

Contents

Preface	xi
Acknowledgments	xiv
1 Introduction and Preliminaries	1
1.1 First-Order Methods in the Modern Era	1
1.2 Limitations of Monotone Operator Theory	2
1.3 Preliminaries	2
Bibliographical Notes	17
Exercises	18
PART ONE: MONOTONE OPERATOR METHODS	
2 Monotone Operators and Base Splitting Schemes	23
2.1 Set-Valued Operators	23
2.2 Monotone Operators	25
2.3 Nonexpansive and Averaged Operators	30
2.4 Fixed-Point Iteration	31
2.5 Resolvents	37
2.6 Proximal Point Method	42
2.7 Operator Splitting	43
2.8 Variable Metric Methods	50
2.9 Commonly Used Formulas	52
Bibliographical Notes	53
Exercises	56
3 Primal-Dual Splitting Methods	66
3.1 Infimal Postcomposition Technique	66
3.2 Dualization Technique	69
3.3 Variable Metric Technique	71
3.4 Gaussian Elimination Technique	74
3.5 Linearization Technique	78
3.6 Discussion	84
Bibliographical Notes	84
Exercises	86

4 Parallel Computing	94
4.1 Computational Complexity via Flop Count	94
4.2 Parallel Computing	96
Bibliographical Notes	103
Exercises	103
5 Randomized Coordinate Update Methods	105
5.1 Randomized Coordinate Fixed-Point Iteration	105
5.2 Coordinate and Extended Coordinate-Friendly Operators	109
5.3 Methods	112
5.4 Discussion	116
Bibliographical Notes	117
Exercises	118
6 Asynchronous Coordinate Update Methods	120
6.1 Asynchronous Fixed-Point Iteration	122
6.2 Extended Coordinate-Friendly Operators and Exclusive Memory Access	130
6.3 Server-Worker Framework	132
6.4 Methods	134
6.5 Exclusive Memory Access	137
Bibliographical Notes	141
Exercises	143
PART TWO: ADDITIONAL TOPICS	
7 Stochastic Optimization	147
7.1 Stochastic Forward-Backward Method	148
7.2 Methods	155
Bibliographical Notes	156
Exercises	157
8 ADMM-Type Methods	160
8.1 Function-Linearized Proximal ADMM	160
8.2 Derived ADMM-Type Methods	167
8.3 Bregman Methods	177
8.4 Conclusion	179
Bibliographical Notes	179
Exercises	184
9 Duality in Splitting Methods	190
9.1 Fenchel Duality	190
9.2 Attouch–Théra Duality	191
9.3 Duality in Splitting Methods	192
Bibliographical Notes	194
Exercises	195
10 Maximality and Monotone Operator Theory	197
10.1 Maximality of Subdifferential	197
10.2 Fitzpatrick Function	198
10.3 Maximality and Extension Theorems	201
Bibliographical Notes	203
Exercises	204

Contents

ix

11 Distributed and Decentralized Optimization	207
11.1 Distributed Optimization with Centralized Consensus	207
11.2 Decentralized Optimization with Graph Consensus	214
11.3 Decentralized Optimization with Mixing Matrices	217
Bibliographical Notes	223
Exercises	225
12 Acceleration	233
12.1 Accelerated Gradient Method	233
12.2 Accelerated Proximal Point and Optimized Halpern Method	236
12.3 When Does an Acceleration Accelerate?	237
Bibliographical Notes	238
Exercises	239
13 Scaled Relative Graphs	242
13.1 Basic Definitions	242
13.2 Scaled Relative Graphs	245
13.3 Operator and SRG Transformations	252
13.4 Averagedness Coefficients	264
Bibliographical Notes	267
Exercises	268
Appendix A Miscellaneous	271
References	273
Index	299