

# Contents

<i>Preface</i>	<i>page</i>	<i>xiii</i>
<b>1 Survey of Experiments</b>		1
1.1 Metals and Band Insulators		1
1.2 Mott Insulators		4
1.3 Ultracold Atoms		8
1.4 The Heavy-Fermion Intermetallic Compounds		13
1.5 The Cuprates		14
<b>Part I Background</b>		
<b>2 Fermi Liquid Theory</b>		21
2.1 Free-Electron Gas		21
2.2 Interacting-Electron Gas		22
2.3 Specific Heat		25
2.4 Compressibility		26
2.5 Dynamic Response Functions		27
2.6 Green's Functions and Quasiparticle Lifetime Problem		28 32
<b>3 Dilute Bose Gas</b>		33
3.1 Bogoliubov Theory		33
3.2 Off-Diagonal Long-Range Order		37
3.3 Path Integral Theory Problem		38 40
<b>4 Bardeen—Cooper—Schrieffer Theory of Superconductivity</b>		43
4.1 The BCS Wavefunction		43
4.2 Off-Diagonal Long-Range Order		45
4.3 Bogoliubov Theory		47
4.4 The Energy Gap Problems		49 52
<b>5 Broken Symmetry and Superfluidity</b>		53
5.1 Ising Model and Surface Tension		54
5.2 <i>XY</i> Model and Helicity Modulus		55

5.3	Superconductors and Gauge Invariance	56
5.4	The London Equation	57
	Problems	58
<b>6</b>	<b>Landau–Ginzburg Theory</b>	<b>59</b>
6.1	Hubbard–Stratonovich Transformation	59
6.2	Expansion near $T_c$	61
6.3	Effective Classical Theory	63
6.4	Classical Dynamics	64
6.5	Magnetic Field	64
	Problems	65
<b>7</b>	<b>Vortices in Superfluids and Superconductors</b>	<b>66</b>
7.1	Neutral Superfluids	66
7.2	Charged Superfluids	69
7.3	Flux Quantization	71
7.4	Vortex Lattices	72
	Problems	73
<b>8</b>	<b>Boson Hubbard Model</b>	<b>74</b>
8.1	Lattice Hamiltonian	75
8.2	Mean-Field Theory	76
8.3	Continuum Quantum Field Theories	79
8.4	Insulators at Non-Integer Filling	83
	Problems	84
<b>9</b>	<b>Electron Hubbard Model</b>	<b>87</b>
9.1	The Superexchange Interaction	89
9.2	Insulating Antiferromagnets and Hard-Core Bosons	91
9.3	The $t$ - $J$ Model and $d$ -Wave Pairing	96
9.4	Paramagnon Theory of Antiferromagnetic Metals	100
	Problems	110
<b>10</b>	<b>Relativistic Scalar Field: Diagrams</b>	<b>114</b>
10.1	Gaussian Integrals	115
10.2	Expansion for Susceptibility	118
	Problems	121
<b>11</b>	<b>Relativistic Scalar Field: Correlation Functions</b>	<b>122</b>
11.1	Spectral Representation	122
11.2	Correlations across the Quantum Critical Point	127
	Problem	132
<b>12</b>	<b>Fermions and Bosons in One Spatial Dimension</b>	<b>133</b>
12.1	Non-interacting Fermions	134

12.2	Interacting Fermions	143
12.3	Bosons in One Dimension	146
	Problems	149

## Part II Fractionalization and emergent gauge fields I

<b>13</b>	<b>Introduction to Gapped Spin Liquids</b>	153
13.1	The RVB State	155
13.2	Topological Properties	155
13.3	Emergent Gauge Fields	156
13.4	Excitations of the $\mathbb{Z}_2$ Spin Liquid	159
<b>14</b>	<b>Fractionalization in the <math>XY</math> Model in 2+1 Dimensions</b>	163
14.1	The Conventional $XY$ Model	164
14.2	The Extended $XY$ Model	165
	Problems	175
<b>15</b>	<b>Theory of Gapped <math>\mathbb{Z}_2</math> Spin Liquids</b>	176
15.1	Parton Formulation	177
15.2	Mean-Field Theory	180
15.3	Excitation Spectrum	183
15.4	Dynamics of Excitations	188
	Problem	194
<b>16</b>	<b><math>\mathbb{Z}_2</math> Gauge Theory</b>	196
16.1	From the Large- $N$ Path Integral to a $\mathbb{Z}_2$ Gauge Theory	196
16.2	Hamiltonian of the $\mathbb{Z}_2$ Gauge Theory	199
16.3	Topological Order at Small $g$	203
16.4	Large- $g$ Limit	206
16.5	Visons and Anyon Condensation	209
16.6	Models of Rydberg Atoms	214
	Problems	219
<b>17</b>	<b>Chern–Simons Gauge Theories</b>	221
17.1	Chern–Simons Theory on a Torus	222
17.2	Quasiparticles and Their Statistics	225
17.3	Coupling to an External Gauge Field	226
17.4	Physics at the Edge	227
	Problems	231

## Part III Band topology

<b>18</b>	<b>Berry Phases and Chern Numbers</b>	235
18.1	Berry Phases	235
18.2	Berry Phase of a Spin	237

18.3	Berry Curvature of Bloch Bands	239
18.4	Chern Insulators	243
	Problem	244
<b>19</b>	<b>Integer Quantum Hall States</b>	<b>246</b>
19.1	Non-relativistic Particles	246
19.2	Relativistic Particles (Graphene)	249
19.3	Edge states	250
19.4	Anomaly Inflow Arguments	254
	Problem	255
<b>20</b>	<b>Topological Insulators and Superconductors</b>	<b>256</b>
20.1	Su–Schrieffer–Heeger Model	257
20.2	Kane–Mele Insulators	261
20.3	Odd-Parity Superconductors	262
<b>Part IV Fractionalization and emergent gauge fields II</b>		
<b>21</b>	<b>Parton Theories</b>	<b>269</b>
21.1	Spin Fractionalization into Bosonic Partons	269
21.2	Spin Fractionalization into Fermionic Partons	270
21.3	Quantum Hall States	272
21.4	Correlated Metals	272
<b>22</b>	<b>The Chiral Spin Liquid</b>	<b>274</b>
22.1	Mean-Field theory	275
22.2	Gauge Fluctuations	276
22.3	Edge States	277
22.4	$SU(2)$ Gauge Theory	278
<b>23</b>	<b>Non-Abelian Ising Anyons</b>	<b>280</b>
23.1	Visons and Majorana Zero Modes	281
23.2	Non-Abelian Statistics	284
23.3	Connections to Odd-Parity Superconductors	286
<b>24</b>	<b>Fractional Quantum Hall States</b>	<b>287</b>
24.1	Partons	287
24.2	Edge Theory of the Fractional Quantum Hall States	290
24.3	Bulk Gauge Theory of the Fractional Quantum Hall States	292
24.4	Moore–Read State	294
<b>25</b>	<b>Dualities of <math>XY</math> Models and <math>U(1)</math> Gauge Theories</b>	<b>295</b>
25.1	$XY$ model in $D = 1$	295
25.2	Vortices in the $XY$ Model in $D = 2$	297
25.3	$U(1)$ Gauge Theory with Monopoles in $D = 3$	304

25.4	Particle–Vortex Duality of the $XY$ model in $D = 3$	306
	Problems	311
<b>26</b>	<b>Applications of Dualities to Spin Liquids</b>	<b>312</b>
26.1	$U(1)$ Spin Liquids	312
26.2	Gapped $\mathbb{Z}_2$ Spin Liquids	319
<b>27</b>	<b>Boson–Fermion and Fermion–Fermion Dualities</b>	<b>328</b>
27.1	Fermion–Boson Duality I	328
27.2	Fermion–Boson Duality II	333
27.3	Fermion–Fermion Duality	334
27.4	Fractional Quantum Hall Effect: Dirac Composite Fermions	337
<b>28</b>	<b>Gapless Spin Liquids</b>	<b>340</b>
28.1	$U(1)$ Spin Liquids on the Square Lattice: Bosonic Spinons	341
28.2	$U(1)$ Spin Liquids on the Square Lattice: Fermionic Spinons	344
28.3	Gapless $SU(2)$ Spin Liquids	344
28.4	Gapless $\mathbb{Z}_2$ Spin Liquid on the Square Lattice	346
<b>Part V Correlated Metals</b>		
<b>29</b>	<b>Kondo Impurity Model</b>	<b>351</b>
29.1	Resonant-Level Model	351
29.2	Adding Interactions	354
29.3	Renormalization Theory	355
29.4	Large- $M$ Theory	359
29.5	Bose Kondo Model	362
	Problems	366
<b>30</b>	<b>The Heavy Fermi Liquid</b>	<b>368</b>
30.1	The Kondo Lattice Heavy Fermi Liquid	369
30.2	The Luttinger Relation	373
<b>31</b>	<b>The Fractionalized Fermi Liquid</b>	<b>381</b>
31.1	The $FL^*$ State in the Kondo Lattice	383
31.2	Emergent Gauge Fields and Generalized Luttinger Relations	386
31.3	Torus Flux Insertion and Generalized Luttinger Relations	388
31.4	The $FL^*$ State in the Single-Band Hubbard Model	395
<b>32</b>	<b>Sachdev–Ye–Kitaev Models</b>	<b>406</b>
32.1	Random Matrix Model: Free Fermions	407
32.2	Large- $N$ Theory of the SYK Model	412
32.3	$G$ – $\Sigma$ Effective Action	425
	Problem	431

---

<b>33</b>	<b>Random Quantum Spin Liquids and Spin Glasses</b>	432
33.1	Classical Ising Spin Glass	433
33.2	Quantum Rotor Spin Glass	435
33.3	Random Heisenberg Magnet	441
<b>34</b>	<b>Fermi Surfaces without Quasiparticles</b>	451
34.1	Onset of Ising Ferromagnetism	452
34.2	Luttinger Relation	461
34.3	Fermi Surface Coupled to a Gauge Field	462
34.4	Pairing Correlations	463
34.5	Transport	466
<b>Appendix A</b>	<b>Coherent-State Path Integral</b>	469
<b>Appendix B</b>	<b>Grassman Path Integral</b>	476
<b>Appendix C</b>	<b>From Spin Berry Phases to Background Gauge Charges</b>	481
<b>Appendix D</b>	<b>Emergent <math>\mathbb{Z}_2</math> Gauge Theories</b>	485
	<i>References</i>	488
	<i>Index</i>	507