Quantitative Risk and Portfolio Management

A comprehensive modern introduction to risk and portfolio management for quantitatively adept advanced undergraduate and beginning graduate students who will become practitioners in the field of quantitative finance. With a focus on real-world application, but providing a background in academic theory, this text builds a firm foundation of rigorous but practical knowledge. Extensive live data and Python code are provided, allowing a thorough understanding of how to manage risk and portfolios in practice. With its detailed examination of how mathematical techniques are applied to finance, this is the ideal textbook for giving students with a background in engineering, mathematics, or physics a route into the field of quantitative finance.

Kenneth J. Winston is a Lecturer in Economics at the California Institute of Technology and an Adjunct Professor of Mathematics at New York University. Having trained as a combinatorist at MIT, he moved into the field of quantitative finance, creating algorithms for equity and option investment strategies. He worked as a Chief Risk Officer at Western Asset Management and Morgan Stanley, and is a founder of the Buy Side Risk Managers Forum. Winston won the 2006 Roger Murray Award at the Institute for Quantitative Research in Finance and is a co-editor of The Oxford Handbook of Quantitative Asset Management (Oxford University Press, 2014).
“This is the book I wish I had had when I started my career in quantitative finance twenty years ago. It is written with the rigor of an academic, the insight of an experienced practitioner, and the didactic style of an empathetic and engaging teacher. Winston connects with his readers through insightful and entertaining discussions of historical background and of how actual financial markets behave or misbehave. At the same time, he provides rigorous but crystal clear and unhurried explanations of technical concepts. His choice of topics reflects current practice. A practitioner will find much to learn and enjoy in this book. A student who masters this material will be well prepared for a career in quantitative finance.”

Colm O’Cinneide, Franklin Templeton Investments

“Ken Winston has created a concise, valuable reference for the quantitatively minded that, in addition to describing our standard approaches for asset pricing and risk management, shows how these tools can and must be extended to reflect the more complicated risks we actually face.”

David Germany, Pitzer College

“This book is a remarkable combination of finance theory, mathematics, and practice. The development of finance theory is deep enough to challenge the most advanced students, yet it is full of applications. The author’s long history of developing risk models is evident in every chapter. The book belongs in the curricula of the best graduate programs in finance and economics.”

Charles Trzcinka, Indiana University

“Few people are as qualified as Ken Winston to provide an academically disciplined practitioner view of how to manage and profit from investment risk-taking. Trained as a mathematician, Ken was the chief risk officer for some of the world’s largest investment managers. Successful risk managers must have excellent quantitative and people skills, and Ken has both. The value of quantitative skill is evident in a game of numbers. People skills are necessary to communicate and successfully enforce limits on managers who too often dream of unachievable profits. Ken drew on both sets of skills to produce this innovative book, already well tested in his classrooms at Cal Tech and NYU. It is an essential read for all aspiring investment managers.”

Larry Harris, University of Southern California

“This is the book that I wish I had been able to have when I switched from applied math/engineering to applied finance more than thirty years ago. In essence, the book fills a very important void: how to approach financial engineering problems from the practitioner’s viewpoint. A must-have for risk managers and investment professionals.”

Arturo Cifuentes, Chile Sovereign Fund
Quantitative Risk and Portfolio Management

Theory and Practice

Kenneth J. Winston
California Institute of Technology
## Contents

List of Images x
List of Figures xi
Preface xv
Acknowledgments xxviii

### 1 What Is Risk?

1.1 Frank Knight’s Formulation 2
1.2 Finite Probability Spaces 3
1.3 Knightian Uncertainty 4
1.4 Making Risky Decisions 6
  1.4.1 Reader Poll 1: St. Petersburg Paradox 6
  1.4.2 Reader Poll 2: The Generous Billionaires 7
  1.4.3 Reader Poll 3: The Probabilistic Thug 8
1.5 Basic Economics Terminology 9
1.6 Basic Capital Markets Terminology 12
1.7 Basic Probability Terminology 15
1.8 Utility Theory 20
  1.8.1 Von Neumann–Morgenstern Utility Theory 22
  1.8.2 VNM Axioms and Theorem 23
  1.8.3 Risk Preferences 26
  1.8.4 Drawbacks of Utility Theory 32

### 2 Risk Metrics

2.1 Caveat Laws 37
  2.1.1 Portfolio Insurance 39
2.2 Volatility 42
2.3 VaR, ES, cVaR, and Spectral Measures 44
  2.3.1 VaR Measures with Discrete Distributions 47
2.4 Risk-Adjusted Reward Measures 50
2.5 Coherent Risk 53
2.6 Risk-Averse Prices 56
2.7 No-Arbitrage 58
  2.7.1 Limits of Arbitrage 59
  2.7.2 Short Sales and Short Squeezes 60
  2.7.3 No-Arbitrage Formulation 66
## Contents

2.8 State Prices and Risk-Neutral Probabilities 66  
2.9 Stochastic Discount Factors 68  
2.10 The Ross Recovery Theorem 72  
  2.10.1 Ross Recovery: Key Assumptions 73  
  2.10.2 Ross Recovery: Matrix Calculations 75  
  2.10.3 Computing the SDF: Ross’s Method 76  
  2.10.4 Computing the SDF: Jackwerth and Menner’s Method 77  
  2.10.5 Ross Recovery Theorem: Empirical Problems 78  

3 Fixed-Income Modeling 81  
3.1 Real, Inflation, and Nominal Rates 82  
3.2 Discounting 85  
3.3 The Risk in the Risk-Free Rate 86  
3.4 Basic Fixed-Income Mathematics 89  
  3.4.1 Generic Pricing Equation 90  
  3.4.2 Price Changes as Rates Change 91  
  3.4.3 Duration Calculations; Convexity 93  
  3.4.4 Approximations and Basis Risk 96  
3.5 Yield Curves 97  
  3.5.1 Features of Yield Curves 99  
  3.5.2 Zero Curves and Par Curves 100  
  3.5.3 Types of Yield Curves 103  
  3.5.4 Yield Curves and Economic Conditions 103  
  3.5.5 Rolldown and Key Rate Durations 105  
  3.5.6 Interpolation and Smoothing Techniques 107  
3.6 Implied Forward Curves 109  
3.7 Stochastic Process Terminology 111  
3.8 Term Structure Models 114  
  3.8.1 Short-Rate Models: General Framework 115  
  3.8.2 Short-Rate Models: Specific Framework 116  
  3.8.3 Expected Value of the Vasicek Model 119  
  3.8.4 Other Short-Rate Models 121  

4 Equity Modeling 123  
4.1 Markowitz Efficient Frontier 123  
  4.1.1 Equality-Constrained Frontier 126  
  4.1.2 Equality-Constrained Frontier: Example 129  
  4.1.3 Inequality Constraints 131  
  4.1.4 Efficient Frontier and Utility Functions 133  
  4.1.5 The Capital Market Line 134  
  4.1.6 Benchmark-Relative 136  
  4.1.7 Efficient Frontiers: Theory and Practice 138
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2 Parameter Estimation Methods</td>
<td>139</td>
</tr>
<tr>
<td>4.2.1 Review of Bayes’s Rule</td>
<td>139</td>
</tr>
<tr>
<td>4.2.2 Shrinkage Estimators</td>
<td>142</td>
</tr>
<tr>
<td>4.2.3 Statistical Tests</td>
<td>147</td>
</tr>
<tr>
<td>4.2.4 Resampled Efficient Frontier</td>
<td>151</td>
</tr>
<tr>
<td>4.3 Black–Litterman</td>
<td>154</td>
</tr>
<tr>
<td>4.3.1 Black–Litterman Example</td>
<td>156</td>
</tr>
<tr>
<td>5 Convex Optimization</td>
<td>159</td>
</tr>
<tr>
<td>5.1 Basic Optimization Terminology</td>
<td>159</td>
</tr>
<tr>
<td>5.2 Convex Properties</td>
<td>161</td>
</tr>
<tr>
<td>5.3 Unconstrained Convex Optimization</td>
<td>165</td>
</tr>
<tr>
<td>5.3.1 Gradient Descent</td>
<td>167</td>
</tr>
<tr>
<td>5.3.2 Newton’s Method</td>
<td>171</td>
</tr>
<tr>
<td>5.4 Constrained Optimization</td>
<td>173</td>
</tr>
<tr>
<td>5.4.1 Lagrange Duality</td>
<td>175</td>
</tr>
<tr>
<td>5.4.2 KKT Conditions with Convexity</td>
<td>178</td>
</tr>
<tr>
<td>5.5 Barrier Methods</td>
<td>180</td>
</tr>
<tr>
<td>6 Factor Models</td>
<td>187</td>
</tr>
<tr>
<td>6.1 The Efficient Market Hypothesis</td>
<td>188</td>
</tr>
<tr>
<td>6.2 The Capital Asset Pricing Model and the Four-Factor Model</td>
<td>190</td>
</tr>
<tr>
<td>6.3 Arbitrage Pricing Theory</td>
<td>192</td>
</tr>
<tr>
<td>6.3.1 APT: Exact Form</td>
<td>193</td>
</tr>
<tr>
<td>6.3.2 APT: Inclusion of Specific Behavior</td>
<td>195</td>
</tr>
<tr>
<td>6.4 Factor Models in Practice</td>
<td>198</td>
</tr>
<tr>
<td>6.5 Principal Components Analysis</td>
<td>200</td>
</tr>
<tr>
<td>7 Distributions</td>
<td>210</td>
</tr>
<tr>
<td>7.1 Central Limit Theorem</td>
<td>211</td>
</tr>
<tr>
<td>7.1.1 Checking Normality: Q-Q and P-P Plots</td>
<td>213</td>
</tr>
<tr>
<td>7.1.2 Jarque–Bera</td>
<td>215</td>
</tr>
<tr>
<td>7.1.3 Causes of Non-Normality</td>
<td>216</td>
</tr>
<tr>
<td>7.1.4 Market Ages</td>
<td>217</td>
</tr>
<tr>
<td>7.2 Student’s T Distribution</td>
<td>220</td>
</tr>
<tr>
<td>7.3 Mixtures of Normals</td>
<td>222</td>
</tr>
<tr>
<td>7.4 Stable Distributions</td>
<td>226</td>
</tr>
<tr>
<td>7.5 Extreme Value Distributions</td>
<td>231</td>
</tr>
<tr>
<td>7.5.1 Block Maxima</td>
<td>234</td>
</tr>
<tr>
<td>7.5.2 Domains of Attraction</td>
<td>236</td>
</tr>
<tr>
<td>7.6 Tail Distributions</td>
<td>237</td>
</tr>
<tr>
<td>7.6.1 Fitting Parameters to Tail Distributions</td>
<td>239</td>
</tr>
</tbody>
</table>
Contents

8 Simulation, Scenarios, and Stress Testing 244
  8.1 Historical Simulation 245
  8.2 Delta-Normal 248
    8.2.1 The Cornish–Fisher Expansion 252
  8.3 Monte Carlo Simulation 253
    8.3.1 Delta-Gamma, Delta-Gamma-Theta Simulations 257
  8.4 Markov Chain Monte Carlo 258
  8.5 Stress Testing and Scenario Analysis 265

9 Time-Varying Volatility 267
  9.1 Historical Volatility 267
  9.2 Options and Volatility 270
    9.2.1 Review of Options Terminology 270
    9.2.2 Market Volatilities and Volatility Markets 273
    9.2.3 Volatility Skews 276
  9.3 Volatility Models 278
    9.3.1 Risk-Neutral Density Recovery 279
    9.3.2 Real-World Density Recovery 280
    9.3.3 Stochastic Volatility Modeling 282
    9.3.4 Local Volatility Modeling 283
    9.3.5 SABR Modeling 284
    9.3.6 VIX® Calculations 286
  9.4 Time Series Terminology 289
  9.5 ARCH and GARCH Modeling 293
    9.5.1 ARCH and GARCH variants 297

10 Modeling Relationships 301
  10.1 Pearson Correlation 301
  10.2 Spearman Correlation 303
  10.3 Conditional Correlation 305
  10.4 Correlations and the Economy 309
  10.5 Implied Correlations 312
  10.6 Copula Functions 316
    10.6.1 Copula Example: Gaussian Copulas 318
  10.7 Historical Estimation of Correlation Matrices 322
  10.8 Time Series Estimates of Covariances and Correlations 325
    10.8.1 Constant and Dynamic Conditional Correlation 325
    10.8.2 Implementation of Dynamic Conditional Correlations 328
    10.8.3 MacGyver Method 333
Contents

11 Credit Modeling

11.1 Basic Credit Risk Concepts 335
11.2 Credit Ratings 337
  11.2.1 Investment Grade and Speculative Grade 340
  11.2.2 Historical Default Frequencies 342
  11.2.3 Sovereign Debt Ratings 348
  11.2.4 Sovereign Spreads in the Eurozone 352
11.3 The Credit Spread Premium Puzzle 354
11.4 The Merton Model 357
  11.4.1 Merton’s Term Structure of Credit Spreads 359
  11.4.2 Enterprise Value Example 361
  11.4.3 Model Improvement: Iterative Enterprise Value 363
  11.4.4 Model Improvement: KMV model 365
  11.4.5 Predictive Power of Default Estimates 368
11.5 Credit Spread Correlates 371
11.6 Credit Spread Metrics 372
11.7 Credit Factor Models 375
11.8 Z-Scores, Reduced-Form, and Hybrid Models 377
11.9 Implied Default Rates 381
11.10 Credit Default Swaps 383

12 Hedging

12.1 Risk Unbundling 387
  12.1.1 Quantity-Adjusting Options 391
12.2 Franchise Preservation 395
  12.2.1 Siegel’s Paradox 397
  12.2.2 Modigliani–Miller and Franchise Hedging 402
  12.2.3 Bank Franchise Hedging 406
12.3 Longevity Risk 419
12.2.4 Asset–Liability Management: Interest Rate Swaps 410
12.2.5 Interest Rate Swaps: Simplified Model 413
12.2.6 Interest Rate Swaps: Market 416
12.2.7 Illiquidity Hedging 420
12.4 Distribution Reshaping 422
  12.4.1 Distribution Reshaping with Options 423
  12.4.2 Option Greeks 429
  12.4.3 Fixed-Income Options 433
12.5 Convexity Hedging 434

Appendix: Code Segments 440
References 591
Index 609
List of Images

1.1 (a) John D. Rockefeller, (b) Andrew Carnegie. page 7
1.2 Probabilistic thug. 9
2.1 Twelve-fingered man. 39
3.1 US Treasury bond. 87
11.1 Bloomberg enterprise value screen. 361
11.2 Bloomberg enterprise value page for Charter. 364
12.1 Northern Rock run in 2007. 408
List of Figures

1.1 Concave upside utility. 
1.2 Convex downside utility. 
1.3 Kahneman’s empirical decision weights $d(p)$. 
2.1 High versus low volatility: same returns. 
2.2 Worst 20 losses out of 1,000. 
2.3 Isoprobability (constant $p$) curves. 
2.4 GME short squeeze. 
3.1 US wholesale prices. 
3.2 Five-year US real rates, 2003-01 to 2022-08. 
3.3 Swiss 10-year nominal rates, 2014-02 to 2022-08. 
3.4 Seasoned 29-year 7% coupon bond price. 
3.5 New 30-year bond duration. 
3.6 First- and second-order approximations. 
3.7 US yearend Treasury curves. 
3.8 US Treasury curve, 2001-01-02. 
3.9 Ten-year, 3-month US Treasury steepness (10.00%neg). 
3.10 UST curve principal components. 
3.11 Nelson–Siegel components. 
3.12 Smoothed US Treasury and short-rate curves. 
3.13 Hull–White curve generation. 
3.14 Hull–White curves, monthly $\sigma = .05$ and .2. 
4.1 Efficient frontier and inefficient portfolios. 
4.2 Franc, pound, yen efficient frontier. 
4.3 Long-only efficient frontier (4.13). 
4.4 Beginning of franc, pound, yen efficient frontier. 
4.5 Capital market line + franc, pound, yen efficient frontier. 
4.6 Resampled minimum variance portfolios. 
5.1 Locally convex disutility function. 
5.2 Normal pdf-based approximation to delta function. 
5.3 Long-only efficient frontier (like Fig. 4.3). 
6.2 Marchenko–Pastur limit pdf. 
7.1 $n = 1,000$ binomial vs. normal pdf. 
7.2 Q-Q plot, CHF 1971–2021, 12,784 observations. 
7.3 P-P plot, CHF 1971–2021, 12,784 observations.
List of Figures

7.4 UK government bond long-term rates, yearend 1703–2021. 219
7.5 a. Overall view of Student’s T densities. b. Left cumulative tails of Student’s T distributions. 222
7.6 Kurtosis of mixtures of normals. 225
7.7 a. Overall view of stable distributions. b. Left cumulative tails of stable distributions. 228
7.8 Inverse standard Normal, Cauchy, and Levy. 230
7.9 Tail cdf approximations, Formula (7.30). 231
7.10 Histogram of 100-sample uniform maxima, 1,000 trials. 232
7.11 Generalized extreme value densities. 233
7.12 Block minima, CHF/USD 1971–2021, 1,278 blocks. 234
7.13 Q-Q plot, CHF 10-day max loss vs. Fréchet, 1971–2021. 235
7.14 Standard normal tails. 238
7.15 Generalized Pareto densities using (7.47). 241
7.16 cdfs of empirical, Gumbel, and fitted distributions. 242
8.1 Histogram of equal-weighted CHF+GBP+ JPY daily log-changes, 1971–2021. 248
8.2 Histogram, delta-normal. 250
8.3 Allowed parameter combinations for Cornish–Fisher. 253
8.4 SPASTT01USM661N (US). 263
8.5 SPASTT01EZM661N (Europe). 263
8.6 SPASTT01JPM661N (Japan). 264
9.1 Virtual US stocks 1926-07–2021-12, annual sigma = 18.42. 268
9.2 Actual US stocks sample std dev 1926-07–2021-12. 269
9.3 Put, call, straddle payoffs. 274
9.5 Moneyness skew SPX options: expiry 2022-03-18, quoted 2021-12-31, compared with 2008-12-31 quotes. 276
9.6 Maturity skew ATM SPX options: strike 4775, money 4782.4502, quoted 2021-12-31, compared with 2008-12-31 quotes. 277
9.7 a. Time skews. b. Money skews. 278
9.8 LVM does not shift the way the market shifts. 285
9.9 Virtual VIX calculations. 289
9.10 GARCH(1,1) fit to US stock market data. 296
9.11 Merton model. 297
9.12 Pseudo-delta functions. 299
10.1 Fisher z-transform. 303
10.2 Simulated 52-week sample correlations. 308
10.3 Simulated 156-week sample correlations. 308
10.4 Historical 156-week sample correlations. 309
10.5 Seasonally adjusted US house price index 1991Q1–2022Q2. 311
10.6 US stock/bond 36-month correlations 1974-01-31 to 2021-12-31 (gray=insignificant). 312
List of Figures

10.7 Histogram of ATM implied vols of SPY components expiring 2022-03-18, quoted 2021-12-31. 313
10.8 Cboe implied correlations COR3M. 315
10.9 GARCH(1,1) annualized standard deviation 1990-07–2021-12. 326
10.10 Historical de-GARCHed 156-week sample correlations. 328
10.11 Integrated correlations $\lambda = 0.01118$, 1990-07-04:2021-12-29. 331
10.12 Objective function as half-life changes. 332
10.13 Mean reverting correlations $\alpha = 0.01447, \beta = 0.97661$, 1990-07-04:2021-12-29. 332
11.1 Moody’s corporate default rates, 1920–2021. 345
11.2 Default rates, Moody’s historical 1920–2021 and Barclays’ model 2009, log-scale. 348
11.3 European spreads over Germany, 10-year rates. 353
11.4 Moody’s smoothed yield spreads over Treasurys. 355
11.5 Term structure of Merton credit spreads, $\sigma^2 - 0.20$. 360
11.6 JNJ capital stack 2021Q3. 361
11.7 CHTR capital stack. 364
11.8 Correlations by decade: AAA/BBB average spd chgs versus Treasury rate chgs. 372
12.1 Total bottle volume of Philippe–Ken Wine Company. 399
12.2 Philippe–Ken Wine Company profit. 400
12.3 Philippe–Ken Wine Company profit – neighborhood. 401
12.4 US swap spreads over Treasurys (by maturity). 412
12.5 Fixed payer failure mitigated by CCP. 416
12.6 a. Payoff of put plus underlying: strike = 90, cost = 3.02. b. Density function with and w/out put. 424
12.7 a. Payoff of underlying minus call: strike = 110, cost = 4.94. b. Density function with and w/out written call. 424
12.8 a. Payoff of put spread plus underlying: HighStrk = 90, HighCst = 3.02; LowStrk = 80, LowCst = 0.96. b. Density function with and w/out put spread. 425
12.9 a. Payoff of call spread plus underlying: HighStrk = 120, HighCst = 2.55; LowStrk = 110, LowCst = 4.94. b. Density function with and w/out call spread. 426
12.10 a. Payoff of underlying plus straddle: Call Strike = Put Strike = 100, Call Cost = 8.92, Put Cost = 6.94. b. Density function with and w/out +straddle. 426
12.11 a. Payoff of underlying minus straddle: Call Strike = Put strike = 100, Call Cost = 8.92, Put cost = 6.94. b. Density function with and w/out –straddle. 427
12.12 a. Payoff of underlying with strangle: Call Strike = 110, Call Cost = 4.94, Put Strike = 90, Put Cost = 3.02. b. Density function with and w/out strangle. 427
12.13 a. Payoff of underlying w/butterfly spread: Low = 80, Mid = 100, High = 100; trade cost = 7.26. b. Density function w/and w/out butterfly. 428
List of Figures

12.14 a. Payoff of underlying with ZC collar: Put Strike = 90, Call Strike = 118; option costs = 3.02. b. Density function with and w/out ZC collar. 428

12.15 Theta as a function of strike price: Price = 100, \( r = 0.02, T = 0.02, \) \( \sigma = 0.20. \) 431

12.16 Gamma, vega as a function of strike price: Price = 100, \( r = 0.02, T = 0.25, \) \( \sigma = 0.20. \) 431