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978-1-009-20904-5 — Quantitative Risk and Portfolio Management
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Excerpt
More Information

1 What Is Risk?

Risk is **lack of information about the future.** A situation is risky if it has widely varying possible outcomes and there's no way to determine with high confidence which outcome will occur. A riskless or risk-free situation is one whose future is known exactly.

We humans yield to no species in our ability to worry, so there is a tendency to focus on whether or not negative outcomes might impend. Indeed, most definitions of the noun "risk" stress the downside: for example, *the possibility of loss, injury, or other adverse or unwelcome circumstance* is the first of several definitions of risk in the *Oxford English Dictionary*.[1] This is a gloomy strain of the more general definition above: a magnitude 8 earthquake is not a risk because we lack information about whether it will be enjoyable. It is a risk because we lack information about whether this unequivocally unpleasant thing will happen.

Expressing your love for the first time to another person involves *the possibility of loss, injury, or other adverse or unwelcome circumstance*: the other person may reject you.[2] So why do people override their pounding pulses and stammer out their feelings? It is because the unknown future also includes positive outcomes: the other person may return your affection.[3] Declaring your feelings is a risky situation, but you do so because you hope that a good outcome will occur.

The *Oxford English Dictionary* encompasses positive outcomes in an alternative definition of risk: *A person or thing regarded as likely to produce a good or bad outcome in a particular respect. Ex.: "The key to their success is information: on-the-ground knowledge of who is a good credit risk."* The phrase "good credit risk" in the example indicates that the good outcome (getting paid back after extending credit) was considered more likely due to on-the-ground knowledge.

1 Oed.com, September 2022. "risk, n." *Oxford English Dictionary*. `www.oed.com/view/Entry/166306`.
2 Madeline Holcombe and Giulia McDonnell, "A Confession of Love Ended in a Professor Attacking Her Friend with a Fire Poker, Police Say." CNN, January 7, 2020. `www.cnn.com/2020/01/07/us/mount-holyoke-professor-attack/index.html`.
3 [Shakespeare 1597], Act II, Scene II.

The philosopher Karl Popper noted that for scientific theories,

> Confirmations should only count if they are the result of **risky predictions**; that is to say, if, unenlightened by the theory in question, we should have expected an event which was incompatible with the theory – an event which would have refuted the theory.[4]

Popper thus captures the essence of this type of risk: a momentous choice about the future has to be made. A scientist has to make a risky prediction (emphasis added above) that can be proved right or wrong; a lender has to decide whether or not making a loan will probably result in repayment; a lover has to choose speech or silence.

So in practice there are two kinds of risks:

- **perils**, where there are no positive outcomes. There is only (a) nothing happens; or (b) bad things happen. Earthquakes, fires, floods, hurricanes, and other natural disasters are risks in this sense.
- **ventures**, where there are both positive and negative outcomes. Investments – such as making loans – are risks in this sense.

Perils are managed by avoidance (stay out of earthquake zones); fortification (build strong structures); and backstops (have emergency services and a pool of money ready to deal with the inevitable damage).

Ventures require a more subtle approach, because both positive and negative outcomes have to be weighed against each other. Should I make a conjecture;[5] extend credit; or declare my love? These are tough questions.

1.1 Frank Knight's Formulation

In 1921, Frank Knight – then an Associate Professor of Economics at the University of Iowa; later the influential head of the University of Chicago's economics department – wrote *Risk, Uncertainty, and Profit*.[6]

Knight delved into the philosophical nature of knowledge itself, but eventually got down to a very pragmatic list of ways in which we can organize our knowledge (or lack thereof) about the future (pp. 224–225):

> 1. **A priori probability**. . . . [O]n the same logical plane as the laws of mathematics.
> 2. **Statistical probability**. Empirical evaluation of the frequency of association between predicates.
> 3. **Estimates**. [T]here is *no valid basis of any kind* for classifying instances.

4 [Popper 1962].
5 [Kim and Pittel 2000].
6 [Knight 1921].

An example of something with a priori probability is the throw of a perfect die. As Knight says, "the mathematician can easily calculate the probability that any proposed distribution of results will come out of any given number of throws." That doesn't help anyone know which face will come up on the next throw of the die; probability theory is silent on that subject. But a priori probability is vocal about the fact that betting even money on the same face coming up 50 times in a row is a bad idea when the die is fair.

A little less information about the future attaches to Knight's second category, *statistical probability*. An example here might be the chance that a 40-year-old male dies in the next year. Life insurance companies have gathered extensive statistics about mortality rates among 40-year-old males. As long as they have a sufficiently large pool of insureds, they can make a reasonable guess as to what to charge for insuring a 40-year-old male life. But there is no *a priori* mathematical model as there is with the throw of a die.

The final category has the least information. Here intuition must be used. Will my beloved return my affection if I declare it? There is no mathematical model, so the first category doesn't apply. Very few people have had the opportunity to build large databases of outcomes of their previous declarations of love, so the second category doesn't apply either. The terrified lover is left only with hunches.

Knight condenses his three categories into two with this more succinct statement (p. 233):

> To preserve the distinction which has been drawn ... between the measurable uncertainty and an unmeasurable one, we may use the term "risk" to designate the former and the term "uncertainty" for the latter.

1.2 Finite Probability Spaces

At about the same time that Frank Knight was developing his ideas, the foundations of modern probability theory were being constructed by people like Émile Borel, Henri Lebesgue, and Andrey Kolmogorov. But Knight's formulation is amenable to a simple finite treatment.

For his "risk" (measurable) category, he assumed a finite number of future outcomes $s_1, \ldots, s_n$ and a known set of associated probabilities $p_1, \ldots, p_n$ (where $\sum_{i=1}^{n} p_i = 1$ and each $p_i \geq 0$). In the more general language of probability theory, $\{s_1, \ldots, s_n\}$ is the **sample space**, often denoted Ω.

Some probabilists treat finite and discrete probability spaces as the poor cousins of continuous probability spaces. But as Carlo Rovelli notes,

> Continuity is only a mathematical technique for approximating very finely grained things. The world is subtly discrete, not continuous.[7]

7 [Rovelli 2018], Chapter 5, p. 84.

It may be mathematically convenient to smooth out the world so that, for example, derivatives can be taken. But in fact Knight's "simple" finite formulation is sufficient to describe all states of the world that could arise in finance, or more generally in human experience.

An American roulette wheel is subject to what is now called **Knightian Risk**. Such a wheel has 38 outcomes (s_i = the ball falls in the slot numbered i for $i = 1, \ldots, 36$; s_{37} = the ball falls in the slot labeled 0; and s_{38} = the ball falls in the slot labeled 00). So the roulette sample space is

$$\Omega_{roulette} = \{s_1, \ldots, s_{38}\} = \{1, \ldots, 36, 0, 00\}. \tag{1.1}$$

Every slot is equally likely, so $p_i = \frac{1}{38}$ for $i = 1, \ldots, 38$.

Casinos make money on their roulette wheels, just as they do at their dice tables. In both cases, bettors can wager on *events*, which are combinations of outcomes. For example, in American roulette a bettor can place a bet on "even," which pays off if the future outcome lies in the event

$$E = \{s_2, s_4, \ldots, s_{36}\}. \tag{1.2}$$

Note that neither s_{37} nor s_{38} (neither 0 nor double-0) is contained in E. So the sum of the probabilities associated with the event E is only $\frac{18}{38} \approx 47.4\%$

If you bet on even and it occurs, you get two dollars for every dollar you bet. If even doesn't occur you lose the money you bet and you get nothing. The expected value of a dollar bet is $\frac{18}{38} * 2 + \frac{20}{38} * 0 = 18/19$, meaning the casino expects to make 5.2 cents for every dollar bet on the even event. Put another way, the casino charges a 5.2 percent fee per 4 minutes[8] to provide the entertainment of betting on the even event.

Knightian Risk is amenable to simple probabilistic calculations like this.

Perhaps the most common mistake made in financial mathematics is to forget that modeling the world of finance with Knightian Risk is a tactic, not a law.

1.3 Knightian Uncertainty

Within the broad umbrella of risk = lack of information about the future, Knight identified a more difficult type: what he called "unmeasurable," and what economists now call Knightian Uncertainty.

Knightian Uncertainty means either

- The sample space $\Omega = \{s_1, \ldots, s_n\}$ is known, but the associated probabilities $p_1, \ldots, p_n$ are not; or
- The sample space Ω is not known.

8 Roulette Life, "How Long Does It Take to Spin 100 Times in to a BM Casino (Average)?," post dated June 15, 2015, by user kav. www.roulettelife.com/index.php?topic=358.0.

Declaring love is a situation where the outcomes are (broadly) known: acceptance or rejection. But there is *no valid basis of any kind* to arrive at accurate probabilities.

Keynes (1937, p. 213) gave examples of Knightian Uncertainty:

> By "uncertain" knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty.... The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatsoever.

In Keynes's examples, the outcomes of whether there will be "a European war" are known: yes or no. But the "prospect" (probability) is not. Or more precisely, in 1937 the probability *was* not; now we know there was a European (in fact, World) war. But in 1937 some future states of the world contained a European war and some didn't. Eventually (by September 3, 1939, when France and Britain declared war on Germany[9]) there were no longer any future states of the world that didn't have a European war in them.

Even deeper uncertainty attached to his last example, "the position of private wealth owners in the social system in 1970" – a date 33 years in the future from the time of his writing. Neither all the outcomes nor all the probabilities were known. Keynes was concerned about whether socialism, capitalism, or communism would prevail, but those broad groupings didn't constitute a precise and exhaustive listing of the outcomes, let alone guide the choice of associated probabilities.

We can be quite certain that if the upper-crust Keynes had tried to list the possible outcomes for "the position of private wealth owners in the social system in 1970," he would not have included the eventual reality. By then, a kind of socialist-tinged capitalism would be in place and one of the larger private wealth owners would be a 30-year-old commoner from Liverpool – John Lennon of the Beatles.[10] Such open-ended outcomes defy the tidy enumeration required for Knightian Risk to obtain.

The financial world is a world of Knightian Uncertainty, not of Knightian Risk.

Despite the fuzziness of Knightian Uncertainty, decisions have to be made. Freezing and doing nothing is a choice. This is the idea behind Pascal's Wager:[11] "Il faut parier." You have to make a bet: even not deciding is deciding.

9 History.com editors, 2009. "World War II," Section 2, Outbreak of World War II (1939). Online article. www.history.com/topics/world-war-ii/world-war-ii-history#section_2.
10 Biography.com editors, Original Published Date April 2, 2014; Last Updated April 14, 2021. "John Lennon: Biography." Online article. www.biography.com/musician/john-lennon.
11 [Hájek 2022].

In many cases, quantitative finance makes a first approach to problems by assuming that they can be described by Knightian Risk. However, we must never forget that this is an approximation, a guide, a forensic tool to help frame our thinking and intuition as we make decisions about an uncertain future.

1.4 Making Risky Decisions

All the focus on classifying what may happen in the future is done so that decisions can be made about what to do now: Set the payoffs on a roulette wheel; price a life insurance policy; determine an interest rate for a loan to a good credit risk; declare love or pine away, mute.

In this chapter, we'll describe a framework for thinking about how risky decisions are made. The framework will reflect the intuitively obvious fact that there are very few purely right or wrong answers; in many situations, personal preferences for safety versus risk play an important role.

For now, consider the following questions about what you might do in the risky situations described below. There aren't any tricks of logic you need to worry about: just answer what seems right to you.

1.4.1 Reader Poll 1: St. Petersburg Paradox

The **St. Petersburg Paradox** was posed by Nicolas Bernoulli in 1713. One resolution of the paradox was proposed by his cousin Daniel Bernoulli, who worked on the problem while he was a mathematics professor in St. Petersburg. D. Bernoulli's solution was published in 1738, although he probably wrote it around 1728.

N. Bernoulli asked how much someone should pay to enter a doubling lottery. In N. Bernoulli's lottery, a coin is tossed. If it comes up heads, the participant gets \$2 and the lottery is over. If it comes up tails, it is tossed a second time. If it then comes up heads, the participant gets \$4 and the lottery is over. Otherwise there's a third toss with an \$8 payoff if heads, and so forth, doubling the payoff every time. The lottery continues until the first head comes up. If the head comes up on the i^{th} toss, the payoff is $\$2^i$.

N. Bernoulli pointed out that the expected value is infinite, since the probability of the lottery ending on the i^{th} toss is 2^{-i}. One might (anachronistically, since Knight was two hundred years in the future) label this Knightian Risk or a priori probability, and evaluate all the outcomes and all the probabilities to find that the expected value to the participant of entering the lottery is:

$$\sum_{i=1}^{\infty} 2^{-i} 2^i = \sum_{i=1}^{\infty} 1 = \infty.$$

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The lottery is risky – you don't know which toss of the coin will be the first heads – but unequivocally positive: you will get at least \$2 if you enter it. So it seems sensible that you would pay at least \$2 to enter, since you'll get that half the time, and more than that the other half of the time. But it seems just as sensible that you would not pay infinity, or even a very large portion of your personal wealth, to enter this lottery.

We'll analyze this in more detail, but for now answer intuitively: *How much would you pay to enter this lottery*?

Make a note of your answer – call it A_{pete}. We'll come back to it later.

1.4.2 Reader Poll 2: The Generous Billionaires

You are walking down the street and you run into John D. Rockefeller and Andrew Carnegie (Image 1.1), both of whom have a net worth well over \$100 billion.

"Hello," Rockefeller says, "We've decided to be generous to the next person we see, and that's you! But we have a disagreement about how to dispense our generosity. So we need you to decide between the following two offers."

Carnegie says, "My offer is very simple. I'm just going to give you this check for \$500, 000, 000."

Rockefeller says, "But my offer is more interesting. I'm going to toss a fair coin – 50 percent chance of either heads or tails. If it comes up heads, I'll give you \$1, 000, 000, 000. But if it comes up tails, I'll give you nothing."

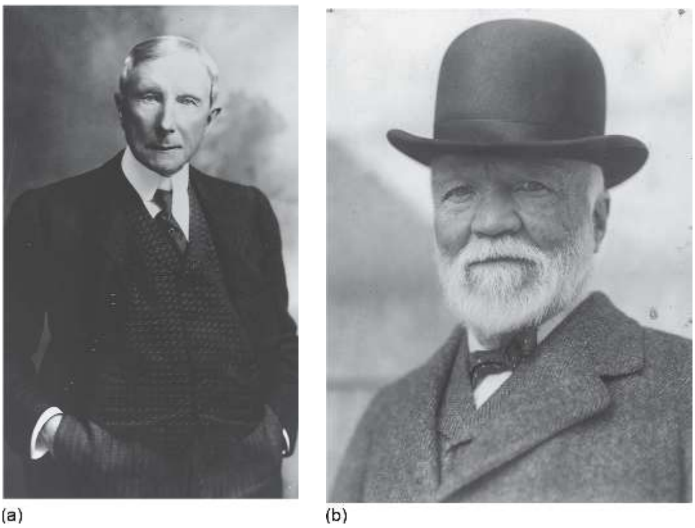

Image 1.1 (a) John D. Rockefeller, (b) Andrew Carnegie. Source: (a) FPG/Archives Photos/Getty Images. (b) Hulton Archive/Stringer/Getty Images.

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You must choose only one. Decide whether you want to take Carnegie's offer (sure thing) or Rockefeller's offer (coin toss).

Make a note of your answer (Rockefeller or Carnegie) – call it $A_{generous}$. We'll come back to it later.

Changing Generosity

If you chose Carnegie's offer (the check for $500,000,000):

> Assume that Rockefeller's offer ($1,000,000,000 or 0 depending on coin toss) remains unchanged, but that Carnegie is only offering a check for $400,000,000. Would you still take the check? What about a check for $200,000,000? What about $100,000,000? At some point you will switch: presumably if Carnegie's offer is only $0.01, you'll take Rockefeller's coin toss instead. So where's your switching point, that is, what amount of sure check from Carnegie will put you just on the edge between that check and Rockefeller's billion-or-zero coin toss? Make a note of this number – call it A_{switch}.

If you chose Rockefeller's offer (billion-or-zero coin toss):

> Assume that Rockefeller's offer remains unchanged, but that Carnegie offers a sure check for $1,000,000,000. At that point you are always better off with Carnegie, so presumably you would take Carnegie's offer over Rockefeller. So somewhere on the way up between a Carnegie offer of $500,000,000 and a Carnegie offer of $1,000,000,000 you switched from Rockefeller to Carnegie. What is your switching point? Make a note of this number – call it A_{switch}.

1.4.3 Reader Poll 3: The Probabilistic Thug

You are walking down a dark street when you run into a thug pointing a gun at you (Image 1.2).

"Hello," the thug says, "Because I am a thug, I am going to do something that one might characterize as unequivocally negative.

"I'm going to give you a choice. In Option A, I will break one of your fingers."

"Ouch," you say.

"But I'm going to give you another choice. In Option B, I will toss a fair coin. If it comes up heads, you can leave unharmed. But if it comes up tails, I will break two of your fingers on the same hand."

Which would you choose?

Image 1.2 Probabilistic thug. Source: Flying Colours Ltd./Digital Vision/Getty Images.

Make a note of this number – call it A_{thug}. We'll come back to it later.

1.5 Basic Economics Terminology

While many of the concepts in this book were formulated by people who won Nobel Prizes in economics,[12] this isn't an economics book and an economics background isn't necessary to understand it. Here we'll give a brief background on some of the rudimentary economics terms that we'll use.

Let's start with a definition: **Economics is the study of how people allocate resources under uncertainty.** Resources include human effort, the results of human effort, and natural resources like water. Uncertainty here comprises both Knightian Risk and Knightian Uncertainty.

The sample spaces we will study will generally encompass a range of allocations of effort and resources. We will be focused on characterizing and dealing with the uncertainty of which allocations will be desirable and which allocations will be undesirable. That is, we will be concerned with the risks of allocation.

The allocation problem studied in economics is essentially a problem of human cooperation. Suppose there were only two people in the world: one a farmer and the other a hunter. The farmer alone may fail to survive a year in which the crop is infested by insects; the hunter alone may fail to survive a year in which game migrates elsewhere. But by pooling their efforts, they might survive on apples in a year when the crop is good and the hunting is bad. Next

12 Alfred Nobel, who died in 1896, didn't include economics in his list of prizeworthy fields. "In 1968, Sveriges Riksbank (Sweden's central bank) established the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel." (`www.nobelprize.org/prizes/facts/nobel-prize-facts/`). All the Nobel Prizes cited in this book are of this type.

year perhaps the apple crop will be bad, but since they cooperated, the hunter will still be alive to bring in enough wild boar meat so they can both survive. By cooperating and diversifying their efforts, they can better deal with the unknowns of the future food supply.

Diversification is central to the risk and portfolio management techniques that will be discussed in this book. Indeed, diversification is central to all of economics and human organization: it does far less good to allocate redundant resources than to allocate nonoverlapping resources. We saw this same idea in Joseph Tainter's observation about the Mayan food supply diversification techniques cited in the Preface.

Current world population is about 8 billion.[13] Human efforts have branched out far beyond farming and hunting. How is it decided which things should get done and which things shouldn't, and who should do what? How can the efforts of 8 billion people be choreographed?

While conspiracy theorists[14] believe otherwise, in fact there is no choreographer. There is no global central decision-making authority that allocates the efforts of the 8 billion people in the world. On the other hand, it is not the case that each person is entirely free to decide what to do. Certainly not in totalitarian countries where a central authority dictates behavior, but not even in countries that are nominally free. The world has a spectrum of systems to allocate human effort ranging from highly centrally planned to highly distributed.

While allocation systems vary widely in different parts of the world, virtually every economic system uses money as some part of the allocation process. If I wanted to become a professional opera singer, in most parts of the world there would be nothing preventing me from giving a recital where I promise to hit all nine high Cs in the aria "Pour mon âme"[15] from Donizetti's *La fille du régiment*. But since I have a terrible singing voice and wouldn't be able to hit a single one of them, no one would pay to attend my recital. If I devoted my efforts to giving recitals to which no one came, I would not have enough money for food, shelter, or clothing. Through (the lack of) money, I would receive the signal that I have to reallocate my efforts to something more useful and less painful to others.

Money isn't the only factor in allocation. If someone offers you a large amount of money to commit a crime, it is to be hoped that primarily ethics and secondarily fear of arrest will cause you to decline. So personal preferences and a moral, legal, and regulatory framework usually also play a big role in the allocation of effort.

13 U.S. and World Population Clock, September 30, 2022. www.census.gov/popclock/.

14 Conspiracy Theory: "a belief that some covert but influential agency (typically political in motivation and oppressive in intent) is responsible for an unexplained event." Under "conspiracy, n," OED Online. September 2022. Oxford University Press. www.oed.com/view/Entry/39766.

15 Anthony Tommasini, "Review: A Tenor Reaches 18 High Cs at the Metropolitan Opera," *The New York Times*, February 8, 2019. www.nytimes.com/2019/02/08/arts/music/review-metropolitan-opera-donizetti-fille-camarena.html.