

## Index

- AAMR (averaged alternating modified reflections), 62
- AC-FPI (asynchronous coordinate-update FPI), 122  
     mathematical definition, 123  
     operational definition, 122
- acceleration, 233
- adapt-then-combine, 221, 224
- ADMM  
     2-1-2, 173, 182  
     2-1-2 with FLiP, 188  
     2-1-2-4-3-4 update, 188  
     3-block, 182  
     block splitting, 171  
     decentralized, 223  
     dual extrapolation parameter, 161  
     dummy variable, 186  
     four-block ADMM, 188  
     function-linearization, 162  
     golden ratio, 162, 181  
     graph coloring, 187  
     Jacobi doubly-linearized, 185  
     Jacobi+1, 185  
     Peaceman–Rachford, 185  
     penalty parameter, 162  
     proximal term, 162  
     scaled form, 163  
     solvability of subproblems, 69, 85, 163
- ADMM (alternating direction method of multipliers), 67, 69
- ADMM-type, 160
- after-read labeling, 143
- AGM (Nesterov’s accelerated gradient method), 233, 238
- algorithm, 95
- ALM, 42
- AMA (alternating minimization algorithm), 70
- Amdahl’s law, 102
- $A$ -norm, 4
- APPM (accelerated proximal point method), 236, 238
- argmin, 7
- ARock assumptions, 124
- asymptotic equivalence, 17
- asymptotic notation, 17
- asynchronous ADMM, 136
- asynchronous coordinate gradient descent, 134
- asynchronous parallel, 121
- atomic operation, 138
- Attouch–Théra duality, 191
- augmented Lagrangian, 14, 54
- augmented Lagrangian method, 42, 180
- Aumann integral, 157
- averaged iteration, 32
- backward-backward method, 61
- Baillon–Haddad theorem, 18, 27
- ball, 5
- Banach fixed-point theorem, 32, 59
- base splitting scheme, 43
- BCV (Bertsekas, O’Connor, and Vandenberghe) technique, 79, 85
- BFS (backward-forward splitting), 44
- biconjugate function, 10
- big- $\mathcal{O}$  notation, 16
- block, 105
- block delay, 123
- Bregman method, 55, 178, 182
- C-FPI (coordinate-update fixed-point iteration), 106
- Cayley operator, 37
- certificates, 192
- Chambolle–Pock, 71, 84

- Chen–Teboulle, 91  
 closed graph theorem, 204  
 cocoercivity, 27  
 combine-then-adapt, 221, 224  
 communication congestion, 121  
 compare-and-swap, 138  
 composition of averaged operators, 265  
 computational tomography, 74  
 Condat–Vũ, 72, 85, 87, 103, 168  
 conic program, 61, 115  
 conjugate function, 10, 25, 39  
   subdifferential, 39  
 consensus, 217  
 consensus set, 226  
 consensus technique, 50, 55, 62, 64, 101  
 consensus tracking, 231  
 constraint qualification, 14  
 contraction mapping algorithm, 32  
 convex-concave, 12, 53, 56, 205  
 coordinate, 105  
 coordinate minimization, 119  
 coordinate proximal-gradient  
   descent/method, 113  
 coordinate selection rule, 106, 118  
 coordinate-friendly method, 109  
 coordinate-friendly operator, 109  
 correspondence, 23
- decentralized ADMM, 216  
 decentralized averaging, 218  
 decentralized FLiP-ADMM, 217  
 decentralized gradient descent, 224  
 decentralized methods, 207  
 delays, 125  
 demipositive, 149, 157, 159  
 DGD (decentralized gradient descent), 221  
 diffusion, 221  
 DIGing, 231  
 directed graph, 224  
 distance to a set, 6  
 distributed ADMM, 209  
 distributed DRS, 210  
 distributed methods, 207  
 distributed proximal gradient method, 208  
 doubly-linearized ADMM, 169  
 doubly-linearized method of multipliers, 88  
 DRS (Douglas–Rachford splitting), 44, 46, 54,  
   61, 193, 258  
 dual ascent, 37, 54, 59, 60  
 dual decomposition, 211, 223, 225, 227  
 dual optimal value, 12  
 dual problem, 12  
 dual proximal gradient, 71  
 duality, 12
- dualization technique, 69, 85  
 DYS (Davis–Yin splitting), 45, 47, 54, 104, 194,  
   265
- edge, 214  
 eigenvalue, 3  
 embarrassingly parallel, 97  
 envelope theorem, 225  
 epigraph, 6  
 epoch, 116  
 Euclidean space, 3  
 exact decentralized methods, 221  
 exclusive access, 122, 125  
 exclusive memory access, 138  
 extended complex plane, 244  
 extended coordinate-friendly operator, 110,  
   130  
 extended real line, 6  
 extended solution set, 196  
 extension theorems, 201  
 EXTRA, 224
- FBS (forward-backward splitting), 43, 192  
 Fejér monotonicity, 34  
 Fenchel conjugate, 10  
 Fenchel duality, 190  
 Fenchel’s identity, 25, 53  
 Fenchel–Moreau theorem, 10, 17  
 Fenchel–Rockafellar dual, 13  
 Fermat’s rule, 8  
 Finito, 114, 119  
 Fitzpatrick function, 198, 203  
 fixed point, 31  
 fixed-point encoding, 48  
 fixed-point residual, 35  
 FLiP-ADMM, 227  
 flop (floating-point operation), 94  
 flop-count operator, 95  
 forward step method, 36, 54, 254, 269  
 forward-Douglas–Rachford, 50, 54  
 FPI (fixed-point iteration), 32, 54  
 function  
   CCP (closed, convex, and proper), 6  
   closed, 6  
   concave, 6  
   convex, 6  
   differentiable, 7, 17  
   domain, 6  
   extended-valued, 6  
   lower semi-continuous, 6  
   proper, 6  
   strictly concave, 6  
   strictly convex, 6  
 function-linearized proximal ADMM, 160

## INDEX

301

- Gaussian elimination technique, 40, 74  
 generalized circles, 244  
 generalized forward-backward, 50  
 gradient descent, 36, 54, 58, 59, 61, 241, 253, 269  
 graph, 214  
   bipartite, 228
- Hahn–Banach sandwich theorem, 205  
 Hogwild, 142
- image of a convex function, 67  
 implementation on a method, 95  
 inconsistent read, 131  
 inconsistent write, 131  
 independence assumption on indices and delays, 125  
 indicator function, 7  
 inexact decentralized methods, 220  
 infimal postcomposition, 66, 86  
 infimal postcomposition technique, 66, 85  
 $+\infty - \infty$ , 29  
 inner product, 3  
 inverse resolvent identity, 41  
 inversion map, 244  
 inversive geometry, 267  
 isotonic regression, 77  
 Iterative Shrinkage-Thresholding Algorithm (ISTA), 49  
 iterative soft thresholding, 56
- Jacobi ADMM, 171
- Kirszbraun–Valentine theorem, 203  
 KKT operator, 29, 40  
 Krasnosel’skiĭ–Mann iteration, 32
- Lagrange multiplier, 13  
 LASSO (least absolute shrinkage and selection operator), 49, 56, 90  
 Legendre–Fenchel transform, 10  
 linearization technique, 78  
 linearized ADMM, 83, 167  
 linearized method of multipliers, 78, 89, 90  
 linesearch, 239  
 Lipschitz continuity, 4  
 little- $o$  notation, 16  
 local averaging matrix, 218  
 logistic regression, 119  
 Lyapunov analysis, 34, 60, 235, 238
- Markov chain, 223  
 matrix  
   positive definite, 3  
   positive semidefinite, 3  
   square, 3  
   square root, 3  
   symmetric, 3  
 matrix inversion lemma, 103  
 maximal cocoercivity, 27  
 maximal monotone extension theorem, 202  
 maximal monotonicity, 26  
 maximal strongly monotonicity, 27  
 maximality, 48, 53  
 method, 95  
 method of multipliers, 42, 54, 59, 204  
 Minkowski sum, 3  
 Minty surjectivity theorem, 38, 53, 199, 203  
 MISO (minimization by incremental surrogate optimization), 114, 119  
 mixing matrix, 218, 223, 230  
   decentralized, 218  
 monotone inclusion problem, 30  
 monotonicity, 25  
 Moreau envelope, 57, 62  
 Moreau identity, 41  
 Moreau–Yosida approximation, 58  
 multi-valued function, 23  
 mutex (mutual exclusion lock), 139
- near-circulant splitting, 181  
 NIDS (Network InDependent Stepsize), 222  
 node, 214  
 norm, 4  
 normal cone operator, 9  
 null space, 3
- OHM (optimized Halpern method), 236, 238  
 operator  
   affine, 28  
   averaged, 31  
   cocoercive, 57  
   composition, 24  
   concatenation, 28  
   continuous, 29, 56  
   contractive, 30  
   differentiable, 29, 57  
   domain, 23  
   extension, 197  
   firmly nonexpansive, 31  
   graph, 23  
   identity, 24  
   image of a set, 23  
   inverse, 24, 56  
   Lipschitz, 24, 57  
   nonexpansive, 30

- range, 23
- set-valued, 23
- single-valued, 23
- sum, 24
- zero, 24
- operator classes, 242
- optimization problem
  - constrained, 8
  - constraint, 8
  - objective function, 8
  - optimization variable, 8
  - solution, 8
  - unconstrained, 8
- PAPC (proximal alternating predictor corrector), 76, 85, 89
- parallel computing, 96
- parallel flop-count operator, 97
- parallel matrix-vector multiplication, 99
- parallelizable method, 98
- parallelizable operator, 98
- parameter server, 132, 143
- paramonotone, 157, 195
- partial inverse, 204
- partial linearization, 169
- partial maximization, 225
- partial minimization, 225
- PD3O (primal-dual three-operator splitting), 81, 85, 88, 89
- PDFP<sup>2</sup>O (primal-dual fixed point algorithm based on proximity operator), 76, 85, 89
- PDHG (primal-dual hybrid gradient), 71, 79, 83, 84, 87, 90, 103, 168
- PG-EXTRA, 222, 224, 231
- point-to-set mapping, 23
- Polyak–Ruppert averaging, 149
- PPM (proximal point method), 42, 54, 58, 258, 269
- preconditioned PDHG, 88
- predictor corrector proximal multiplier method, 181
- primal decomposition, 210, 223, 225, 227
- primal optimal value, 12
- primal problem, 12
- primal-dual method, 66
- product space trick, 55
- projected gradient method, 46, 54
- projected subgradient method, 155
- projection, 16, 39
- projection onto convex sets, 58
- proximable, 16
- proximal ADMM, 81, 86
- proximal augmented Lagrangian method, 43
- proximal gradient method, 46
- proximal method of multipliers, 43
- proximal method of multipliers with function linearization, 74
- proximal minimization, 42
- proximal operator, 15
- proximal subgradient method, 155, 157
- proximal term, 51, 78
- proximal-gradient method, 54
- PRS (Peaceman–Rachford splitting), 44, 54, 61
- pseudononexpansive operators, 269
- race condition, 130
- Radon transform, 74
- randomized coordinate gradient descent/method, 112
- randomized primal-dual block coordinate update method, 180
- range, 3
- RC-FPI (randomized coordinate-update fixed-point iteration), 106
- readers-writers lock, 140
- reduction, 98, 208, 216
- reflected resolvent, 37
- regularity condition, 10
- representation function of an operator class, 251
- representative function, 198
- resolvent, 37, 48
- robust stochastic approximation, 159
- saddle point, 12
- saddle subdifferential, 29, 56
- Schur complement, 4
- SDCA (stochastic dual coordinate ascent), 114
- second-order cone, 118
- self-dual property of DRS, 193
- seminorm, 4
- separable constraint, 110
- separable operator, 110
- serial method, 98
- server-worker framework, 132
- set
  - affine, 5
  - affine hull, 5
  - boundary, 5
  - closure, 5
  - convex, 3
  - interior, 5
  - relative boundary, 5
  - relative interior, 5
- set-valued mapping, 23

## INDEX

303

- SFB (stochastic forward-backward method), 148
- SGD (stochastic gradient descent), 147
- shared memory system, 132
- Sherman–Woodbury–Morrison formula, 103
- singular value, 4
- Slater’s constraint qualification, 14
- smoothness, 11
- soft-thresholding operator, 16
- spherical triangle inequality, 243
- split Bregman, 179
- SRG (scaled relative graph), 245
  - of an operator, 245
  - composition, 263
  - intersection, 252
  - inversion, 257
  - of a matrix, 247
  - of an operator class, 248
  - product, 263
  - scaling, 253
  - sum, 261
  - translation, 253
- stack operator, 217
- staleness, 121
- Stewart’s theorem, 243, 267
- stochastic approximation, 157
- stochastic Condat–Vũ, 156
- stochastic coordinate gradient descent/method, 112
- stochastic matrix, 220
- stochastic projected subgradient method, 147
- stochastic proximal simultaneous gradient method, 155
- stochastic proximal subgradient method, 147, 155
- stochastic subgradient method, 147, 158
- strictly convex function, 56
- strong convexity, 11
- strong duality, 13
- strong monotonicity, 26
- strongly-convex accelerated gradient method, 240
- subdifferentiability, 9
- subdifferential, 8, 25, 39
  - maximality, 26
- subdifferential of conjugate, 25
- subgradient, 8
- subgradient inequality, 8
- subgradient method, 155
- subspace, 4
- summability argument, 34
- supermartingale convergence theorem, 108
- support-vector machine (SVM), 102
- synchronization barrier, 120
- synchronous parallel, 120
- termination criterion, 35
- total duality, 13, 47
- trip-ADMM, 175
- unbounded delay, 142
- Uzawa method, 37
- variable metric FBS, 52
- variable metric methods, 51
- variable metric PPM, 51, 55
- variable metric technique, 71
- variational inequality, 167
- vector delay, 123
- weak duality, 13