

# Index

- Abel's test, 302, 378
- Abel's theorem, 313
- absolute extrema, 154
- absolute value, 8
- alternating harmonic series, 293, 296, 313
- alternating series test, 294, 313
- anti-derivative, 173
- arc length, 242
- Archimedean property, 16, 17, 60, 268
- asymptote
  - horizontal, 123
  - slant, 123
  - vertical, 123
- average
  - of a function, 114
  - of numbers, 113
- beta function, 306
- binary operation, 2, 22
- binomial coefficients, 14
- binomial series, 316
- binomial theorem, 14
- bisection method, 99
- bound
  - lower, 16
  - upper, 16
- bounded, 16
- bounded above, 16
- bounded below, 16
- boundedness theorem, 110, 112
- catenary, 70, 243, 247
- catenoid, 247
- Cauchy condensation test, 293
- Cauchy principal value, 202
- Cauchy product of power series, 319
- Cauchy's mean value theorem, 220, 224, 227
- Cauchy-Schwarz inequality, 302
- ceiling function, 29
- chain rule, 141, 179
- codomain, 22
- comparison test
  - for series, 288
- comparison theorem
  - for improper integrals, 122, 198
  - for integrals of bounded functions, 56
  - for integrals of step functions, 48, 50
- completeness axiom, 14, 41, 50, 62
- complex power series, 339
- composition, 25
- continuity
  - at a point, 91
  - on an interval, 94
  - one-sided, 93
  - uniform, 126
- continuous function
  - integrability, 113
- continuously differentiable, 218
- convexity, 162
- convexity test, 162
- critical point, 155, 237
- Darboux integral, 78
- Darboux sums, 77
- Darboux's theorem, 158

- De Moivre's formula, 335
- derivative, 130
  - one-sided, 134, 168
- derived power series, 310
- difference of sets, 25
- differentiable
  - at a point, 130
  - on an interval, 134
- differential equation
  - autonomous, 209
  - Bernoulli, 213
  - linear, 206
  - logistic, 204, 209, 213
  - separable, 203
- Dirichlet function, 50, 92, 322
- Dirichlet kernel, 327
- Dirichlet's test, 302, 378
- disc of convergence, 340
- discontinuity
  - essential, 94
  - jump, 93, 226
  - removable, 93, 226
- discs method for volume of a solid of revolution, 248
- divergence test, 287, 338
- division, 5
- division algorithm for polynomials, 35
- domain, 22
- eccentricity, 244
- equilibrium
  - semistable, 211
  - stable, 210
  - unstable, 210
- Euclidean
  - line, 1
- Euler's identity, 341
- Euler's number, 65
- exponential function, 65, 147
- extreme value theorem, 110, 112, 154, 219
- factor theorem, 36
- factorial, 14
- Fermat's theorem, 154, 156, 220
- field, 2
- first derivative test, 159
- first fundamental theorem, 152, 173
- first fundamental theorem of calculus, *see* first fundamental theorem
- fixed point theorem, 100
- floor function, *see* greatest integer function
- folium of Descartes, 143
- Fourier coefficients, 325, 343
- Fourier series, 325, 343, 348
- function, 21
  - bijective, 24
  - bounded, 49
  - concave, 161
  - convex, 161, 168
  - decreasing, 32
  - even, 31
  - increasing, 32
  - injective, 23
  - inverse, 24, 33
  - log convex, 304
  - monotonic, 32
  - odd, 31
  - one-one, 23
  - onto, 23
  - periodic, 33
  - piecewise continuous, 113
  - piecewise monotonic, 60
  - rational, 36, 93
  - real, 27
  - smooth, 315
  - step, 46
  - strictly concave, 161
  - strictly convex, 161, 169
  - strictly decreasing, 32
  - strictly increasing, 32
  - surjective, 23
  - weakly convex, 304
- Gabriel's horn, 249

Gamma function, 200, 304  
 Gaussian integral, 198, 201, 306  
 Gaussian quadrature, 257  
 geometric series, 284, 312, 338, 340  
 graph, 27  
 greatest, 8  
 greatest integer function, 28, 93  
 greatest lower bound, 19  
 harmonic series, 286, 293  
 Heaviside step function, 28, 61, 90, 93  
 hyperbolic functions, 69, 182, 346

image  
 of a function, 22  
 of a point, 22

implicit differentiation, 142

improper integrals  
 of the first kind, 197  
 of the second kind, 199

indeterminate forms, 122, 223, 228

inequalities between means, 170

inequality  
 Bernoulli's, 169  
 Bessel's, 330, 344, 348  
 Hölder's, 170, 171  
 Jensen's, 169, 171  
 Minkowski's, 170, 171  
 Young's, 170

infimum, 19

inflection point, 162

integers, 10

integrable function, 50

integral  
 indefinite, 95  
 lower, 77  
 of a bounded function, 50  
 of a step function, 46  
 upper, 77

integral test, 290

integration by parts, 184

interior point, 20

intermediate value property, 61  
 intermediate value theorem, 97, 99, 102, 115, 237, 254, 284  
 interpolating polynomial, 39, 222, 260  
 interval, 19  
 closed, 20  
 interval of convergence, 309  
 inverse hyperbolic functions, 149  
 irrational numbers, 10, 14, 18, 348

L'Hôpital's rule, 224, 226, 227, 230, 278

Lagrange's mean value theorem, *see* mean value theorem for derivatives

least, 8  
 least squares line, 40  
 least upper bound, 19  
 left derivative, 134  
 left-continuous, 93  
 limit  
 left-hand, 89  
 of a complex sequence, 337  
 of a function at a point, 80  
 of a sequence, 267  
 one-sided, 89  
 right-hand, 89

limit comparison test  
 for improper integrals, 201  
 for series, 289

local extrema, 154

logarithmic differentiation, 148

lower Darboux sum, 77  
 lower sum, 49  
 LUB Property, 19

Maclaurin polynomial, 233  
 Maclaurin series, 316

maximum, 8  
 absolute, 153  
 local, 154, 159, 237

mean  
 arithmetic, 170  
 geometric, 170

- harmonic, 170
- mean value inequality, 218, 233
- mean value theorem
  - for integrals, 114
  - for derivatives, 219, 221, 328
  - for weighted integrals, 115, 254
- midpoint rule, 253
- minimum, 8
  - absolute, 153
  - local, 154, 159, 237
- monotone convergence theorem, 198
  - for functions, 117
  - for sequences, 273
- monotone function, 32, 95
  - integrability, 59
  - intermediate value property, 61
- monotonicity theorem, 156, 160, 218
- multiplicity of a root of a polynomial, 36
- natural logarithm, 64
- natural numbers, 9
- nested interval property, 41, 127
- Newton–Raphson method, 281
- one-one correspondence, 24
- order axioms, 6
- ordinary differential equation, 202
- orthogonal trajectory, 213
- $\pi$ 
  - as area, 74
  - as ratio of lengths, 244
- $p$ -series test, 291
- partial fractions, 192
- partial sums, 285
- partition, 45
  - adapted to a step function, 46, 48
- pointwise convergence
  - of a sequence of functions, 322
- power
  - rational, 67
  - real, 67
- power series, 308
- powers, 10
- pre-image
  - of a point, 22
- principle of mathematical induction, 10, 13
- principle of strong mathematical induction, 11, 13
- racetrack inequality, 218
- radius of convergence, 309, 340
- range
  - of a function, 22
- ratio test
  - for sequences, 276
  - for series, 299, 310, 340
- rational numbers, 10
- rectifiable, 242
- reduction formulas, 186
- refinement of a partition, 47
- remainder theorem, 237, 239, 253, 308
- Riemann condition, 51, 113, 240
- Riemann integral, 262
- Riemann lemma, 330
- Riemann rearrangement theorem, 297
- Riemann sum, 240, 244, 253
- right derivative, 134
- right-continuous, 93
- Rolle's theorem, 220, 222, 224, 239, 257
- root test
  - for sequences, 276
  - for series, 299, 310, 340
- roots of unity, 336
- saddle point, 159, 237
- sandwich theorem, 87, 88, 90, 118, 224, 230, 269, 337
- sawtooth function, 328
- second derivative test, 162
- second fundamental theorem, 174, 251
- second mean value theorem for integration, 189
- sequence, 266

- bounded, 271
- bounded above, 271
- bounded below, 271
- convergent, 267
- decreasing, 272
- divergent, 267
- increasing, 272
- monotone, 272
- unbounded, 271
- series, 284
  - absolutely convergent, 295, 338
  - conditionally convergent, 296
  - convergent, 285, 338
  - divergent, 285, 338
  - telescoping, 286
- set
  - countable, 41
  - uncountable, 41
- shell method for volume of a solid of revolution, 250
- sign function, *see* signum function
- signed area, 44, 47, 49
- signum function, 28, 83
- Simpson's rule, 255
- small span theorem, 111, 113, 127, 241
- solid of revolution, 247
- span, 111
- spheroid, 373
- square wave function, 325
- squeeze theorem, *see* Sandwich theorem
- Stirling's approximation, 279
- Stirling's formula, 280, 303
- subsequence, 274
- substitution method, 179, 203
- subtraction, 5
- sup norm, 346
- support line, 169
- supremum, 19
- surface area, 246
- surface of revolution, 245
- tag of a partition, 240
- tangent line, 130
- Taylor polynomial, 233, 307
- Taylor series, 316
- Taylor's theorem, 235
- term-by-term differentiation, 310
- term-by-term integration, 311
- torus, 249
- transitivity, 7
- triangle inequality, 8, 9
- trichotomy, 7, 62
- trigonometric polynomial, 323
- trigonometric series, 324, 343
- unbounded, 16
- uniform convergence
  - of a sequence of functions, 346
  - of a series of functions, 347
- unit step function, *see* Heaviside step function
- upper Darboux sum, 77
- upper sum, 49
- variation of parameters, 207
- Weierstrass M-Test, 347
- weighted average, 115
- well ordering principle, 13
- whole numbers, 9