

# Index

- abelian group, 3
- abelian Lie algebra, 99
- adjoint representation, 12
- adjoint transformation rule for  $SO(3)$  UR's, 63
- alternating subgroup, 5, 7, 30
- associativity as a system of (nonlinear) PDE's, 85
- automorphism, 7
- automorphism among Lie algebras, 99
- axis-angle parametrisation of  $SO(3)$ , 57
- axis-angle parametrisation of  $SU(2)$ , 69
  
- Baker–Campbell–Hausdorff formula, 98
- Bargmann invariants, 216
- Bargmann representation of the CCR's, 165
- Bargmann's construction of all the UIR's of  $SU(1, 1)$ , 271
- Braid relations, 30
- Brauer–Weyl construction, 146
  
- canonical coordinates of first kind, 87
- canonical subgroup chain, 141
- canonical transformations, 10
- Cartan subalgebra basis, 115
- Cartan's theorem, 109
- Cartan–Weyl form, 112
- Cartan–Weyl structure, 111
- Cartesian tensors, 66
- Casimir polynomials, 140
- character of a representation, 13
- characters of  $SO(2)$ , 54
- characters of  $SO(3)$ , 64
- characters of  $SU(2)$ , 76
- class function, 14, 22
- classification of CSLA's, 114
- Clebsch–Gordan coefficients, 24
- Clebsch–Gordan problem, 23
- Clebsch–Gordan series, 24
- column subgroup, 37
  
- commutative (noncommutative) ring, 24
- commutator subalgebra, 100
- commutator subgroup, 6
- compact group, 11
- complete reduction, 16
- complete reduction of the two regular representations, 163
- completely symmetric functions, 46
- complex conjugate representation, 12
- complex extension of Lie algebras, 108
- complex form of  $Sp(2n, \mathbb{R})$ , 195
- complex Lie algebras, 107, 108
- complex simple Lie algebras, 107
- complexification of real Lie algebras, 107
- conjugacy class, 4
- conjugacy classes of  $A_n$ , 32
- conjugacy classes of  $S_n$ , 31
- conjugacy classes of  $SO(3)$ , 60
- conjugacy classes of  $SU(2)$ , 73
- conjugate partition, 33
- conjugate subgroup, 5
- contagredient representation, 12
- coset space  $SO(3)/SO(2)$ , 61
- cosets, 6
- covariance condition, 244
- covariant description of Lie algebras, 114
- cycle, 5
- cycle of eight structure for spinor UIR's, 153
- cycle structure, 5
  
- decomposable representation, 14
- decomposable representation of a Lie algebra, 104
- diagonal representation, 175
- diagonal representation for operators, 181
- dimension of a representation, 10
- Dirac algebra, 149

- Dirac's unitary expansor representations of  $SO(3, 1)$ , 261
- direct product, 9
- direct product representation, 13
- direct sum representation, 13
- dominant weight, 132
- Dynkin diagram, 114
- elementary symmetric functions, 46
- elementary UIR's, 139
- equivalent representations, 11
- essentially nonorthogonal representation, 11
- Euclidean group  $E(3)$ , 235, 237
- Euler angle parametrisation of  $SO(3)$ , 58
- Euler angle parametrisation of  $SU(2)$ , 69
- exceptional groups, 129
- exponential map, 89, 98
- factor algebra, 100
- factor group, 6
- faithful representation, 10
- finite dimensional irreducible representations of  $SL(2, \mathbb{C})$  and  $SO(3, 1)$ , 259
- Fubini–Study metric, 215
- fundamental highest weights, 134
- fundamental UIR's, 134
- Galilei group  $\mathcal{G}$ , 235, 245
- gamma-matrices, 146
- generalised coherent states a la Klauder, 176
- generalised coherent states a la Perelomov, 176
- generalised Huyghens kernel, 193
- generators of  $E(3)$  and their commutation relations, 238
- generators of  $Mp(2)$ , 191
- generalised Huyghens kernel in  $n$  dimensions, 204
- global decompositions of  $Sp(2n, \mathbb{R})$ , 198
- group, 1
- group  $O(2)$ , 55
- group  $O(3, 1)$ , 254
- group  $SL(2, \mathbb{C})$ , 252, 257
- group  $SO(2)$ , 53
- group  $SO(3)$ , 56
- group  $SO(3, 1)$ , 253
- group  $SU(2)$ , 68
- group algebra, 24
- group of permutations, 2
- groups related to spacetime, 235
- harmonic oscillator coherent states, 174
- Heisenberg CCR's, 186
- Heisenberg–Weyl group, 176
- helicity, 241
- highest weight, 132
- highest weight vector, 134
- homogeneous Euler parametrisation of  $SU(2)$ , 69
- homogeneous Lorentz group  $SO(3, 1)$ , 235, 252
- homomorphism, 8
- homomorphism among Lie algebras, 99
- homotopy group of  $SO(3)$ , 60
- hook-length formula, 34
- idempotent elements in a ring, 25
- indecomposable representation, 14
- indecomposable representation of a Lie algebra, 104
- induced representations, 160
- inducing construction, 170
- inequivalent representations, 11
- inner automorphism, 8
- integrability conditions, 90
- intertwining operator, 15
- invariant integration over  $SO(3)$ , 61
- invariant integration over  $SU(2)$ , 73
- invariant Lie subalgebra, 100
- invariant subgroup, 6
- invariant uncertainty principles, 205
- irreducible characters of  $S_n$ , 42
- irreducible representation, 14
- irreducible representation of a Lie algebra, 104
- irreducible representations of  $S_n$ , 32, 42
- irreducible submodules, 25
- irreducible tensor operator, 236
- isomorphism, 8
- isomorphism among Lie algebras, 99
- Iwasawa decomposition, 187
- Jacobi condition, 93
- k-cycle, 28
- kernel of homomorphism, 8
- Kostka numbers, 46
- Lagrange's theorem, 6
- left and right regular representations, 21
- left coset, 6
- left handed (right handed) spinors, 148
- lens subgroup, 198
- Levi splitting theorem, 109
- Levi–Civita tensor, 155

- Lie algebra, 81, 93
- Lie algebra  $SL(2, \mathbb{C})$ , 258
- Lie algebra in the defining representation of  $Sp(2n, \mathbb{R})$ , 200
- Lie algebra of  $\mathcal{P}$ , 263
- Lie bracket, 93
- Lie group, 81
- Lie subalgebra, 100
- lightlike UIR's of  $\mathcal{P}$ , 273
- linear canonical transformations, 185
- little group, 172, 241
- local coordinates, 81
  
- Majorana's infinite component relativistic wave equations, 261
- metaplectic group  $Mp(2)$ , 189
- metric tensor, 110
- Minkowski metric, 262
- module over a ring, 25
- monomial symmetric functions, 45
- multiplication table, 4
- multiplicity, 17
- multiplicity problem, 130
- multispinors of  $SU(2)$ , 77
- Murnaghan–Nakayama rule, 43
  
- negative, 113
- neutral elements, 213, 235
- neutral elements in  $\mathcal{P}$ , 264
- neutral elements in classical mechanics, 232
- nilpotent group, 7
- nilpotent Lie algebra, 101
- nonunitary representation, 11
- nonabelian, 3
- nonfaithful representation, 10
- normal subgroup, 6
  
- one parameter subgroups, 84, 87
- orthogonal idempotents, 25
- orthogonality and completeness of the  $SO(3)$   $D$ -matrices, 65
- orthogonality and completeness of the  $SU(2)$   $D$ -matrices, 76
- orthogonality properties of Young symmetrisers, 41
- orthonormal basis for the Schwinger representation of  $SU(2)$ , 166
- outer automorphism, 8
- overcompleteness of coherent states, 175
  
- partition of  $n$ , 5
- Pauli triplets, 146
- Pauli–Lubanski four-vector, 265
- Peter–Weyl theorem, 170
- Poincaré group  $\mathcal{P}$ , 235, 262
- Poincaré–Bloch sphere, 215
- positive root, 113
- power symmetric functions, 46
- presentation of a group, 30
- primitive idempotents, 25
- principal series UIR's of  $SL(2, \mathbb{C})$ , 261
- projection map, 215
- proper orthochronous homogeneous Lorentz group, 254
- properties of  $SO(3)$  representations, 62
- properties of  $SU(2)$  representations, 74
- properties of the structure constants, 92
- pure Lorentz transformations, 254, 271
  
- quantum variance matrices, 205
- quotient group, 6
  
- rank of complex simple Lie algebras, 111
- ray representations, 213, 230
- real orthogonal representation, 11
- real symplectic group  $Sp(2n, \mathbb{R})$ , 185
- realisation of a group, 9
- reciprocity theorem, 173
- reconstruction of the Lie group from its Lie algebra, 84
- reducible representation, 14
- reducible representation of a Lie algebra, 104
- regular representation, 21
- representation  $A_l^{(2l+1)}$  of  $B_l$ , 155
- representation of a group, 10
- representations of CSLA's, 129
- representations of Lie algebras, 102
- resolution of unity, 175
- Riemann metric, 215
- Riemann space, 215
- right cosets, 6
- root space, 111
- root vectors, 111
- row subgroup, 37
  
- Schur functions, 36
- Schur's Lemma, 15
- Schwinger representations as induced representations, 173

- Schwinger representation of  $SO(3)$ , 167
- Schwinger representation of  $SU(3)$ , 167
- Schwinger representation of  $SU(2)$ , 78
- Schwinger representation of  $SU(2)$ , 161
- Schwinger representation of a group, 160
- self adjoint representation, 13
- semidirect product, 9
- semisimple group, 7
- semisimple Lie algebra, 102
- semistandard Young tableau, 35
- Shirokov–Foldy form, 272
- signature of a permutation, 29
- simple group, 7
- simple Lie algebra, 102
- simple root, 113
- simple weight, 132
- solvable group, 7
- solvable Lie algebra, 101
- spherical harmonics, 178
- spherical tensor operators, 236
- spinor generators, 151
- spinor UIR's, 145
- spinor UIR's and metaplectic UR of  $Sp(2n, \mathbb{R})$ , 208
- spinor UIR's of  $B_I$ , 153
- spinor UIR's of  $D_I$ , 152
- spinor UIR's of the real orthogonal groups, 153
- stability group, 172
- standard Young tableau, 34
- Stone–von Neumann theorem, 190
- structure constants, 91, 231
- structure constants of  $SU(2)$  and  $SO(3)$ , 75
- subgroup, 5
- superposition principle of quantum mechanics, 218
- supplementary series of UIR's of  $SL(2, \mathbb{C})$ , 261
- symmetric functions, 45
- symmetric group, 27
- symmetry condition, 220
- symmetry operation in quantum mechanics a la Wigner, 218
- symplectic metric matrix, 119, 194
- tabloid, 33
- tangent vectors to the one parameter subgroup, 88
- timelike UIR's of  $\mathcal{P}$ , 271
- topology of  $\mathcal{P}$ , 262
- topology of  $SO(2)$  manifold, 54
- topology of  $SO(3)$  manifold, 57
- topology of  $SU(2)$  manifold, 69
- transpositions, 29
- UIR's of  $\overline{\mathcal{P}}$ , 268
- UIR's of  $E(3)$ , 240
- UIR's of  $O(2)$ , 56
- UIR's of  $O(3)$ , 68
- UIR's of  $SO(2)$ , 54
- UIR's of  $SO(3)$ , 64
- UIR's of  $Sp(2, \mathbb{R})$ , 189
- UIR's of  $SU(2)$ , 75
- unit rays, 215
- unital ring, 24
- unitary irreducible representation, 14
- unitary representation, 11
- useful subgroups of  $Sp(2n, \mathbb{R})$ , 196
- weight vector, 130
- weights in the adjoint representation, 130
- Weyl basis, 148
- Weyl group, 132
- Weyl unitary trick, 189
- Weyl, Jordan and Schwinger constructions, 77
- Wigner boost, 271
- Wigner rotation, 243, 269
- Wigner symmetry, 218
- Wigner's theorem, 213
- Williamson normal form, 208
- Williamson theorem, 207
- winding number, 54
- Young frames, 32
- Young subgroups of  $S_n$ , 36
- Young symmetrisers, 37
- Young tableaux, 33