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K-Theory and Representation Theory

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Preface

In July 2021, three of us, Nigel Higson, Roger Plymen, and Haluk Şengün, organized an online summer school on *K*-theory and representation theory. The original intention was for this to be an in-person event. However, the constraints of the pandemic forced us to have the event online.

The lecturers, Anne-Marie Aubert, Peter Hochs, Bram Mesland, and Hang Wang, gave their minicourses comprising four lectures each. Much time has elapsed, and the lecturers have taken the opportunity to considerably expand their lecture notes into their present forms. It is our sincere hope that these lecture notes will prove useful to beginning graduate students who wish to enter this area, particularly because much of the material collated here is scattered around the literature.

In Chapter 1, Mesland and Şengün set the scene by describing the basics of the theory of C^* -algebras associated with locally compact groups. The important notions of Hilbert C^* -modules, C^* -correspondences, and their associated generalized induction functors are discussed. After the introduction of Morita equivalence, examples of C^* -correspondences that capture well-known constructions from representation theory are given.

The authors then move onto a detailed discussion of *K*-theory and its powerful generalization *KK*-theory, finishing with a basic discussion of Dirac induction. This is hopefully a good foundation for Chapter 3. A first course in functional analysis is a useful prerequisite.

In Chapter 2, Hochs describes the classification of tempered representations of semisimple Lie groups. The treatment starts from the very basic notions, supporting the reader with concrete examples throughout the journey. The structure theorem of the reduced group C^* -algebra of a real linear reductive group and its *K*-theory is described based on the recent accounts of Nigel Higson and collaborators.

In Chapter 3, Wang describes with many examples the role of the Dirac operator in the representation theory of semisimple Lie groups. This topic has a long history with roots going back to holomorphic induction. She develops the subject carefully with links to the previous chapters. A first course in differential geometry would be a useful prerequisite.

In Chapter 4, Aubert presents an essentially self-contained introduction to the representation theory of p -adic groups. After a formulation of the Aubert–Baum–Plymen–Solleveld conjecture, her chapter culminates in a discussion of her recent work with Afgoustidis, which provides, under a suitable hypothesis, a structure theorem for the reduced group C^* -algebra of a (linear) reductive group over any local field of characteristic zero.