

Index

- [·], *see* equivalence class
- $\mathfrak{F}(\mathcal{M})$ -linear, 46, 93
- Π , *see* second fundamental form
- \mathcal{W} , *see* Weingarten map
- ∇ , *see* connection
- \odot , *see* Hadamard product
- \mathbb{R} -linear, 93
- \sim , *see* equivalence relation
- \succ , *see* positive definite
- \succeq , *see* positive semidefinite

- A, *see* assumption
- acceleration, 101, 199
- accumulation point, 52
- action (of a vector field), 85
- adaptive regularization with cubics, 148
- adjoint (of a linear map), 22
- affine connection, *see* connection
- affine invariant metric, 318
- algorithms
 - backtracking, *see* line search
 - conjugate gradients, 120
 - Riemannian gradient descent, 54
 - Riemannian Newton's method, 118
 - Riemannian trust regions, 127
 - truncated conjugate gradients, 140
- ambient space, 200
- ARC, *see* adaptive regularization with cubics
- Armijo, *see* line search
- assumption
 - A4.1 (lower-bounded f), 55
 - A4.2 (sufficient decrease), 55
 - A4.3 (Lipschitz-type first order), 57
 - A6.1 (H_k first order), 128
 - A6.2 (H_k second order), 129
 - A6.3 (Cauchy step), 130
 - A6.4 (eigenstep), 130
 - A6.5 (lower-bounded f), 131
 - A6.6 (Lipschitz-type first order), 131
 - A6.7 (Lipschitz-type second order), 131
 - A6.8 (Lipschitz-type gradient norm), 137
 - A6.9 (retraction distortion), 142
- atlas, 178
 - maximal, 178
 - topology, 183
- automatic differentiation, 75

- backtracking, *see* line search
- base (of a tangent vector), 36
- basis (for a topology), 183
- basis (of a linear space), 20
- Beltrami–Klein model, 169
- bounded, metric space, 254
- Brockett cost function, 11
- bump function, 94
- Bures–Wasserstein metric, 242

- canonical inner product, *see* Frobenius inner product
- canonical projection, 209
- Cartan–Hadamard manifold, 298
- Cauchy point, 130
- CG, *see* conjugate gradients
- chart, 176
 - compatible, 178
- cheap gradient principle, 75
- C^k , 179
- Clairaut–Schwarz theorem, 23
- closed manifold, 204
- closed set, *see* topology
- commutator, 85
- compact manifold, 185
- compact set, 185
- compatible charts, 178
- complete, metric space, *see* metrically complete
 - complete
- complete, Riemannian manifold, *see* geodesically complete
 - complete
- complex circle, phases, 153
- complex differentiable, 70
- condition number, 125, 312
- conjugate gradients, 120
- connected components, 252
- connected components ($O(n)$), 158
- connected, topological space, 252

- connection, 82, 93, 197
 - Euclidean, 85
 - Riemannian, 85
 - symmetric, 85
 - torsion free, 85
- constrained optimization, 2, 174, 175
- continuous map, 182
- contraction mapping theorem, 64
- contravariant, 292
- convergent sequence, 52
- convex function, 299
 - geodesically, *see* geodesically convex function
 - strictly, 299
 - strongly, 299
- convex set, 298
 - geodesically, *see* geodesically convex set
- coordinate frame, 45
- coordinate representative, 177, 179
- coordinate vector field, 191, 193
- coordinates, 176
- cost function, 51
- cotangent vector field, 48
- covariant, 292
- covariant derivative, 82, 93, 197
- covariant derivative, induced, *see* induced covariant derivative
- covariant derivative, total, 288
- covector, 292
- critical point
 - first order, 53
 - second order, 116
 - second order, strict, 116
- curve segment, 253
 - length, 253
 - minimizing, 254
 - piecewise, 253
 - regular, 253
 - smooth, 253
- defining function, *see* local defining function
- derivation, 85, 90, 113, 187
- descend (quotient), 216
- diffeomorphism, 26
- differentiable structure, *see* smooth structure
- differential
 - complex, 70
 - embedded submanifold, 33
 - linear space, 23
 - matrix factorizations, 72
 - matrix function, 71
- dimension
 - linear space, 20
 - manifold, 29, 179
- disjoint union, 36
- distance, 253
- distance, Riemannian, 253
- Ehresmann completeness, 250
- eigenstep, 130
- embedded submanifold, *see* submanifold, embedded
- embedding space, 200
- entrywise product, *see* Hadamard product
- equivalence class, 10
- equivalence relation, 10
- essential manifold (camera pose), 175
- Euclidean connection, 85
- Euclidean gradient, 23
- Euclidean Hessian, 23
- Euclidean inner product, 20
- Euclidean metric, 40, *see also* Euclidean inner product
- Euclidean norm, 20
- Euclidean space, 20
- exponential map, 256
 - on a sphere, 39
- extension (smooth map), 32
- extension lemma, 95
- extrinsic acceleration, 101, 110
- extrinsic curvature, 110
- fiber, 210
- field
 - scalar, 32
 - vector, 37, 191
- field, tensor, 286
- finite differences, 100, 283
- finite subcover, 185
- first-order critical point, *see* critical point
- first-order stationary point, *see* critical point
- Fisher–Rao metric, 44
- fixed point, 64
- fixed-rank matrices, 160
- foot, *see* base (of a tangent vector)
- Fréchet derivative, 71
- Frobenius inner product, 21
- Frobenius norm, 21
- function, *see* map
- general linear group, 213
- geodesically complete, 254, 255
- geodesically concave function, 302
- geodesically convex function, 302
 - strictly, 302
 - strongly, 302
- geodesically convex program, 304
- geodesically convex set, 301
 - strongly, 306
 - totally, 306
- geodesically linear function, 302
- geodesically quasiconvex function, 303
- geodesics, 102, 199
 - horizontal, 240
 - maximal, 256
 - minimizing, 254
 - on a sphere, 39

- geometric program, 316
- global convergence, 62
- global frame, 46
- gradient
 - Euclidean, 23
 - Riemannian, 41, 194
- Grassmann manifold
 - embedded geometry, 245
- group, 212
 - rigid motion, 175
 - special Euclidean, 175
 - translations, 213
- group action, 213
 - free, 214
 - isometric, 226
 - proper, 214
 - smooth, 213
- group, Lie, 213
- H*-conjugate, 121
- Hadamard manifold, *see* Cartan–Hadamard manifold
- Hadamard product, 21
- hairy ball theorem, 46
- Hausdorff, *see* topology
- Heine–Borel property, 254
- Hessian
 - Euclidean, 23
 - Riemannian, 90, 198
 - signature, 106
 - symmetric, *see* Clairaut–Schwarz theorem
- homeomorphism, 183
- Hopf–Rinow theorem, 255
- horizontal curve, 239
- horizontal distribution, 247
- horizontal geodesic, 240
- horizontal lift, *see* lift, horizontal
- horizontal space, 217
- horizontal vector field, 219
- hyperbolic space, 169
- hyperboloid model, 169
- IFT, *see* inverse function theorem
- image (of a linear map), 20
- inclusion map, 200
- induced covariant derivative, 97, 198, 289
- induced metric, 40
- injectivity radius, 257
- inner product, 20
- inner product (pseudo), 168
- interior, relative, 305
- interior, set, 304
- intrinsic, 3
- intrinsic acceleration, *see* acceleration
- invariance, 206
- inverse function theorem, 26
- Jacobian, 119
- kernel, 20
- KKT conditions, 174
- Koszul formula, 89
- Krylov subspace, 123
- Kurdyka–Łojasiewicz inequality, 78
- Lagrange multipliers, 174
- length, curve segment, 253
- Levi-Civita, *see* Riemannian connection
- LICQ, 174
- Lie bracket, 85
- Lie group, 213
- lift (of a map), 216
- lift, horizontal
 - of a vector, 218
 - of a vector field, 219
 - of a vector field on a curve, 239
- limit point, 52
- line search, 54, 59
- linear manifold, 24
- linear map, 20
- linear operator, *see* linear map
- linear space, 19
- Lipschitz continuous
 - Euclidean gradient, 58
 - on a metric space, 266
 - Riemannian gradient, 268
 - Riemannian Hessian, 271
 - tensor field, 290
 - tensor field of order two, 271
 - vector field, 268
- Lipschitz-type assumption, 59
- local convergence, 62
 - at least linear, 63
 - at least quadratic, 63
 - R-linear, Q-linear, 63
 - superlinear, 63
- local defining function, 24, 201
- local frame, 45
 - orthonormal, 48
- local section, 216
- log-barrier function, 314
- Log-Euclidean metric, 318
- logarithmic map, 258
- manifold, 184
- manifold*, 179
- Manopt
 - checkgradient, 75–77
 - checkhessian, 145
 - dexpm, dlogm, dsqrtm, 71
 - egrad2rgrad, 42, 153
 - ehess2rhess, 106–108, 153
 - tangent2ambient, 167
 - manifolds (factories), 150
- map
 - continuous, *see* continuous map
 - diffeomorphic, *see* diffeomorphism

- homeomorphic, *see* homeomorphism
- linear, *see* linear map
- open, *see* open map
- proper, *see* proper map
- quotient, *see* quotient map
- smooth, *see* smooth map
- matrices of fixed rank, 160
- matrix function, 70
- matrix-free solver, 120
- maximal atlas, 178
- maximal geodesic, 256
- metric, 194
 - affine invariant, 318
 - Bures–Wasserstein, 242
 - Euclidean, *see* Euclidean metric
 - Fisher–Rao, 44
 - induced, *see* induced metric
 - Log-Euclidean, 318
 - product, *see* product metric
 - Riemannian, *see* Riemannian metric
 - Sasaki, 294
- metric (on a manifold), 39
- metric projection, 38, 110
- metric space, 253
- metric topology, 253
- metrically complete, 254
- minimizer
 - global, 51
 - local, 51
 - local, strict, 51
- minimizing curve segment, 254
- minimizing geodesic, 254
- minimum, *see* minimizer
- Minkowski pseudo-inner product, 168
- Möbius band, 50
- musical isomorphism, 45, 47

- natural projection, *see* canonical projection
- neighborhood, 20, 31
- Newton equations, 118
- Newton step, 118
- Newton’s method, 118
- norm
 - Euclidean, *see* Euclidean norm
 - operator, *see* operator norm
- normal space, 107, 156
- null space, *see* kernel

- objective function, 51
- oblique manifold, 8, 153
- one-form, 46
- open cover, 185
- open map, 211
- open set, *see* topology
- open submanifold, 24, 31
- operator, *see* linear map
- operator norm, 22

- optimality conditions
 - first-order, necessary, 53
 - second-order, necessary, 116
 - second-order, sufficient, 117
- optimization algorithms, *see* algorithms
- optimizer, *see* minimizer
- orbit, 213
- orbit space, 214
- orientable manifold, 50
- orthogonal group, 158, 213
- orthogonal projector
 - to a manifold, 110
 - to a tangent space, 42
- orthogonal vectors, 20
- orthonormal basis, 20
- orthonormal local frame, 48
- orthonormal matrices, 154
- orthonormal vectors, 20

- parallel frame, 264
- parallel tensor field, 288
- parallel translation, *see* parallel transport
- parallel transport, 263
- parallel vector field on a curve, 263
- parallelizable manifold, 46
- parameterization (local), 176
- partition of unity, 50, 185
- PCA, *see* principal component analysis
- phases, 153
- Poincaré ball model, 169
- Poincaré half-space model, 169
- polar retraction, 156
- Polyak–Łojasiewicz inequality, 78, 311
- positive definite, 22
- positive semidefinite, 22
- posynomial, 316
- principal component analysis, 6, 8
- product connection, 84
- product manifold
 - connection, 84
 - connection, Riemannian, 90
 - differentials, 35
 - distance, 255
 - embedded submanifolds, 30
 - exponential map, 262
 - general manifolds, 185
 - geodesically convex sets, 305
 - geodesics, 102
 - gradient, 44
 - Hessian, 92
 - induced covariant derivative, 100
 - metric, 40
 - parallel transport, 265
 - product of spheres, *see* oblique manifold
 - retraction, 39
 - table, summary, 151
 - tangent bundle, 37

- product metric, 40
- projector, *see* orthogonal projector
- proper embedding, 50, 201
- proper map, 214
- pullback, 53

- Q-factor, 155
- quotient manifold, 10, 209
- quotient map, 209
- quotient set, 10, 209
- quotient space, 209
- quotient topology, 209

- range, *see* image (of a linear map)
- Rayleigh quotient, 5, 9, 34
 - global minimizers, 92, 238
 - gradient, 43
 - Hessian, 91, 236
- real projective space, 180
- regular submanifold, *see* submanifold, embedded
- restriction, 22, 32
- retraction, 38, 192
 - differential, 279, 282
 - metric projection, 38, 111
 - polar factor, 156
 - projective, 114
 - Q-factor, 155
 - second order, 104, 111, 199
 - third order, 292, 293
 - topological, 202
 - truncated SVD, 163
- RGD, *see* Riemannian Gradient Descent
- Ricci identity, 292
- Riemannian
 - connection, 85, 197
 - distance, 253
 - gradient, 41, 194
 - gradient descent, 54
 - Hessian, 90, 198
 - isometry, 317
 - manifold, 40, 194
 - metric, 40, 194
 - Newton's method, 118
 - product manifold, 40
 - quotient manifold, 225
 - submanifold, 40
 - submersion, 225
 - trust region method, 127
- rigid motion group, 175
- root, *see* base (of a tangent vector)
- rotation group, *see* special orthogonal group
- RTR, *see* Riemannian trust regions

- Sasaki metric, 294
- scalar field, 32
- SDP, *see* semidefinite program
- second fundamental form, 108
- second-countable, *see* topology
- second-order critical point, *see* critical point
- second-order stationary point, *see* critical point
- section, local, 216
- section, of a vector bundle, 277
- section, zero, *see* zero section
- sectional curvature, 169
- self-adjoint linear map, 22
- semidefinite program, 14
- shape space, 175
- simplex, 13, 31, 44
- singular value, 22
- smooth at a point (map), 22, 32, 179
- smooth manifold, *see* manifold
- smooth map
 - k -times differentiable, 179
 - embedded submanifold, 32
 - extension, 32
 - linear space, 22
 - manifold, 179
- smooth structure, 178
- span, 20
- special Euclidean group, 175, 213
- special orthogonal group, 159, 213
- speed, 103, 253
- sphere
 - product of spheres, *see* oblique manifold
- star shaped, 112, 256
- stationary point, *see* critical point
- Stiefel manifold, 154
- sublevel set, 59
- submanifold
 - embedded in a linear space, 24
 - embedded in a manifold, 35, 200, 201
 - immersed, 50, 200
 - linear, 24
 - open, 24, 31, 184, 201
 - properly embedded, 50
 - regular, *see* embedded Riemannian, 40, 202
- submersion, 211
- submersion, Riemannian, 225
- subsequence, 52
- subspace, *see* linear space
- subspace topology, 31
- successful step, 134
- sufficient decrease, 55
- symmetric linear map, *see* self-adjoint linear map

- tangent bundle, 36, 191
- tangent covector, 292
- tangent space, 29, 188
- tangent vector, 29, 188
- tangent–cotangent isomorphism, *see* musical isomorphism
- tCG, *see* truncated conjugate gradients

- tensor
 - fixed rank, 174
- tensor bundle, 287
- tensor field, 286, 287
- tensor field on a curve, 289
- topological manifold, 204
- topological space, 182
- topology, 20
 - atlas, 183
 - Hausdorff, 52, 183
 - manifold, 182
 - metric, 253
 - quotient, 209
 - second countable, 183
 - submanifold, 31
 - subspace, 31, 183
- total space, 209
- trace (of a matrix), 21
- trace inner product, *see* Frobenius inner product
- transporter, 277
- truncated conjugate gradients, 140
- trust region, 127
- trust-region method, 127
- trust-region subproblem, 127
- tubular neighborhood, 50, 114, 202
- unsuccessful step, 134
- vector bundle, 277
- vector field, 37, 191
 - horizontal, 219
- vector field on a curve, 97, 198
- vector space, *see* linear space
- vector transport, 282
- velocity, 101, 193
- vertical space, 217
- Weingarten map, 108
- zero section, 257