Quantum Models of Cognition and Decision

Quantum Models of Cognition and Decision, Second Edition presents a fully updated and expanded version of this innovative and path-breaking text. It offers an accessible introduction to the intersection of quantum theory and cognitive science, covering new insights, modeling techniques, and applications for understanding human cognition and decision making. In it, Busemeyer and Bruza delve into such topics as the noncommutative nature of judgments, quantum interference as a general principle governing human decision making, contextuality in modeling human cognition, and thought-provoking speculation about what a quantum approach to cognition might reveal about the ultimate nature of the human mind. Additions include new material on measurement, open systems, and applications to computer science. Requiring no prior background in quantum physics, this book comes complete with a tutorial and fully worked-out applications in important areas of cognition and decision.

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Quantum Models of Cognition and Decision

Principles and Applications

SECOND EDITION

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老子云：“有物混成，先天地生”.....。

Laozi said: Something formed out of the chaos first, and it existed before the heaven and earth were created....

The first author dedicates this book to his wife, Meijuan Lu, who has always encouraged and supported him.

The second author dedicates this book to C. J. van Rijsbergen and S. Malin.

Contents

Preface

Preface to the Second Edition

The following rationale was written more than a decade ago, and is still largely intact. In the intervening period quantum cognition has matured into a field that is now recognized in its own right within cognitive psychology. It has gained attention from researchers from allied fields wishing to apply and further develop the ideas that were set out in the first edition. These have primarily been computer scientists, but also (computational) linguists, philosophers, economists, and even management scientists. Somewhat ironically, researchers inspired by quantum cognition have made a mark by subsequently publishing in the physics literature. When the first edition was published, we had no inkling that such developments would take place.

In this the second edition of the book, we have completely rewritten and reorganized all of the chapters. Although we cover many of the same topics presented in the first edition, we include many of the advances that have been made during the past decade since the first edition appeared. To make the second edition more accessible, while at the same time duly representing the advances, we alternate chapters containing elementary/conceptual presentations with chapters presenting more advanced material. An introductory reading would cover Chapters 1, 2, 4, 6, 8, 10, 12, and 14. More advanced topics are presented in Chapters 3, 5, 7, 9, 11, and 13.

The literature of quantum cognition and related areas has grown formidably large. Consequently, we are conscious that this second edition is inevitably incomplete, and intend no disrespect to the research that we have missed, or didn't cover.

Rationale

The purpose of this book is to introduce the application of quantum theory to cognitive and decision scientists. At first sight it may seem bizarre, even ridiculous, to draw a connection between cognition and decision making – research lying within the realm of day-to-day human behavior – on the one hand, and quantum mechanics – a highly successful theory for modeling subatomic phenomena – on the other. Yet there are good scientific reasons for doing so, which is leading a growing number of researchers to examine quantum theory as a way to understand perplexing findings and stubborn problems within their own fields; hence this book. Given the nascent state of this field, some words of justification are warranted. The research just mentioned is not concerned with modeling the brain using quantum mechanics, nor is it directly concerned with the idea of the brain as a quantum computer. Instead it turns to quantum theory as a fresh conceptual framework for explaining empirical puzzles, as well as a rich new source of alternative formal tools. To convey the idea that researchers in this area are not doing quantum mechanics, various modifiers have been proposed to describe this work such as quantum-like models of cognition, cognitive models based on quantum structure, or generalized quantum models.

There are two aspects of quantum theory which open the door to addressing problems facing cognition and decision in a totally new light. The first is known as “contextuality” of judgments and decisions, which is captured in quantum theory by the idea of “interference”: The context generated by making a first judgment or decision interferes with subsequent judgments or decisions to produce order effects, so that judgments and decisions are non-commutative. The second aspect relates to quantum entanglement. Entanglement is a phenomenon whereby making an observation on one part of the system instantaneously affects the state in another part of the system, even if the respective systems are separated by space-like distances. The crucial point about entanglement relevant to this book is that entangled systems cannot be validly decomposed and modelled as separate subsystems. This opens the door to developing quantum-like models of cognitive phenomena which are not decompositional in nature. For example, the semantics of concept combinations would seem to be non-compositional, and quantum theory provides formal tools to model these as non-decomposable interacting systems. Similar applications appear in human memory. Most models consider words as separate entities; new models are made possible by going beyond this assumption and, for example, modeling a network of word associates as a non-decomposable system.

It is important to note the authors are agnostic toward the quantum mind hypothesis, which assumes there are quantum processes going on in the brain. We motivate the use of quantum models as innovative abstractions of existing problems. That's all. These abstractions have the character of idealizations in the sense that there is no claim as to the validity of the idealization "on the ground." For example, modeling concept combinations as quantum entangled particles involves no claim as to whether there is associated physical entanglement going on somewhere in the brain. This may seem like an easy way out, but is not that different than idealizations employed in other areas of science. For example, in neural dynamical models of the brain, continuous state and time differential equations are used to model growth of neural activation, even though actually there are only a finite number of neurons and each one only fires in an all-or-none manner. In short, we apply mathematical structures from quantum mechanics to cognition and decision without attaching the physical meaning attributed to them when applied to the human behavioral phenomena. In fact, many areas of inquiry that were historically part of physics are now considered part of mathematics, including complexity theory, geometry, and stochastic processes. Originally they were applied to physical entities and events. For geometry this was shapes of objects in space. For stochastic processes, this was statistical mechanics of particles. Over time they became generalized and applied in other domains. Thus what happens here with quantum mechanics mirrors the history of many if not most branches of mathematics.

The cognitive revolution that occurred in the 1960s was based on classical computational logic, and the connectionist/neural network movements of the 1970s were based on classical dynamic systems. These classical assumptions remain at the heart of both cognitive architecture and neural network theories, and they are so commonly and widely applied that we take them for granted and presume them to be obviously true. What are these critical but hidden assumptions upon which all traditional theories rely? Quantum theory provides a fundamentally different approach to logic, reasoning, probabilistic inference, and dynamic systems. For example, quantum logic does not follow the distributive axiom of Boolean logic; quantum probabilities do not obey the law of total probability; quantum reasoning does not obey the principle of monotonic reasoning; and quantum dynamics can evolve along several trajectories in parallel rather than be slave to a single trajectory as in classical dynamics. Nevertheless, human behavior itself does not obey all of these restrictions. This book will provide an exposition of the basic assumptions of classical versus quantum models of cognition and decision theories. These basic assumptions will be examined, side by side, in a parallel and elementary manner. For example, classical systems assume that measurement merely observes a pre-existing property of a system; in contrast, quantum systems assume that measurement actively creates the existence of a property in a system. The logic and mathematical foundation of classical and quantum theory will be laid out in a simple and elementary manner that uncovers the mysteries of both theories. A view of classical theory will emerge to be seen as a possibly overly restrictive case of the more general quantum theory. The fundamental implications of these contrasting assumptions will be examined closely with concrete examples and applications to cognition and decision making. New research programs in cognition and decision making, based on quantum theory, will be reviewed.

Book Chapters

Chapter 1 provides the motivation for why one might be interested in applying quantum theory to cognition and decision making. Chapter 2 provides a simple and intuitive introduction to the basic ideas of quantum probability theory, alongside with a comparison with classical probability theory. Only linear algebra is needed for this introduction, which will be presented and explained in a simple tutorial manner and no background in physics is required. Chapter 3 provides a more advanced and rigorous review of quantum probability theory along with recent mathematical advances in the field of quantum cognition. Chapter 4 provides a variety of interesting applications using simple calculations, and computer programs that provide clear and concrete ideas about how to use quantum theory to compute probabilities for cognitive and decision making applications. Chapter 5 presents more advanced and rigorous applications of quantum probability to these topics. Chapter 6 provides an elementary introduction to quantum dynamics and makes a side-by-side comparison with Markov dynamics. Chapter 7 describes more advanced dynamics including open systems that combine quantum and Markov dynamics. Chapter 8 describes information processing using controlled-U gates as well as an example application to judgmental heuristics. Chapter 9 describes three different ways to perform learning with quantum systems. Chapter 10 describes the fundamental issue of contextuality and presents empirical research on the contextuality of human judgments. Chapter 11 presents some directions for developing quantum applications to language. Chapter 12 presents quantum variants of the well-known Bayesian networks as well as quasi-probabilities. Chapter 13 reflects on some philosophical implications of quantum cognition. The final chapter looks toward the future.

In our experience thus far, people either love or hate these ideas, but no one remains unaffected. We challenge you to make up your own mind.

Other Matters

The appendices contain a list of the notation we use and proofs of two key results, as well as some computer code for models that are not too complex. In the text, we provide boxes that describe simple code in both Matlab and Julia (the latter is very similar to Matlab). The boxes are numbered x.y.z where the letter x refers to the chapter, y refers to the section, and z refers to the subsection. A website that converts the Matlab to Python is located at

https://bit.ly/4cSe7QV

For longer and more complex programs written mostly in Matlab the reader is referred to

https://jbusemey.pages.iu.edu/quantum/HilbertSpaceModelPrograms.htm

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