NOETHER SYMMETRIES IN THEORIES OF GRAVITY

This volume summarizes the many alternatives and extensions to Einstein's General Theory of Relativity and shows how symmetry principles can be applied to identify physically viable models. The first part of the book establishes the foundations of classical field theory, providing an introduction to symmetry groups and the Noether theorems. A quick overview of general relativity is provided, including discussion of its successes and shortcomings; then several theories of gravity are presented, and their main features are summarized. In the second part, the "Noether Symmetry Approach" is applied to theories of gravity to identify those which contain symmetries. In the third part of the book, these selected models are tested through comparison with the latest experiments and observations. This constrains the free parameters in the selected models to fit the current data, demonstrating a useful approach that will allow researchers to construct and constrain modified gravity models for further applications.

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Noether Symmetries in Theories of Gravity

With Applications to Astrophysics and Cosmology

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CAMBRIDGE
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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

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www.cambridge.org
Information on this title: www.cambridge.org/9781009208741
DOI: 10.1017/9781009208727

First published 2023

A catalogue record for this publication is available from the British Library.

A Cataloging-in-Publication data record for this book is available from the Library of Congress.

ISBN 978-1-009-20874-1 Hardback

Cambridge University Press & Assessment
978-1-009-20874-1 — Noether Symmetries in Theories of Gravity
Francesco Bajardi , Salvatore Capozziello
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Dedicated to all women who,
like Emmy Noether,
are fighting for Science and Knowledge.

Contents

Preface

This book aims to provide evidence of the importance of symmetries in Gravitation and Cosmology. In particular, we pay attention to the search for symmetries in the context of theories of gravity, outlining how their existence contributes to solve dynamics, to select physically viable models, and to achieve, eventually, exact solutions. We will take into account large classes of extensions and alternatives to Einstein's General Relativity (GR) considering, inside the gravitational action, functions of the Ricci scalar and other curvature invariants, higher-order geometric terms, minimal and non-minimal couplings to scalar fields, torsion, and non-metricity invariants. As often pointed out throughout the text, in particular in Parts I and II of the book, there are several physically motivated reasons to consider alternatives and extensions to GR. Indeed, although GR obtained a lot of confirmations and successes in astrophysics and cosmology, it suffers some shortcomings from the theoretical and experimental points of view that do not allow one to obtain a comprehensive and self-consistent picture of gravitational interaction, and then of cosmology, ranging from ultraviolet (UV) to infrared (IR) scales. Alternative and extended theories of gravity aim to fix such problems by relaxing some of the hypotheses lying behind Einstein's theory. Specifically, testing theories of gravity through observational cosmology is a good starting point in order to find consistent models and discard others. However, this approach cannot be satisfactory from a fundamental physics point of view. In fact, since the final form of the gravitational interaction is substantially unknown, we need general selection criteria, based on first principles, to constrain the starting action. The existence of Noether symmetries can be a useful approach to this issue for several reasons. For example, extending GR leads to hardly solvable field equations, so that, several times, the given theory ends up losing any predictive power. In this perspective, in order to reduce dynamics, it is possible to introduce the so-called *Noether Symmetry Approach*, by means of which theories of gravity containing symmetries can be selected. The corresponding conserved quantities can be used to reduce dynamics.

It is interesting to point out that, if symmetries exist, the related conserved quantities often have a physical meaning, as we will discuss throughout the book. To probe this statement, in Part II of the book, we apply the Noether Symmetry Approach to several theories of gravity, focusing on cosmological backgrounds and deriving exact solutions. Furthermore, Noether symmetries can be searched for in spherical and cylindrical symmetry, providing black hole solutions with further features besides the Schwarzschild radius.

It is worth noticing that the Noether approach can also be used in Hamiltonian formalism in view of Quantum Cosmology applications. In this perspective, it is possible to demonstrate that the existence of Noether symmetries allows one to select "observable" universes, according to the Hartle criterion, which means that a *Wave Function of the Universe*, giving rise to physically viable initial conditions, can be obtained.

The book is organized in three parts. Part I is devoted to preliminary concepts. We introduce the notion of symmetry group, conserved quantity, and the Noether Theorem. Then we discuss some applications of the latter, as well as the free-particle Lagrangian, the harmonic oscillator, the Dirac and electromagnetic fields, as well as the invariance under translations and rotations. In the two final chapters of the Part I, we outline the main aspects of some modified theories of gravity, pointing out the differences with respect to GR. In Chapter 5 we outline the *Canonical Quantization* of gravity, focusing on the application to a homogeneous and isotropic universe and constructing the formalism of *Quantum Cosmology*. We also analyze the link between Noether symmetry and canonical quantization, as well as the capability of the former to select physically viable models.

In Part II, we show that the First Noether Theorem can be formalized as a method to select models with symmetries, even without knowing a priori the form of the symmetry generator. Then, we use the Noether Symmetry Approach to select exact solutions for cosmological models. Specifically, in Chapter 7, we consider geometric extensions of GR, Teleparallel Equivalent of General Relativity, and Symmetric Teleparallel Equivalent of General Relativity, showing, in other words, that the methods work for any representation of gravity. In Chapters 8 and 9 second-order curvature invariants and higher-order terms in the gravitational action are respectively treated. In Chapter 10, non-minimal couplings between geometry and scalar fields are considered.

In Chapter 11, we deal with nonlocal terms in curvature and torsion in gravitational action. In fact, it is worth noticing that nonlocality can play a major role in selecting effective theories useful for both Quantum Gravity and Cosmology, as we will discuss. As reported, Noether symmetries are a useful criterion to fix such nonlocal terms. In Chapter 12 the Noether Symmetry Approach is considered within the context of Bianchi Universes. Therefore, starting from a scalar-tensor action (where the scalar field is minimally coupled with the geometry), several symmetries are provided. These results are useful to show that more than one Noether symmetry can be achieved for the same point-like Lagrangian. In Chapter 13, we apply the Noether approach to spherically symmetric Lagrangians in order to find out black hole solutions. In particular, we focus on $f(R)$, $f(T)$, and

$f(\mathcal{G})$ theories respectively in Sections 13.1, 13.2 and 13.3, and show how it is possible to generate the axial symmetry from the spherical one.

In the Part III, we conclude by considering applications of theories of gravity to classical and quantum cosmology, and to astrophysics through the Noether symmetries. Specifically, we show that the range of parameters from Solar System tests, stellar mass-radius diagram, and galaxy rotation curves can be fixed by the presence of Noether symmetries. It is worth noticing that not all models selected by the Noether approach are necessarily viable because some of them could suffer theoretical shortcomings or could be ruled out by experiments. However, the criterion of searching for symmetries allows one to select classes of reliable models that can eventually be constrained by experimental observations. The conserved quantities allow one, indeed, to reduce the complexity of the field equations and to find solutions whose physical meaning can be, in principle, investigated.

This book is not meant to be only a summary of solutions of theories of gravity with symmetries, but it is aimed at stressing the importance of symmetries, at comparing cosmological and astrophysical solutions of different theories, and at developing a coherent treatment that emphasizes the importance of symmetries in Physics of Gravitation. All theories discussed here take into account *scalar fields* of different origin. These scalar fields are often geometric invariants like curvature R, torsion T, or the Gauss–Bonnet term $\mathcal{G}$. The intent is to show that different dynamical (and geometrical) quantities can be related to each other, providing equivalent descriptions for the gravitational interaction. Therefore, comparing their symmetries and solutions in different backgrounds can constitute a sort of paradigm to address shortcomings of gravitational interaction at any scale of energy. This could contribute to constructing a comprehensive cosmic history from UV to IR epochs. Clearly, the discussion is sometimes not complete. We apologize for this and refer to the literature reported in the bibliography for detailed information.

Acknowledgments

We want to acknowledge many collaborators and colleagues who helped us to develop the topics presented in this book with useful exchanges of ideas, discussions, and suggestions.

One of us (SC) wants to dedicate this book to the beloved memory of Ruggiero de Ritis who, with the patience and dedication of a mentor and a friend, introduced him to the amazing world of symmetries.

Furthermore, we acknowledge J. Alcaniz, L. Amendola, K. Atazadeh, S. Bahamonde, K. Bamba, J. D. Barrow, F. Bascone, S. Basilakos, M. Benetti, T. Bernal, D. Borka, V. Borka Jovanović, A. Borowiec, W. G. Boskoff, M. Bouhmadi-López, U. Camci, V. F. Cardone, M. Capriolo, S. Carloni, R. Cianci, G. Cristofano, R. D'Agostino, F. Darabi, V. De Falco, A. De Felice, M. De Laurentis, C. Deliduman, I. De Martino, M. Demianski, K. F. Dialektopoulos, P. K. S. Dunsby, E. Elizalde, H. Farajollahi, V. Faraoni, L. Fatibene, M. Francaviglia, N. Frusciante, R. Garattini, G. Gionti, T. Harko, M. Jamil, J. C. Hidalgo, P. Jovanović, T. S. Koivisto, G. Lambiase, J. Levi Said, L. Lobato-Graef, F. S. N. Lobo, O. Luongo, C. A. Mantica, V. I. Man'ko, A. A. Marino, G. Marmo, P. Martin-Moruno, S. Mendoza, J. P. Mimoso, L. G. Molinari, H. Motavali, R. Myrzakulov, G. L. Nashed, S. Nesseris, S. Nojiri, A. N. Nurbaki, F. Occhionero, S. D. Odintsov, V. Oikonomou, G. J. Olmo, A. Paliathanasis, M. Paolella, L. Perivolaropoulos, E. Piedipalumbo, A. Ravanpak, M. Roshan, C. Rubano, V. Salzano, E. N. Saridakis, P. Scudellaro, A. Stabile, C. Stornaiolo, S. V. Sushkov, A. Troisi, M. Tsamparlis, S. Tsujikawa, Z. Urban, D. Vernieri, S. Vignolo, and A. M. Wojnar.

We wish to thank particularly the Editorial Director of Cambridge University Press, Simon Capelin, who proposed us to write the book, and the Commissioning Editors, Nicholas Gibbons and Vince Higgs, for their continuous support and suggestions that allowed us to realize this book.

Amalie Emmy Noether: A Life for Mathematics

Amalie Emmy Noether in 1905

Amalie Emmy Noether can be considered as one of the most prominent mathematicians of all time. With the theory of algebraic invariants and her theorems on symmetries and conservation laws, she revolutionized Mathematics and Physics.

She was born on March 23, 1882 in the German city of Erlangen. After completing secondary school, she could not immediately access the University, which was then closed to women. She managed to do so only in 1904 when the University of Erlangen definitively opened the doors to female enrollment. She quickly recovered the lost time, graduating in just three years with a thesis on the theory of algebraic invariants. The difficulties did not end there. After moving to Göttingen, she lived for more than ten years with various jobs, semiofficial and poorly paid lessons, and scientific assistance to some established professors. Despite the difficulties, she collaborated with the top-level mathematicians of the epoch, such as Felix Klein, David Hilbert, and Hermann Weyl, and, finally, she discovered some of the mathematical bases of Einstein's GR.

Emmy Noether was already distinguished by addressing several open questions of Mathematics, but it was the publication of two original theorems in the field of the calculus of variations, which we call *Noether's First and Second Theorems*, that brought her to the attention of the scientific community in 1918. By these results, Emmy Noether was capable of demonstrating the deep link between symmetries and conservation laws. In other words, Noether's theorems state that conserved quantities correspond to symmetries of Lagrangians defining dynamics of physical systems ranging from Classical to Quantum Mechanics up to GR and Quantum Field Theory. The theorems not only have important theoretical implications: they also offer a practical tool to derive the conserved quantities starting from the symmetries observed in a physical system. If a new theory is proposed, the Noether Theorems guarantee that, if symmetries exist, conserved quantities also must exist. In some sense, these theorems represent also a sort of protocol for experiments: the theory is proved true, if conserved quantities are observed.

The impact of the Noether Theorems in GR is enormous. After the Einstein and Hilbert formulation, many problems remained open. In particular, the lack of local energy conservation challenged the whole theory. Hilbert pointed out "the failure of the energy theorem" in GR, not being able to demonstrate the local energy conservation. The problem was so serious that it directly questioned the foundations of the theory. Emmy Noether, in a short time, provided a brilliant solution that had wide-reaching consequences not only for GR but also for the whole evolution of modern Physics, as Hilbert himself acknowledged in 1924.

Despite the relevance of her research, the doors of the academy did not open for Emmy Noether. Her main work on variational invariants was presented in 1918, at a meeting of the Royal Society of Sciences in Göttingen by her tutor Felix Klein. She was not only not admitted as a member of the Society but, furthermore, being a woman, she was not allowed to be present in the audience.

In 1919, she applied again for a teaching position at Göttingen. She had been refused years before, despite support by Hilbert. However, thanks to her outstanding results, she obtained the position. She slowly managed to get out of her "gender precariousness." Within a few years, she began to get paid for her lessons, which attracted so many students from all over Germany, and which she had long been forced to offer for free and on behalf of her mentor, David Hilbert, who concretely helped her despite the prejudices. It is worth noticing that, until 1923, she never had a proper salary.

Besides her famous theorems, Noether's research works concerned areas of Mathematics other than the calculus of variations, and most of them were developed during these fruitful years. In the meantime, the "Göttingen School," became attractive for the most prestigious physicists and mathematicians coming from all over Europe. Noether's contributions were outstanding, in particular, in abstract algebra: from the discovery of rings, today called Noetherian Rings, to the noncommutative algebra, passing through the theory of ideals and the algebraic theory of numbers. In some way, most of the language and results of modern Algebra can be attributed to Emmy Noether or traced from her fundamental contributions. Her students, the "Noether Boys," gave her the affectionate nickname of "Mother Algebra."

Despite the successes, her career was suddenly broken by the advent of Nazism. She was of Jewish origin and, in 1933, she was forced to flee to the United States, where she got a position at Bryn Mawr College in Pennsylvania. But once again, being a woman, she was prevented from accessing the most prestigious research centers, such as Princeton University, where Einstein, Weyl, and Gödel were welcomed at the same time.

She died on April 14, 1935 from the consequences of an infection contracted during a surgical operation for the removal of a carcinoma. Einstein wrote an obituary for her in *The New York Times*: "Noether was the most significant creative mathematical genius thus far produced since the higher education of women began." Weyl defined her as the most important woman in the history of Mathematics, equivalent to what Marie Curie had been for Physics. Starting from the 1950s, Noether's theorems were recognized as a milestone for Physics, and "Fräulein Emmy" is considered an absolute genius of Mathematics for all time.

Notation

We will set $\hbar = c = 8\pi G = 1$ unless otherwise indicated and we will use the following notation:

1. For the indexes:
   - Greek indexes $\{\alpha, \beta, \gamma ... = 0,1,2,3\} \rightarrow$ label the four-dimensional curved space-time coordinates,
   - Latin indexes $\{a, b, c... = 0,1,2,3\} \rightarrow$ label the four-dimensional flat space-time coordinates,
   - Middle indexes $\{i, j, k... = 1,2,3\} \rightarrow$ label the spatial coordinates in curved space-time,
   - Symmetrization over the indexes will be indicated by the curly brackets, and anti-symmetrization by the square brackets.
2. Let A_μ be a generic four-vector, we adopt the following notation:
   - $D_\nu A_\mu = A_{\mu;\nu}$ is the covariant derivative in terms of the Levi–Civita connection,
   - $\partial_\nu A_\mu = A_{\mu,\nu}$ is the partial derivative,
   - Christoffel connections will be indicated equivalently by $\Gamma^\alpha_{\beta\gamma}$ or $g^{\alpha\sigma}\{\sigma, \beta\gamma\}$,
   - $\nabla_\nu A_\mu$ is the covariant derivative in terms of any connection except for the Levi–Civita connection.
3. We use the symbol $\mathscr{L}$ for Lagrangian density, while the Lagrangian will be denoted by $\mathcal{L}$.
4. For the Einstein tensor, we use the notation $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, with $R_{\mu\nu}$ being the Ricci tensor and $R = R^\mu_{\ \mu}$ the Ricci scalar.
5. The derivative with respect to a given variable will be indicated by the subscript variable or sometimes by the subscript variable in the partial derivative.
6. Derivatives of a given function f with respect to temporal and spatial coordinates will be denoted by $\dot{f}$ and f', respectively, unless otherwise indicated.
7. D labels the number of space-time dimensions, while spatial dimensions are denoted by d.

8. $\Box$ stands for the d'Alembert operator $\Box = g_{\mu\nu}\nabla^\mu\nabla^\nu$.
9. X represents the generator of a certain symmetry, while $X = X + \dot{\eta}^i \partial_{\dot{q}^i}$ is the Noether vector.

The adopted metric signature is $(+, -, -, -)$, unless otherwise indicated.

We will introduce other particular symbols during the discussion.

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Cambridge University Press & Assessment
978-1-009-20874-1 — Noether Symmetries in Theories of Gravity
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Acronyms

- ADM: *Arnowitt–Deser–Misner*
- AdS: *Anti de Sitter*
- CFT: *Conformal Field Theory*
- CMB: *Cosmic Microwave Background*
- DEC: *Dominant Energy Condition*
- EH: *Einstein–Hilbert*
- EL: *Euler–Lagrange*
- EoS: *Equation of State*
- FLRW: *Friedman–Lemaître–Robertson–Walker*
- GR: *General Relativity*
- GWs: *Gravitational Waves*
- IDGs: *Infinite Derivative Theories of Gravity*
- IKGs: *Integral Kernel Theories of Gravity*
- IR: *Infrared*
- LG: *Lorentz Group*
- LT: *Lorentz Transformation*
- NEC: *Null Energy Condition*
- PN: *Post-Newtonian*
- PPN: *Parametrized Post-Newtonian*
- QFT: *Quantum Field Theory*
- RLG: *Restricted Lorentz Group*
- SEC: *Strong Energy Condition*
- STEGR: *Symmetric Teleparallel Equivalent of General Relativity*
- SUSY: *Supersymmetry*
- TEGR: *Teleparallel Equivalent of General Relativity*
- TOV: *Tolman–Oppenheimer–Volkoff*
- UV: *Ultraviolet*
- WDW: *Wheeler–DeWitt*
- WEC: *Weak Energy Condition*