

## Quantitative Risk and Portfolio Management

A comprehensive modern introduction to risk and portfolio management for quantitatively adept advanced undergraduate and beginning graduate students who will become practitioners in the field of quantitative finance. With a focus on real-world application, but providing a background in academic theory, this text builds a firm foundation of rigorous but practical knowledge. Extensive live data and Python code are provided, allowing a thorough understanding of how to manage risk and portfolios in practice. With its detailed examination of how mathematical techniques are applied to finance, this is the ideal textbook for giving students with a background in engineering, mathematics, or physics a route into the field of quantitative finance.

**Kenneth J. Winston** is a Lecturer in Economics at the California Institute of Technology and an Adjunct Professor of Mathematics at New York University. Having trained as a combinatorist at MIT, he moved into the field of quantitative finance, creating algorithms for equity and option investment strategies. He worked as a Chief Risk Officer at Western Asset Management and Morgan Stanley, and is a founder of the Buy Side Risk Managers Forum. Winston won the 2006 Roger Murray Award at the Institute for Quantitative Research in Finance and is a co-editor of *The Oxford Handbook of Quantitative Asset Management* (Oxford University Press, 2014).

“This is the book I wish I had had when I started my career in quantitative finance twenty years ago. It is written with the rigor of an academic, the insight of an experienced practitioner, and the didactic style of an empathetic and engaging teacher. Winston connects with his readers through insightful and entertaining discussions of historical background and of how actual financial markets behave or misbehave. At the same time, he provides rigorous but crystal clear and unhurried explanations of technical concepts. His choice of topics reflects current practice. A practitioner will find much to learn and enjoy in this book. A student who masters this material will be well prepared for a career in quantitative finance.”

**Colm O’Cinneide**, *Franklin Templeton Investments*

“Ken Winston has created a concise, valuable reference for the quantitatively minded that, in addition to describing our standard approaches for asset pricing and risk management, shows how these tools can and must be extended to reflect the more complicated risks we actually face.”

**David Germany**, *Pitzer College*

“This book is a remarkable combination of finance theory, mathematics, and practice. The development of finance theory is deep enough to challenge the most advanced students, yet it is full of applications. The author’s long history of developing risk models is evident in every chapter. The book belongs in the curricula of the best graduate programs in finance and economics.”

**Charles Trzcinka**, *Indiana University*

“Few people are as qualified as Ken Winston to provide an academically disciplined practitioner view of how to manage and profit from investment risk-taking. Trained as a mathematician, Ken was the chief risk officer for some of the world’s largest investment managers. Successful risk managers must have excellent quantitative and people skills, and Ken has both. The value of quantitative skill is evident in a game of numbers. People skills are necessary to communicate and successfully enforce limits on managers who too often dream of unachievable profits. Ken drew on both sets of skills to produce this innovative book, already well tested in his classrooms at Cal Tech and NYU. It is an essential read for all aspiring investment managers.”

**Larry Harris**, *University of Southern California*

“This is the book that I wish I had been able to have when I switched from applied math/engineering to applied finance more than thirty years ago. In essence, the book fills a very important void: how to approach financial engineering problems from the practitioner’s viewpoint. A must-have for risk managers and investment professionals.”

**Arturo Cifuentes**, *Chile Sovereign Fund*

# Quantitative Risk and Portfolio Management

## Theory and Practice

**Kenneth J. Winston**

California Institute of Technology



Shaftesbury Road, Cambridge CB2 8EA, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

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