An Elementary Course on Partial Differential Equations

Differential equations are very important due to the extensive applications in various fields such as pure and applied mathematics, physics, engineering, biology, and economics. At present differential equations form the foundation of mathematical modelling applied to solving real-life problems that may not be solved directly. This subject is a part of higher mathematics curriculum and taught at undergraduate and postgraduate levels. Differential equations are taught as two courses, namely ordinary differential equations and partial differential equations.

This book is designed to serve as a textbook for the first course on partial differential equations, which is often taught after a first course on ordinary differential equations. There are numerous books on this subject but most such standard books run very quickly through the elementary partial differential equations, which is not sufficient for undergraduate students. The authors have utilized their teaching experience of several years to fill this gap wherein special attention is paid to elementary partial differential equations. This book covers important techniques such as Lagrange's method, Charpit's method, Jacobi's method, Monge's method, Monge–Ampere type non-linear equations, Fourier method, reduction of first and second order linear equations in canonical forms, derivation of first and second order equations, higher-order equations with constants coefficients, and their reduction to variable coefficients, and so on.

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To our parents and teachers

Contents

Preface

This book is designed to serve as a textbook for the first course on partial differential equations. After the introduction of the semester system in various universities of India, normally there are two courses of differential equations at undergraduate levels, one is on ordinary differential equations (ODEs), whereas the other is exclusively on partial differential equations (PDEs). Generally, most of the books used at the undergraduate level include ODEs as well as PDEs, and all such books emphasise more on ODEs, which, consequently, affect the coverage of the PDEs. On the other hand, all the standard textbooks on PDEs cover the contents of graduate as well as postgraduate levels, wherein often undergraduate materials are discussed in haste. Overall, it is necessary to write a separate book exclusively for undergraduate students of Indian as well as foreign universities. With a view to have a detailed discussion on elementary topics of PDEs, we endeavour to write this book.

In fact, this book is based on our lecture notes, which we have given to our students at Aligarh Muslim University (AMU) over the last several years. We hope that this book will meet the requirement as well as expectations of undergraduate students. While preparing this book, we have examined deeply the syllabi of all such courses of B.Sc. (Mathematics) of all Indian universities. The course contents covered by this book can be described as the union of all syllabi prescribed by various Indian universities as well as UGC curriculum up to undergraduate level. The course contents are presented in such a manner that they can be equally useful in various competitive examinations such as UPSC, UGC-CSIR NET, and GATE. On the other hand, from the application point of view, the book also contains the relevant contents of Mathematical Physics/Engineering Mathematics/Applied Mathematics, which are the parts of course contents in B.Sc. (Physics) as well as B.E./B.Tech. We attempt to strike a balance between theory and problems. Consequently, our book remains equally useful to both pure as well as applied mathematicians. We have made every attempt to have a simpler and lucid presentation without scarifying theoretical rigour. The book provides different types of examples, updated references, and applications in diverse fields. All methods of solution and necessary concepts are arranged in the form of theorems. The proofs of theorems may be omitted for an undergraduate course or a course in other disciplines.

This book consists of six chapters organised in a natural order. The topics are discussed in a systematic way. The aim of the first chapter is an attempt to make this book as self-contained as possible, wherein we have collected the necessary background material from ODEs, multivariable calculus, and geometry required for the subsequent chapters. Chapter 2 deals with basic concepts and necessary notions regarding PDEs and their originations. The object of Chapter 3 is to study some basic techniques to solve certain types of PDEs. Chapters 4, 5, and 6 are devoted, respectively, to first-, second-, and higher order PDEs.

All suggestions and constrictive criticisms for further improvement of the book will be received thankfully by us (at emails aafu.amu@gmail.com and mhimdad@gmail.com) to serve the cause of imparting correct and valuable information to learners.

Aftab Alam
Mohammad Imdad

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Aftab Alam
Mohammad Imdad

Symbols

NOTATIONS

$\neq$	non-equality
$:=$	equal by definition
$\equiv$	equivalence
$\varnothing$	empty set
$\mathbb{N}$	the set of natural numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	n-dimensional Euclidean space
$\in$	belongs to
$\notin$	does not belong to
$\exists$	there exists
$\Rightarrow$	implies
$\Leftrightarrow$	logical equivalence

ACRONYMS

iff	if and only if
w.r.t.	with respect to
Eq., Eqs	equation, equations
PDE, PDEs	partial differential equation/equations
ODE, ODEs	ordinary differential equation/equations
BC	boundary condition
IC	initial condition
BVP	boundary value problem
IVP	initial value problem
IBVP	initial-boundary value problem
LHS	left-hand side
RHS	right-hand side
AE	auxiliary equation
CF	complementary function
PI	particular integral
IF	integrating factor