Introduction 1
   1.2 Creation of a Matrix 2
   1.3 Nature of the Matrix 4
   1.4 Matrix Operation 5
   1.5 Vector Algebra 9
   1.6 Applications 10
      1.6.1 Coordinate conversion (Cartesian to cylindrical coordinate system) 10
      1.6.2 Coordinate conversion (Cartesian to spherical coordinate system) 12
      1.6.3 Orthogonal vectors 13
      1.6.4 Centre of mass of a system 14
      1.6.5 Electrical circuits (Mesh analysis) 15
      1.6.6 Electrical circuits (Nodal analysis) 16
      1.6.7 Force on a test charge 18
      1.6.8 Principal axes of moment of inertia 19
      1.6.9 Matrix representation of differential operator 21
      1.6.10 Position–Momentum commutation 23
      1.6.11 Matrix representation of the Laplace operator 25
      1.6.12 Wave function for stationary states 28
   1.7 Exercises 30

2. Plotting and Graphics Design 37
   2.1 Introduction 38
   2.2 Formatting of the Coordinate Axes 39
## Contents

2.2.1 Font size 41
2.2.2 Font colour 42
2.2.3 Typeface 44
2.2.4 Axis position 46
2.2.5 Tick marks 47
2.2.6 Logarithmic axes 49
2.2.7 Polar plot 50

2.3 Formatting of the Line Styles 53
2.3.1 Thickness 53
2.3.2 Line style 54
2.3.3 Line colour 56

2.4 Formatting of the Markers 57
2.4.1 Marker style 57
2.4.2 Marker: Size and colour 58
2.4.3 Thickness and line mode 59

2.5 Formatting of the Title 60
2.6 Formatting of the Legend 61

2.7 Applications 64
2.7.1 Trajectory of a projectile 64
2.7.2 Superposition of collinear harmonic oscillations 65
2.7.3 Beats 67
2.7.4 R-C Circuit 70
2.7.5 R-L Circuit 71
2.7.6 Maximum power transfer theorem 73
2.7.7 Diode characteristics 75
2.7.8 Specific heat of solids 77
2.7.9 Spectral radiance of a blackbody radiation 79
2.7.10 Miller indices 80
2.7.11 Linear interpolation 83
2.7.12 Gradient of a scalar field 86

2.8 Exercises 88

3. Least Square Curve Fitting 95
3.1 Introduction 95
3.2 Fitting of Linear Data 96
3.3 Fitting of Non-Linear Data 99
3.4 Polynomial Fitting 105
3.5 Fitting with Built-in Scilab Function—‘datafit’ 111
3.6 Applications 112
3.6.1 Refractive index of water 113
3.6.2 Spring constant 115
3.6.3 Cauchy’s constant of a prism 118
3.6.4 RC Time constant 120
## Contents

3.6.5 Coefficient of electronic heat capacity and Debye’s temperature 123
3.6.6 Lennard–Jones potential 125
3.6.7 Spectral radiance of blackbody radiation 127

3.7 Exercises 130

4. Ordinary Differential Equation 135

4.1 Introduction 136
4.2 Euler’s Method 136
4.2.1 First order differential equation 136
4.2.2 Second order differential equation 140
4.3 Modified Euler’s Method 142
4.4 Second Order Runge–Kutta Method 144
4.5 Fourth Order Runge–Kutta Method 146
4.5.1 First order differential equation 146
4.5.2 Second order differential equation 149
4.6 Comparison of the Four Methods 152
4.7 Finite Difference Method 155
4.8 Applications 160
4.8.1 Radioactive decay 160
4.8.2 Orthogonal trajectory 166
4.8.3 Square wave ↔ Triangular wave 169
4.8.4 Sinusoidal wave 171
4.8.5 Freely falling object 176
4.8.6 Atwood’s machine 179
4.8.7 Simple pendulum 184
4.8.8 Mass–spring system 187
4.8.9 Series L–C–R circuit 191
4.8.10 Schrödinger equation 196
4.8.11 Lagrangian dynamics 213
4.9 Exercises 222

5. Integration and Differentiation 229

5.1 Introduction 229
5.2 Built-in Scilab Functions for Integration 230
5.2.1 intg 230
5.2.2 integrate 231
5.3 Trapezoidal Rule 231
5.4 Simpson’s 1/3 – Rule 233
5.5 Simpson’s 3/8 – Rule 234
5.6 Differentiation 235
5.7 Applications 237
5.7.1 Integration in cylindrical coordinates 237
5.7.2 Total charge 242
5.7.3  Electric flux density 242
5.7.4  Planck’s law for blackbody radiation 244
5.7.5  Specific heat of solids 245
5.7.6  Dirac delta function (Shifting property) 247
5.7.7  Cornu’s spiral and Fresnel’s diffraction pattern 248
5.7.8  Arc length 252
5.7.9  Motion of an object 253
5.8  Exercises 255

6. Special Functions 259

6.1  Introduction 259
6.2  Bessel Function of the First Kind 260
6.3  Legendre Polynomial 265
6.4  Laguerre Polynomial 270
6.5  Hermite Polynomial 273
6.6  Improper Integrals – Quadrature Methods 277
6.6.1  Gauss–Legendre quadrature 277
6.6.2  Gauss–Laguerre quadrature 279
6.6.3  Gauss–Hermite quadrature 280
6.7  Applications 281
6.7.1  Simple pendulum 281
6.8  Exercises 282

7. Fourier Analysis 285

7.1  Introduction 285
7.2  Periodic Functions 286
7.3  Fourier Series 290
7.4  Harmonics 292
7.5  Fourier Series Expansion of Periodic Functions 294
7.5.1  Fourier series expansion of \( x^2 \) 294
7.5.2  Fourier series expansion of saw-tooth wave 297
7.5.3  Fourier series expansion of a square wave 299
7.5.4  Fourier series expansion of a triangular wave 301
7.5.5  Fourier series expansion of output of half wave rectifier 303
7.6  Fast Fourier Transform 305
7.6.1  FFT of a sine wave 305
7.6.2  FFT of the sum of two cosine wave signals 305
7.6.3  FFT of a noisy signal 307
7.6.4  FFT of a square wave 308
7.6.5  FFT of a Gaussian curve 309
7.7  Summary 311
7.8  Exercises 311
8. Algebraic and Transcendental Equations

8.1 Introduction

8.2 Equation Solver in Scilab
   8.2.1 Division operator
   8.2.2 Built-in Scilab function – ‘linsolve’
   8.2.3 Built-in Scilab function – ‘fsolve’

8.3 Gauss–Seidel Method

8.4 Gaussian Elimination Method

8.5 ‘pivoting’ Gaussian Elimination Method

8.6 Bracketing Method: Bisection Method

8.7 Bracketing Method: Regula Falsi Method

8.8 Open Method: Secant Method

8.9 Open Method: Newton–Raphson Method

8.10 Applications
   8.10.1 Trajectory of a particle
   8.10.2 Matrix inverse
   8.10.3 Determinant of a matrix
   8.10.4 Fraunhofer diffraction pattern
   8.10.5 Bound state of proton and neutron
   8.10.6 Central angle of an elliptical orbit
   8.10.7 Bearing angle of a boat

8.11 Exercises

Appendix

1 Matrices and Vector Spaces (vectors.sci)
   1.1 Distance between two points
   1.2 Coordinate conversion

2 Plotting and Graphics Design (plot.sci)
   2.1 Formatting of coordinate axes
   2.2 Formatting of the line styles
   2.3 Formatting of the markers
   2.4 Formatting of the legend

3 Least Square Curve Fitting
   3.1 Exponential Fitting
   3.2 Polynomial Fitting

4 Ordinary Differential Equation (differentiation.sci)
   4.1 Euler’s Method (for first order differential equation)
   4.2 Euler’s Method (for second order differential equation)
   4.3 Modified Euler’s Method
   4.4 Second order Runge–Kutta Method (for first order differential equation)
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>Fourth Order Runge–Kutta Method (for first order differential equation)</td>
<td>354</td>
</tr>
<tr>
<td>4.6</td>
<td>Fourth Order Runge–Kutta Method (for second order differential equation)</td>
<td>354</td>
</tr>
<tr>
<td>4.7</td>
<td>Finite Difference Method</td>
<td>355</td>
</tr>
<tr>
<td>5</td>
<td>Integration and Differentiation (integrate.sci)</td>
<td>356</td>
</tr>
<tr>
<td>5.1</td>
<td>Trapezoidal Rule</td>
<td>356</td>
</tr>
<tr>
<td>5.2</td>
<td>Simpson’s 1/3 – Rule</td>
<td>356</td>
</tr>
<tr>
<td>5.3</td>
<td>Simpson’s 3/8 – Rule</td>
<td>356</td>
</tr>
<tr>
<td>5.4</td>
<td>Line Integral in Cylindrical Coordinates</td>
<td>356</td>
</tr>
<tr>
<td>5.5</td>
<td>Surface Integral in Cylindrical Coordinates</td>
<td>357</td>
</tr>
<tr>
<td>5.6</td>
<td>Volume Integral in Cylindrical Coordinates</td>
<td>357</td>
</tr>
<tr>
<td>6</td>
<td>Special Functions (special_func.sci)</td>
<td>358</td>
</tr>
<tr>
<td>6.1</td>
<td>Legendre Polynomials</td>
<td>358</td>
</tr>
<tr>
<td>6.2</td>
<td>Laguerre Polynomials</td>
<td>359</td>
</tr>
<tr>
<td>6.3</td>
<td>Hermite Polynomials</td>
<td>359</td>
</tr>
<tr>
<td>6.4</td>
<td>Gauss–Legendre Quadrature</td>
<td>360</td>
</tr>
<tr>
<td>6.5</td>
<td>Gauss–Laguerre Quadrature</td>
<td>361</td>
</tr>
<tr>
<td>6.6</td>
<td>Gauss–Hermite Quadrature</td>
<td>361</td>
</tr>
<tr>
<td>7</td>
<td>Fourier analysis (fourier.sci)</td>
<td>361</td>
</tr>
<tr>
<td>7.1</td>
<td>Periodic Functions</td>
<td>361</td>
</tr>
<tr>
<td>7.2</td>
<td>Fourier Series</td>
<td>362</td>
</tr>
<tr>
<td>8</td>
<td>Algebraic and Transcendental Equations (numerical_techniques.sci)</td>
<td>363</td>
</tr>
<tr>
<td>8.1</td>
<td>Gauss–Seidel Method</td>
<td>363</td>
</tr>
<tr>
<td>8.2</td>
<td>Gaussian Elimination Method</td>
<td>364</td>
</tr>
<tr>
<td>8.3</td>
<td>‘pivoting’ Gaussian Elimination Method</td>
<td>365</td>
</tr>
<tr>
<td>8.4</td>
<td>Inverse of a Matrix</td>
<td>366</td>
</tr>
<tr>
<td>8.5</td>
<td>Determinant of a Matrix</td>
<td>367</td>
</tr>
<tr>
<td>8.6</td>
<td>Bisection Method</td>
<td>367</td>
</tr>
<tr>
<td>8.7</td>
<td>Regula Falsi Method</td>
<td>368</td>
</tr>
<tr>
<td>8.8</td>
<td>Secant Method</td>
<td>368</td>
</tr>
<tr>
<td>8.9</td>
<td>Newton–Raphson Method</td>
<td>369</td>
</tr>
</tbody>
</table>

References 371
Index 373